Physics Part IA
Cambridge University, 2012-2013

Notes for Experimental Physics

Dimensional analysis:

The dimension of a quantity specifies the nature of that quantity. It can be presented in two forms – specific and abstract:

- Specific dimensions: Commonly referred to as *units.* The standard specific dimensions of mass, length and time are kg, m and s, respectively.
- Abstract dimensions: These do not specify the particular unit in which a value is given, only its fundamental nature. The abstract dimensions of mass, length and time are expressed as [M], [L] and [G], respectively.

Dimensional analysis:

Since dimensions specify the nature of a quantity, it is possible to deduce how given quantities are related to other quantities by comparing their dimensions. This is called dimensional analysis:

Example: Determining how the period T of a simple pendulum depends on mass m and length l :

- Abstract dimension of T is [T], m is [M] and l is [L]
- This necessitates the presence of another, constant quantity in the expression which incorporates units of time – this is the gravitational constant, g , which has abstract dimension $[L][T]^{-2}$

Expressing T generally in terms of m, l and g :

$$T \propto m^{\alpha} l^{\beta} g^{\gamma}$$

$$\therefore [T] = [M]^{\alpha} [L]^{\beta} \left([L][T]^{-2} \right)^{\gamma} = [M]^{\alpha} [L]^{\beta+\gamma} [T]^{-2\gamma}$$

$$\therefore \alpha = 0, -2\gamma = 1 \therefore \gamma = \frac{-1}{2} \therefore \beta - \frac{1}{2} = 0 \therefore \beta = \frac{1}{2}$$

$$\therefore T \propto l^{\frac{1}{2}} g^{\frac{-1}{2}}$$

$$\therefore T \propto \sqrt{\frac{l}{g}}$$

Limitations of dimensional analysis:

- Dimensional analysis can obviously only be applied to quantities which have dimensions
- It therefore cannot give the constant of proportionality in an equation, since such constants are dimensionless numbers
- The relationship between quantities obtained via dimensional analysis *will not be unique* if some or all of the constituent quantities can be combined to give a dimensionless quantity. Such a dimensionless quantity could be multiplied into the

equation or excluded without affecting the overall dimension, and hence there exist an infinite number of possibilities for the form of such an equation

Experimental error and uncertainty:

The final result of any experiment is always expressed in the form $A \pm \Delta A$, where A is the value of the result obtained and ΔA is its associated uncertainty. Uncertainty is caused by *error,* which is intrinsic to every experimental procedure, and can be one of two types – random or systematic.

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Types of error:

Random error: The effect of natural fluctuations in measured values from the central/mean value:

- Equally likely to be positive or negative
- Can be detected by analysing the spread of data
- Can be reduced by repeating measurements to reduce the spread of the data
- Generally affects the <u>precision</u> of the result

Systematic error: The effect of an experimental arrangement which differs from that assumed:

- Not equally likely to be positive or negative
- May vary or remain constant throughout the experiment
- Not a result of the natural spread of data, hence cannot be reduced by repeating measurements
- If its source and size are identified, it can be removed from the final result
- Systematic error generally affects the <u>accuracy</u> of the result

Distribution of experimental data:

By the Central Limit Theorem, experimental data collected with a large number of trials is always normally-distributed, thus follows a Gaussian distribution of the form:

$$f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Where μ is the mean of the distribution and σ is its standard deviation. This leads to the following properties of experimental data:

- The mean obtained from the data $\bar{x}$ is an approximation of the true mean of the distribution μ, such that $\bar{x}\approx\mu$
- The maximum value of the distribution is at μ, thus $\bar{x}$ is the best indication of the true value represented by the data
- Data are therefore always represented by $\bar{x}$ for all trials conducted
- Experimental uncertainty is the uncertainty in this mean value, σ_{mean}

$$Result=\bar{x}\pm\sigma_{mean}$$

Where $\bar{x}$ and σ_{mean} are always given to the same number of decimal places, and σ_{mean} is given to two significant figures if it begins with a 1 or a 2, and one significant figure otherwise.

Obtaining the value of $\bar{x}$ from experimental data:

Since experimental data X usually have discrete values x, the mean is simply given by: