1201 Algbra 1 Notes

Based on the 2016 autumn lectures by Prof F E A Johnson

The Author(s) has made every effort to copy down all the content on the board during lectures. The Author(s) accepts no responsibility whatsoever for mistakes on the notes nor changes to the syllabus for the current year. The Author(s) highly recommends that the reader attends all lectures, making their own notes and to use this document as a reference only.

**	Man. 03/10/16
a a a r r a ann gaile agus an ann ann agus ann ann ann agus ann ann ann agus ann ann ann agus ann ann ann agus	Algebra MATHIZO
ner ka a a a a a a a a a a a a a a a a a a	Prof. Francis Johnson
CHAPTER I.	§ Elementary Linear Algebra § (Equations)
<u>i:1</u>	A note about linear equations and notation
that and the specific parts is a specific plant is supported as the specific parts and the specific parts and the specific parts are specific parts.	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
	H. Anton x²+y²-Z²=1 surface (x)
	Linear Algebra & Geometry , we need 6 variables to illustrate the relationship Linear Algebra U. good Linear Algebra V. good
	of letters. So we subscript variables.
	$X = (x_1)$ $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ $X = \begin{pmatrix} x_1 \\ x_4 \\ x_4 \end{pmatrix}$ Column Vector
	It can also be $x_1 + x_2 - x_3 - x_4 + 2x_5 - 3x_6 = 1$
**************************************	· The general equation in n-dimensions looks like
e-e-e-e-e-e-e-e-e-e-e-e-e-e-e-e-e-e-e-	$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$ where $x_1, \cdots x_n$ are variables
1	Think as follows:
	a = (a,, a,, an) represents "constants"
	· How about 2 equations in n unknowns?
	$a_1x_1+a_2x_2+a_3x_3+\cdots+a_nx_n=b$
30 -30-3 -1-1-10-0	$C_1 \chi_1 + C_2 \chi_2 + C_3 \chi_3 + \dots + C_n \chi_n = d$
	Eventually, we will run out of letters.
	Answer: Use double indices for coefficients.
the distribution of the section of t	$\int a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \dots + a_{1n} x_n = b_1$ General System of
	$S = \sqrt{a_{21} \chi_1 + a_{22} \chi_2 + a_{23} \chi_3 + \dots + a_{2n} \chi_n} = b_2 \qquad m \text{ linear equations}$
1000 mm	in n unknowns.
	$a_{m_1}x_1 + a_{m_2}x_2 + a_{m_3}x_3 + \cdots + a_{m_n}x_n = b_m$

Cayley C. 1830 : · Go back to a single equation. $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$ $a = (a_1, a_2, ..., a_n)$ Column Vector Row Vector $Q.X = Q_1X_1 + Q_2X_2 + \dots + Q_nX_n$ · Define = Darxr So a single equation looks like g.x=b ·In a general system S , these coefficients are arranged as an $m \times n$ matrix 1≤k≤m $\begin{vmatrix} a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \end{vmatrix} = \begin{pmatrix} a_{ij} \end{pmatrix}$ OR (Okt) Note: The 1st index represents the row number The 2nd index represents the column number. · FXAMPLE: $x_1 - x_2 + 2x_3 - x_4 = 1$ $2x_1 + x_2 - x_3 - x_4 = 2$ (3 equations in 4 variables) Coefficient matrix = 1.2 Matrix Product Cayley's Great Idea l≤i≤m A = (Q4) m×n matrix l≼j≤n 1 sisn n×p matrix B = (bjk)l≤k≤p AB is then the $m \times p$ matrix defined as follows (AB) ik = (ith row of A)(kth column of B) = (ai , ai2, ai3, ai4, -ain) bok/ = Qubik + Qubak += + Qinbak

	EXAMPLE: $A = \begin{pmatrix} 1 & 0 & 5 \\ -1 & 7 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 7 \\ 7 & 7 \end{pmatrix}$
and the second s	[2×3] [3×2]
errinantissiyaya ayanmanimin partippeeeesaa aasaa aasaa	
erinteren et let terme de trette en de de del delegte til stylle på de	$AB = \begin{pmatrix} 26 & 9 \\ 9 & 54 \end{pmatrix} \qquad BA = \begin{pmatrix} -7 & 49 & 14 \\ 3 & 14 & 29 \end{pmatrix}$
in the second	
~ ^ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~	Fri. 07/10/16
the state of the s	Algebra MATH1201
THE STATE OF THE S	Prof. Francis Johnson
	Recap: $(a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1)$ $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ variable factor
de en	$S = \begin{cases} a_{21} \chi_1 + a_{22} \chi_2 + \cdots + a_{2n} \chi_n = b_2 \\ \vdots \\ x_n \end{cases} \qquad \begin{cases} \chi_n / n \times 1 \text{ matrix} \end{cases}$
	$\begin{array}{c} \vdots \\ \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \\ \begin{pmatrix} b_2 \\ b_3 \end{pmatrix} \\ \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \\ \begin{pmatrix} b$
eccentralismi (which puris) presente a cress, et e assission, a est	
errere e e e anno e en en en el compone por persone e e e e e en en en en en en en en en	$A = (a_{ij})$ $ s \le n$ coefficient matrix $S = A \times = b$
emana a chaeann ar minimin (minig) (gapen yng yn ganau yn ar na an ar an a	$A = (aij) si \in M Multiplication \\ A = (bik) si \in O $
	$A = (a_{ij}) \qquad B = (b_{jk}) \qquad (\leq k \leq p)$ no, of columns $A = no$. of rolls B
**************************************	So product AB is defined.
2000	$(AB)_{ik} = \sum_{i=1}^{n} A_{ij} B_{jk}$
Petro habitant de la la mandal mandal mandal mandal de la petro	EXAMPLE: $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 0 & -1 \end{pmatrix}$ $B = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$
	12 -1
entromethologische er er er voor er er voor en entster en en ook en ook bevel voor en ook en ook bevel voor de	$AB = \begin{pmatrix} 0 & 3 \end{pmatrix} \qquad 2 \times 2 \text{matrix}$
	$BA = \begin{pmatrix} 3 & -1 & -1 \\ 3 & 1 & -2 \\ 0 & -2 & 1 \end{pmatrix}$ 3×3 matrix
mare e ve em mont else als es e em est e corent a casa de la casa e tra casa.	$BA = \begin{pmatrix} 0 & -2 & 1 \end{pmatrix} 3 \times 3 \text{matrix}$
1:3	Basic Facts about Matrix Multiplication
aga a a a a a a a a a a a a a a a a a a	(0) If A is $m \times n$, B is $n' \times p$, "iff" = if and only if
	then AB is defined iff $n=n'$
tanema com um es e e e e e e e e e e e e e e e e e e	(1) Suppose A is m×n, B is n×p,
The Control of Control of the Contro	then AB is defined, but BA is defined iff $p=m$
no di kalamban connellito parance curanne rumanum nera	(2) If AB, BA both defined, then in general they have different sizes.
	If AB, BA have the same size, then A and B are both square $(n \times n \text{ say})$
militari e e e e e e e e e e e e e e e e e e e	(3) If A is $n \times n$, B is $n \times n$, both products are defined, but in general AB \neq BA
1900-ren en e	EXAMPLE:

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$\Rightarrow AB = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad BA = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$
In this case, $BA = -AB$, BUT this is very unusual.

$$A = (A_{ij}) \qquad | 1 \le i \le m \qquad B = (B_{ij}) \qquad | 1 \le i \le m \qquad B = (B_{ij}) \qquad | 1 \le i \le m \qquad B = (B_{ij}) \qquad | 1 \le i \le m \qquad B = (B_{ij}) \qquad | 1 \le i \le m \qquad B = (B_{ij}) \qquad | 1 \le i \le m \qquad B = (B_{ij}) \qquad | 1 \le i \le m \qquad B = (B_{ij}) \qquad | 1 \le i \le m \qquad B = (B_{ij}) \qquad | 1 \le i \le m \qquad B = (B_{ij}) \qquad | 1 \le i \le m \qquad B = (B_{ij}) \qquad | 1 \le i \le m \qquad B = (B_{ij}) \qquad | 1 \le i \le m \qquad | 1 \le i \le m$$

·	Formal Def: $(I_n)_{ij} = \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$
Professional Management (Control of Managemen	· Traditional Notation:
	$\delta \dot{y} = \begin{cases} 1 & i=j \\ \delta \dot{y} = \end{cases}$ Kronecker Delta
	btj=[o i≠j Kronecker Delta
el (l'Amateur s'este ans a s'este de l'hand a de l	· <u>Properties of In</u>
	Let A be m×n,
and the contract of the section of t	then AIn is defined and AIn=A
en transport de la companya de la co	Likewise, if B is $n \times p$, $I_n B = B$
metholisecture actument actual	- Prove that $AIn = A$ if A is $m \times n$.
	$Proof: A = (Aij) si \leq m$ $ sj \leq n $
	$I_{n} = (\delta_{jk}) \leq j \leq n$ $ \leq k \leq \rho$
	Use definition.
	$(AI_n)_{ik} = \sum_{j=1}^{n} A_{ij}(I_n)_{jk}$
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	$= \sum_{i=1}^{n} A_{ij} \delta_{jk} \qquad k \text{ is fixed while } j \text{ varies from } l \text{ to } n$
	so picked out the fixed terms (1)
er e e room on amerikanskipskipskipskipskipskipskipskipskipskip	= Aik Sijk + PAij Sjk When j # k, Sjk = 0
**************************************	$= A_{ik} + 0$
	= Ack
andria) and a construction of the second probability of the second position (second second second second second	$\therefore (AIn) \epsilon_k = A \epsilon_k$
	$\Rightarrow AI_n = A$
**	$- EXAMPLE: \int 2x_1 + 3x_2 = 1$
nne - e e e e e e e e e e e e e e e e e	$\left(x_1 + 2x_2 = 2\right)$
	Three Basic Operations:
**************************************	(I) Add $\Lambda Eq^n(i)$ to $Eq^n(j)$ where $\Lambda$ is some number
the destruction (in the property of the structure of the	(II) Multiply Eg ⁿ (i) by 1 ≠0
	(III) Permute the order in which the equations are written.
nn n namara at far fi gara, a a a carran, a garan ann an aiginn gan garan f	Add $(-2)Eg^{n}(2)$ to $Eg^{n}(1): -x_{2}=-3$
	$x_1 + 2x_2 = 2$
ann a ann a dh' a ann a a dheala a a a a ann a a ca a a a a a a a a a a	Add $2Eg^{n}(1)$ to $Eg^{n}(2)$ : $\begin{cases} -x_2 = -3 \\ x_1 = -4 \end{cases}$
et de la come est e e secon e commune, e e e e e e e e e e e e e e e e e e	Swap Eq ⁿ (1) and Eq ⁿ (2): $\chi_1 = -4$ $\left[-\chi_2 = -3\right]$
di de en establica en entra minima, por por por porto por porto por escapo de la compansión de la compansión d	\\\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\

THEY I'V THE	Multiply Eq ⁿ (2) by -1: $\chi_1 = -4$ $\chi_2 = 3$	
1.4	Elementary Row Operations	
	·Consider a system of linear equations:	
	$\chi_1 + \chi_2 + \chi_3 = 1$	
	$\chi_1 - \chi_2 + \chi_3 = 3$	
(man)	$\chi_1 + \chi_2 - \chi_3 = -1$	
	· What can we do? Three things:	
	(I) & (i,j; λ) Adds λ Row(j) to Row(i)	
400-000 A B B B B B B B B B B B B B B B B B	i.e. new RaW (i) = ald Raw(i) + 1 Row(j)	
	All other rows stay the same.	مسلما والمعارض المعارض
و من	(I) D(i; λ) Multiplies Row(i) by λ, provided λ≠0.	
na a a a a a an an an an an an an an an	$(II) \mathcal{F}(i,j)$ Swaps Row(i) and Row(j)	***************************************
	· We are going to show that we can perform these three operations	o o o nome and an analysis and a supplementary functions and and a supplementary functions.
	by multiplying on LEFT by a suitable matrix.	oo
	$A = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \qquad j=1  \dot{t}=2$	
addition of a first of a second comment [] ( ) is defined	$\mathcal{E}(2,1;\lambda)$ Adds $\lambda Row(1)$ to $Row(2)$	
	/100/ab) /ab)	
e a e e e e e e e e e e e e e e e e e e	$\begin{pmatrix} 1 & 0 & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} = \begin{pmatrix} a & b \\ \lambda a + c & \lambda b + d \\ e & f \end{pmatrix}$	
	& (1,3; M)	
m∏ (v) (v m v v v v v v v v v v v v v v v v v	(10µ)(ab) (a+ue b+uf)	
-21	$\begin{pmatrix} 1 & 0 & \mu \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a + \mu e & b + \mu f \\ c & d \\ e & f \end{pmatrix}$	**************************************
	QUESTION. Which matrix perform & (i,j,2)?	i de la companya de l
and a second control of the second control o	Elementary Matrices	····
and a second and a	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	a de la calenda
and a second	Greek letter (0 0) (0 0) (1 0) (0 1)	anning and a second
and the second s	epsilon $\mathbf{\xi}(1,1)$ $\mathbf{\xi}(1,2)$ $\mathbf{\xi}(2,1)$ $\mathbf{\xi}(2,2)$	
w.w.	$3 \times 3: \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	
411 • \\ \	1000/ (000/ (001/	
	$\mathcal{E}(1,1)$ $\mathcal{E}(1,3)$ $\mathcal{E}(2,1)$ $\mathcal{E}(3,3)$	a v ar han sannar va 3 sensitempere (peri Graff ref sheld direction)
nad Toppe Gref (1/2) Friend Philippe (1/4)	- Informal Def:	

 $\in (i,j)$  is  $n \times n$  matrix which has I in position (i,j) and

Os everywhere else.

	- Formal Def: $ \xi r \leqslant \eta $ $\mathcal{E}(i,j) = \left(\mathcal{E}(i,j)rs\right)  \xi s \leqslant r$
······ <u> </u>	
energy	$F(i,j)_{rs} = SirSjs$
eddimen a new enderliet en a menen en en en en en en	Check:
COCCAO (CACACAMACAMACAMACA) que e e e e e e e e e e e e e e e e e e	$ \frac{E(i,j)_{rs}}{c} = \begin{cases} 1 & r=i \text{ and } s=j \\ 0 & \text{otherwise} \end{cases} $
	QUESTION: Let A be m×n
** * * * * * * * * * * * * * * * * * *	Let $\in (i,j)$ be elementary $m \times m$
an marina an a	What is $\in (i,j)A$ ?
i m giya lahakis unakis suma ara-ara ka kunaka kuna	EXAMPLE: $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} = \begin{pmatrix} e & f \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$
	€(1,3) A
	"takes out the j th now & put it into the i th now & kills everything else"
}*************************************	"takes out the j th row & put it into the i th row & kills everything else" $- \text{Proposition}: \text{ Let } A = (a_{st})  \text{i} \leq t \leq n$
ditata e menuna ancora e accomanda antonio a	Let $\in (i,j)$ be elementary $m \times m$ matrix.
errorinante fina fina fina por	€(i,j)A is the matrix where
	$i^{th} row = j^{th} row of A$
r 11 111111 11 11111 11111 11111 11111 1111	All other rows = 0
a a c e se se seminario de la companio de la compa	$Proof: [\in (i,j)A]_{rt} = \sum_{s=1}^{n} \in (i,j)_{rs} A_{st} $ $(AB)_{rt} = A_{rs}.B_{st}$
TTO COLOR AND	= \sum_{\sigma_i} \delta_i \delta_j \delta_i \delta_j \delta_i \delta_j \delta_i \delta_j \delta_i \de
	$= \sum_{s=1}^{m} \delta_{ir} \delta_{js} Ast$ $= \sum_$
	the case where say - Sindit + O
mmand vernalistikki juung maa semasta an asaa as a vuun	$S_{jj}=1 = S_{ir}A_{jt}+0 \qquad \text{when } S\neq j  \partial_{j}\tau=0  \text{by def}$ $(A_{it}  \Gamma=i)$
ant that the street they go fore greaters for every selective sections of selections.	$= \begin{cases} A_{i}t & r=i \\ 0 & r\neq i \end{cases}$ Q.E.D.
- Control of the state of the s	
e 1888 - 1888 - 1889 - 1890 - 1890 - 1890 - 1890 - 1890 - 1890 - 1890 - 1890 - 1890 - 1890 - 1890 - 1890 - 189	
- 000 C C C C C C C C C C C C C C C C C	Mon. 10/10/16
t to all the second	Algebra I MATHIZOI
e en	Prof. Francis Johnson
etista e marca (e) eta erre a carrena a jengan esperator	· Recap: <u>Flementary Row Operations</u>
<u> </u>	<u>Basic Matrices</u>
hanana( v  ma'ah(=>>>>>>>>>=))/(a)= / aha	· Corollary: If $\wedge$ is a "number",
a ggyana <b>n</b> t dalitich de nyongon (net dalitya) a genegal felologia	then $\lambda \in (i,j)A$ is a matrix whose $i^{th}$ row = $\lambda$ ( $j^{th}$ row of A)

= ith row of A + x. jth row of A QED

• Def :  $E(i,j;\Lambda) = I_m + \lambda E(i,j)$ e.g. m=3, then:  $E(2,3;\Lambda) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \lambda \\ 0 & 0 & 1 \end{pmatrix}$ · Th: Let A be m×n, then the matrix  $E(i,j;\lambda)A$  is the matrix obtained from A by performing the operation & (i,ji)  $\mathcal{E} \equiv \text{operation}$   $\mathbf{E} \equiv \text{matrix}$ (Curly letters) (Straight letters) Proof:  $[I_m + \lambda \in (i,j)]A = A + \lambda \in (i,j)A$ so  $k^{th}$  row of  $[I_m + \lambda \in (i,j)]A = k^{th}$  row of A, where  $k \neq i$  and k is a numbe (This is because  $k^{th}$  row of  $\lambda \in (i,j) A = 0$ ) and ith row of [Im +  $\Lambda \in (i,j)]A = i^{th}$  row of  $A + i^{th}$  row of  $\Lambda \in (i,j)A$ EXAMPLE:  $E(2,3,\Lambda) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \lambda \end{bmatrix}$   $A = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$  $\Rightarrow E(2,3;\lambda)A = \begin{pmatrix} a & b \\ c+\lambda e & d+\lambda f \end{pmatrix}$ · Def: Let A be an m×n matrix,

we say that A is invertible when there exists a matrix  $B(m \times m)$ s.t.  $AB = I_m$  and  $BA = I_m$ .

When this happens, write  $\beta = A^{-1}$ 

EXAMPLE:  $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$   $B = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ 

 $\Rightarrow AB = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$   $BA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 

In this case, A is invertible and  $A^{-1} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ 

A = E (1,2;2)

B = E(1,2,-2)

· Prop: Ε(i,j; λ) is invertible

and  $E(i,j;\lambda)^{-1} = E(i,j;-\lambda)$ 

Proof: coming soon U

Matrix (i≠j) Operation

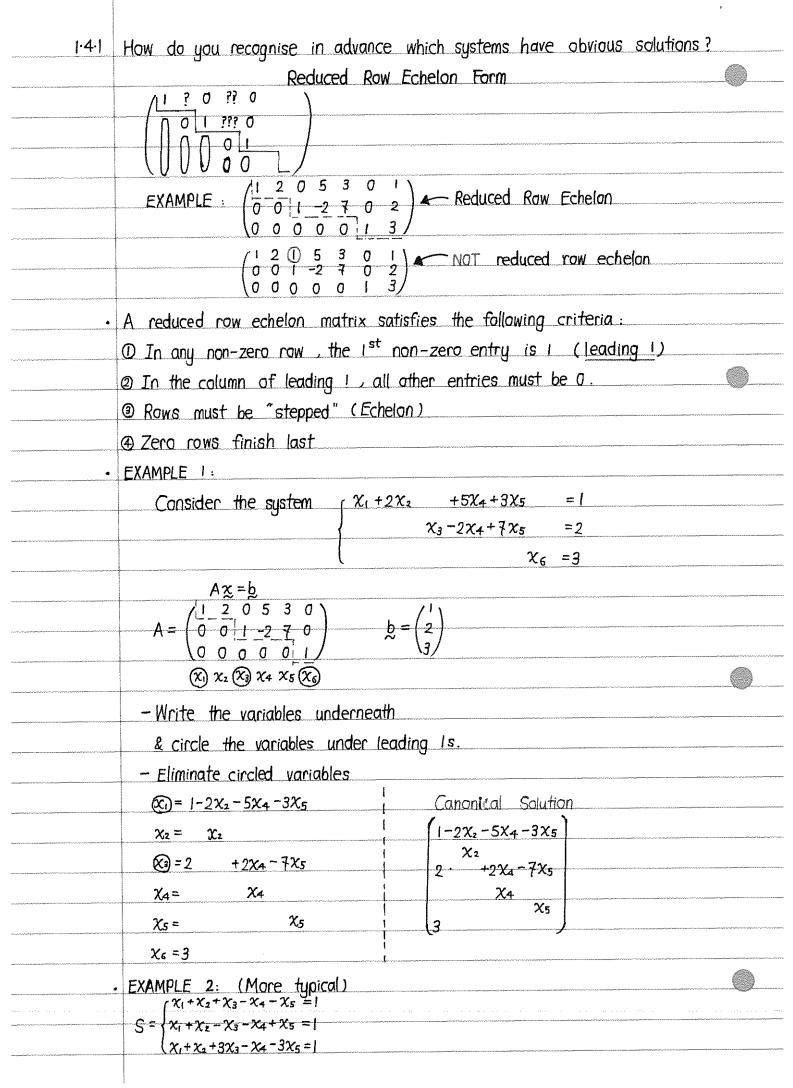
 $\sim$   $E(i,j;\lambda) = I_m + \Lambda \in (i,j)$ 0 E(i,j; 1)

 $\Delta(i; \lambda) = I_m + (\lambda - i) \in (i, i)$  $a \mathcal{L}(ij\lambda) \quad (\lambda \neq 0)$ 

```
EXAMPLE: A = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} perform \mathcal{D}(2; \lambda)
                               \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} + \begin{pmatrix} 0 & - & 0 \\ (\lambda - 1)c & (\lambda - 1)d \end{pmatrix} = \begin{pmatrix} a & b \\ \lambda c & \lambda d \\ e & f \end{pmatrix}
                          A+(x-1)e(i,i)A1
               - k^{th} row of \epsilon(i,i)A=0 (k\neq i)
                  i^{th} now of \in (i,i)A = i^{th} now of A
              - k^{th} row of (\Lambda - 1) \in (1,1) A = 0
                                                                  (k≠i)
                  i^{th} row of (\lambda - i) \in (i, i) A = (\lambda - i)(i^{th} row of A)
           This can be transformed into
                          [Im + (\lambda - 1) \in (i,i)]A
             - k^{th} row of [I_m + (\lambda - i) \in (i, i)]A = k^{th} row of A (k \neq i)
                   i^{th} row of [I_m + (\lambda - 1) \in (i, i)]A = i^{th} row of A + (\lambda - 1)(i^{th}) row of A
                                                                    = \pi.(i^{th} \text{ row of } A)
            -\mathcal{D}(i;\lambda) multiplies i^{th} row of A by \lambda \neq 0
               \mathcal{D}(i, \frac{1}{\pi}) multiplies i^{th} now of A by \frac{1}{\pi} \neq 0
               \triangle (i; \land) invertible and
                         \Delta(i;\Lambda)^{-1} = \Delta(i,\pi)
\mathfrak{P}^{(i,j)} interchanges i^{th} & j^{th} row
    - We expect that P(i,j) = matrix obtained from Im by swapping i^{th} \& j^{th} rows.
     EXAMPLE: M=4, then P(2,3) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
    - Does it work?
               I_{m} - \in (i,i) - \in (j,j) + \in (i,j) + \in (j,i)
               P(i,j)^{-1} = P(i,j)
Fri. 14/10/16
                            MATHI401 Algebra I
                            Prof. Francis Johnson
          Operation
                                                                         Matrix
          E (ijin)
                                                               E(i,j;\lambda) = I_m + \lambda \in (i,j)
```

	$\mathcal{D}(i;\Lambda)  (\Lambda \neq 0) \qquad \sim \qquad \Delta(i;\Lambda) = \underline{I}_{m} + (\Lambda - 1) \in (i,i)$	and a second and a second as a second
	$\mathcal{P}(i,j) = \operatorname{Im} - \mathcal{E}(i,i) - \mathcal{E}(j,j) + \mathcal{E}(i,j)$	)+() €(j):
	(1+1)	
	· On Exercise 2,	g (
	we need to show $\{(i,j)\in(k,l)=\{(i,l),j=k\}$	
	0 ; j + k	
	Assume this is true, we have	
000° 00° 00° 00° 00° 00° 00° 00° 00° 00	corollary $\in (i,j) \in (i,j) = 0$ $i \neq j$	
	$\in (i,i) \in (i,i) = \in (i,i)$	
	· Prop If i + j , then	
	E(i,j;λ) is invertible	
	and $E(i,j;\Lambda)^{-1} = E(i,j;-\Lambda)$	
and the first and the second of the second o	Proof: $E(i,j;N)E(i,j;-\lambda)$	**
والمراقب وال	$= \left[ \operatorname{Im} + \Lambda \in (i,j) \right] \left[ \operatorname{Im} - \Lambda \in (i,j) \right]$	
and the state of t	$= \operatorname{Im}^{2} + \Lambda \operatorname{Im} \in (i,j) - \Lambda \operatorname{Im} \in (i,j) - \Lambda^{2} \in (i,j) \in (i,j)$	en reconstruction and the contract of the cont
gangaganan di EEE di	$= I_{m} - \Lambda^{2} \in (i,j) \in (i,j) \qquad \qquad \in (i,j) \in (i,j) = 0 \qquad i \neq j$	105 m 5 C 16 1 16 1 C 16 m 16 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	= Im	
ar ar 1 co 100 100 100 100 100 100 100 100 100 10	$\Rightarrow E(i,j;\Lambda)E(i,j;-\Lambda) = I_m = E(i,j;-\Lambda)E(i,j;\Lambda) \qquad QED$	
	· Prop. If ∧≠0, then	
	$\triangle$ ( $i$ : $\hbar$ ) is invertible	t and an at the television is a sure a page more community of the last
garawa sarahas sarahan wan wasan (m) m)	and $\Delta(i; n)^{-1} = \Delta(i; \frac{1}{n})$	
	Proof: $\Delta(i; \Lambda) = I_m + (\Lambda - i) \in (i, i)$	to a section of the s
	$\Delta (i; \frac{1}{\Lambda}) = \operatorname{Im} + (\frac{1}{\Lambda} - 1) \in (i, i) = \operatorname{Im} + (\frac{1 - \Lambda}{\Lambda}) \in (i, i)$	e e e e e e e e e e e e e e e e e e e
	$\Rightarrow \Delta(i, n) \Delta(i, \frac{1}{n})$	at a final series are assumed to the series and the series and the series and the series are series as the series and the series are series as the series are series are series as the series are series are series as the series are series ar
**************************************	$= \left[ \operatorname{Im} + (\lambda - 1) \in (i, i) \right] \left[ \operatorname{Im} + \left( \frac{(-\lambda)}{\lambda} \right) \in (i, i) \right]$	and the second s
	$= \operatorname{Im}^{2} + (\lambda - 1)\operatorname{Im} \in (i,i) + (\frac{1-\lambda}{\lambda})\operatorname{Im} \in (i,i) + (\lambda - 1)(\frac{1-\lambda}{\lambda}) \in (i,i) \in (i,i)$	
an - 1 - 1 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2	$= \operatorname{Im} + \left( \lambda - 1 + \frac{1 - \lambda}{\lambda} \right) \operatorname{Im} \in (i, i) + (\lambda - 1) \left( \frac{1 - \lambda}{\lambda} \right) \in (i, i)$	
	$= I_{m} + \frac{(\Lambda - 1)^2}{\Lambda} I_{m} \in (i, i) - \frac{(\Lambda - 1)^2}{\Lambda} \in (i, i)$	t o des processos processos processos de processos de contra entre contra contra de 1 de 1 de 1 de 1 de 1 de 1
00000000000000000000000000000000000000	$=I_{m}$ QED	· · · · · · · · · · · · · · · · · · ·
	· Prop. If i≠j, then	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
	P(i,j) is invertible	
page and global and an analysis and an analysi	and $P(i,j)^{-1} = P(i,j)$ self-inverse	
	Proof: P(i,j)P(i,j)	
	$= \left[ I_{m} - \varepsilon(\hat{\imath}, \hat{\imath}) - \varepsilon(\hat{\jmath}, \hat{\jmath}) + \varepsilon(\hat{\imath}, \hat{\jmath}) + \varepsilon(\hat{\jmath}, \hat{\imath}) \right] \left[ I_{m} - \varepsilon(\hat{\imath}, \hat{\imath}) - \varepsilon(\hat{\jmath}, \hat{\jmath}) + \varepsilon(\hat{\imath}, \hat{\jmath}) + \varepsilon(\hat{\jmath}, \hat{\jmath}) \right]$	(i)]
manda que fatiramente e en entre en en		

	-∈(j,i)-0-∈(i,j)+	0+E(i,i)-E(j,i)-0+E(j,j)+0
·voori=)/ilimoreorous-veenn	= Im	QEO
	1.4 How to solve a system of line	ear equations?
	$\int a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \cdots + a_{11}$	
	$S = \begin{cases} Q_{21} \chi_1 + Q_{22} \chi_2 + Q_{23} \chi_3 + \cdots + Q_{2n} \end{cases}$	$S = \{A x = b\}$
·^~~~	$a_{m_1}x_1 + a_{m_2}x_2 + a_{m_3}x_3 + \cdots + a_{m_n}$	in Xn = bm
······································	· PropSuppose that I operate	on the left by an invertible matrix P, then the
atendra et eller i de la company et en eller i de la company et en eller i de la company et en eller i de la c	solutions don't change.	
**************************************	$S = \{A \underset{\sim}{\times} = \underset{\sim}{b}\} \qquad S'$	$' = \{PAx = Pb\}$
	- If $\stackrel{\times}{\sim}$ is a solution to $S$ ,	, then $\approx$ is also a solution to $s'$ .
····	- Conversely , if % is a so	elution to S',
***************************************	PAX = Pb	
v	– Multiply on the left by P	~
197 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 - 17 (1 -	P ⁺ PA≈=P ⁺ Pb	where $P^+P = I_m$
~~~~	So, Az=b	
	-So x is a solution to s	
······································	· (0) For some systems, solutions	s are obvious.
**************************************	EXAMPLE: $\left\{ \begin{array}{l} x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \right\} $ satisfying	
	$\begin{cases} \begin{array}{c} (x_3) \\ (x_3) \end{array} & \text{out to sign } g \end{cases}$ $\chi_1 = 1 - \chi_2 - \chi_3$	
	$ \frac{\chi_1 = 1 - \chi_2 - \chi_3}{\left(1 - \chi_2 - \chi_3\right)} $	
	$ \begin{array}{c} \left(\begin{array}{c} \chi_{3} \\ \chi_{3} \end{array} \right) & \text{Soft of } y \text{ and } y a$	$x_1 + x_2 + x_3 = 1$ has infinitely many solutions)
	$ \begin{array}{c} \left(\begin{array}{c} \chi_{3} \\ \chi_{3} \end{array} \right) & \text{solitorizing} \\ \chi_{1} = 1 - \chi_{2} - \chi_{3} \\ \chi_{2} = \left(\begin{array}{c} 1 - \chi_{2} - \chi_{3} \\ \chi_{3} \end{array} \right) \\ \chi_{3} = \left(\begin{array}{c} \chi_{3} \\ \chi_{3} \end{array} \right) & \text{(1)} \end{array} $ $ \begin{array}{c} \chi_{3} = \chi_{2} \\ \chi_{3} = \chi_{3} \\ \chi_{4} = \chi_{5} \\ \chi_{5} = \chi_{5} \\ \chi_{7} = \chi_{7} \\ \chi_{$	$x_1+x_2+x_3=1$ } has infinitely many solutions) we can get all possible solutions. (2-dimension)
	$ \begin{array}{c} \left(\begin{array}{c} \chi_{3} \\ \chi_{3} \end{array} \right) & \text{satisfying} \\ \chi_{1} = 1 - \chi_{2} - \chi_{3} \\ \chi_{2} = \left(\begin{array}{c} 1 - \chi_{2} - \chi_{3} \\ \chi_{2} \\ \chi_{3} \end{array} \right) \\ \text{As } \chi_{2} \text{and } \chi_{3} \text{vary} , \end{array} $	$x_1 + x_2 + x_3 = 1$ has infinitely many solutions) we can get all possible solutions. (2-dimension) $x_3 = 1$
	$ \begin{array}{c} \left(\begin{array}{c} \chi_{3} \\ \chi_{3} \end{array} \right) & \text{some give} \\ \chi_{1} = 1 - \chi_{2} - \chi_{3} \\ \chi_{2} = \left(\begin{array}{c} 1 - \chi_{2} - \chi_{3} \\ \chi_{3} \end{array} \right) \\ \chi_{3} = \left(\begin{array}{c} \chi_{3} \\ \chi_{3} \end{array} \right) & \text{ond} \chi_{3} \text{ vary} \\ \chi_{1} + \chi_{3} + \chi_{4} + \chi_{5} + \chi_{6} - \chi_{6} \end{array} $	$x_1 + x_2 + x_3 = 1$ has infinitely many solutions) we can get all possible solutions, (2-dimension) $x_3 = 1$ $x_4 = 2$
	$\chi_{1} = 1 - \chi_{2} - \chi_{3}$ $\chi_{2} = \begin{pmatrix} 1 - \chi_{2} - \chi_{3} \\ \chi_{3} \end{pmatrix}$ $\chi_{3} = \begin{pmatrix} 1 - \chi_{2} - \chi_{3} \\ \chi_{2} \end{pmatrix}$ $\chi_{4} = \begin{pmatrix} 1 - \chi_{2} - \chi_{3} \\ \chi_{2} \end{pmatrix}$ $\chi_{5} = \begin{pmatrix} 1 - \chi_{2} - \chi_{3} \\ \chi_{2} \end{pmatrix}$ $\chi_{6} = \begin{pmatrix} 1 - \chi_{2} - \chi_{3} \\ \chi_{2} \end{pmatrix}$ $\chi_{7} = \begin{pmatrix} 1 - \chi_{2} - \chi_{3} \\ \chi_{2} \end{pmatrix}$ $\chi_{8} = \begin{pmatrix} 1 - \chi_{2} - \chi_{3} \\ \chi_{2} \end{pmatrix}$ $\chi_{1} = 1 - \chi_{2} - \chi_{3}$ $\chi_{2} = \chi_{3} + \chi_{4} + \chi_{5} + \chi_{6} - \chi_{6}$ $\chi_{2} = \chi_{3} - \chi_{4} + \chi_{5} - \chi_{6} + \chi_{6}$	$x_1 + x_2 + x_3 = 1$ has infinitely many solutions) we can get all possible solutions, (2-dimension) $x_3 = 1$ $x_4 = 2$
	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$x_1 + x_2 + x_3 = 1$ has infinitely many solutions) we can get all possible solutions. (2 - dimension) $x_3 = 1$ $x_4 = 2$
	$ \begin{array}{c} \chi_{1} = 1 - \chi_{2} - \chi_{3} \\ \chi_{2} = \begin{pmatrix} 1 - \chi_{2} - \chi_{3} \\ \chi_{3} \end{pmatrix} \\ \chi_{3} = \begin{pmatrix} 1 - \chi_{2} - \chi_{3} \\ \chi_{3} \end{pmatrix} \\ \chi_{4} = \begin{pmatrix} 1 - \chi_{2} - \chi_{3} \\ \chi_{3} \end{pmatrix} \\ \chi_{5} = \begin{pmatrix} 1 - \chi_{2} - \chi_{3} \\ \chi_{2} - \chi_{3} \end{pmatrix} \\ \chi_{6} = \begin{pmatrix} 1 - \chi_{1} + \chi_{2} + \chi_{5} + \chi_{6} - \chi_{6} \\ \chi_{1} + \chi_{2} + \chi_{3} + \chi_{4} + \chi_{5} - \chi_{6} + \chi_{7} \\ \chi_{2} - \chi_{3} - \chi_{4} - \chi_{5} - \chi_{6} + \chi_{7} \\ \chi_{2} + \chi_{3} + \chi_{4} - \chi_{5} + \chi_{6} - \chi_{7} \end{array} $	$x_1 + x_2 + x_3 = 1$ has infinitely many solutions) we can get all possible solutions (2-dimension) $x_3 = 1$ $x_4 = 2$ (infinitely many solutions)
	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$x_1 + x_2 + x_3 = 1$ has infinitely many solutions) we can get all possible solutions. (2-dimension) $x_3 = 1$ $x_4 = 2$



 $(\hat{X}_2) + 2x_3 + 3x_4 + 4x_5 = 1$

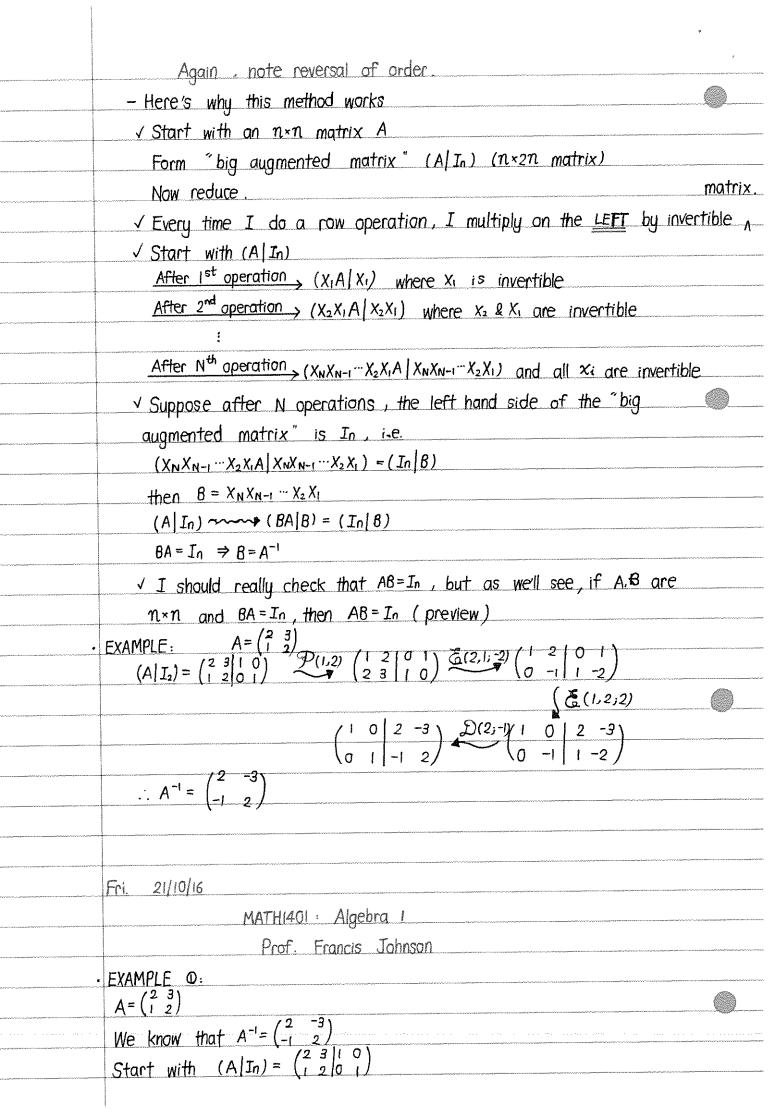
	/ X3+2X4+3X5 \	an and a second an	errore e established de la companya
Canonical solution:	$1-2x_3-3x_4-4x_5$	(mensus un visco quantifolima as sensos visus si mantifolita anno es si mensus en o punto librar en es si mens	
	$\begin{pmatrix} \chi_3 & \chi_4 & \chi_5 \end{pmatrix}$		e e de se se sum escoramization amb del trade mét d'és de l'été d'és de l'été d'és de l'été d'és de l'été d'és
n salawa milayo ka maa maa maa maa maa maa maa maa maa	Legister) tradust artist automorphis (et al. 14 de terre de tradus artistas automorphis de legiste de l'estrat de tradus de tradus artistas automorphis de tradus automorphis de tradus artistas artistas automorphis de tradus artistas arti		
			ran persona (1) (the lightest and an instruction and a service of a section)
Mon. 17/10/16			
	MATHI201: Algebra		
***************************************	Prof. Francis (John	<u>SO()</u>	
EXAMPLE 4:			
$S = \begin{cases} \chi_1 + \chi_2 + \chi_3 + \chi_4 = \\ \chi_1 + \chi_2 - \chi_3 + \chi_4 = \end{cases}$		en service de la company d	
$\begin{cases} \chi_1 + \chi_2 - \chi_3 - \chi_4 = 1 \\ \chi_1 + \chi_2 - \chi_3 - \chi_4 = 1 \end{cases}$		олго на нашине възване на на възване на на населната на населна на населна на населна на н	
A & = b	$A = \begin{pmatrix} 1 & 1 & -1 & 1 \end{pmatrix}$	<u>b</u> = (1)	
	1 1 -1 -1		annon mulatatata di 1994 di 2004, di 2000 di 2
augmented matrix (111111		
augmented matrix		na ann an an aire an aire bhaill (an 1861). An ann an ann ann an an an an Ann an Ann an Ann an Ann an Ann an A	
8(2,1;-1) (1 1 1 1			
8(3,1;-1) 0 0 -2 0 0 0 -2 -2	10		
/1101			
$\frac{\mathcal{D}(2,\frac{1}{2})}{\mathcal{D}(3,-\frac{1}{2})} \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	0		mande al _a unit all al e e e e e e e e e e e e e e e e
$ \begin{array}{c} \underline{8}(1,2,1) \\ \underline{8}(3,2,1) \end{array} $			ess of the second secon
8(3,2,1) (0001)	0		
$ \underbrace{\mathcal{D}(2;-1)}_{0 \ 0 \ 0} \left(\begin{array}{c} 1 \ 1 \ 0 \ 0 \\ 0 \ 0 \ 0 \end{array}\right) $		tanan mengangkan kelangkan kelangkan mengangkan kelangkan mengangkan kelangkan di terbuh sasi an ana ana ana d	and the second of the second o
0001	<u> </u>		
$\underbrace{\{(1,3,-1)}_{0,0} \left(\begin{array}{c} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$	magazinan magazinan saran		
	10/2 19 III I COUCL	d form	
(X) X, (X) (Q)		· Write variables underneath	
(Ø) + X _z	=	& circle variables under leading	15
	=0	· Eliminate	
(& =	:0	1	د سده میشد که با است و مسیده و در
	$\begin{pmatrix} 1-x_2 \\ x_2 \end{pmatrix}$	* If you start with 4 variable	s,()
		you should end up with 4.	

Generalisation:

 $= I_n$

If An, An-1, "A₂, A₁ are all invertible $n \times n$ matrices, then the product · AnAn-1 "A₂A₁ is invertible and its inverse $(A_{N}A_{N-1} - A_{2}A_{1})^{-1} = A_{1}^{-1}A_{2}^{-1} - A_{N-1}^{-1}A_{N-1}^{-1}$

QED



	$\frac{\mathcal{P}(1,2)}{23 10} \stackrel{(12 01)}{=} $ Reduce & keep in	ecord
	E(2,1,-2) (2 0 1)	and the control of th
<i>*</i>	$\mathcal{E}_{0}(1,2;2), \begin{pmatrix} 1 & 0 & & 2 & -3 \\ 0 & -1 & & -2 \end{pmatrix}$	
	$\underbrace{\mathcal{D}(2;-1)}_{>} \begin{pmatrix} 1 & 0 & & 2 & -3 \\ 0 & 1 & & -1 & 2 \end{pmatrix}$	
·~~~~	∴ Δ(2;-1) E(1,2;2) E (2,1;-2) P(1,2) (A I)	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
············		
	$(I \Delta(2;-1)E(1,2,2)E(2,1;-2)P(1,2))=A^{-1}$	eesse ka mika joo ka saa k
	So I've written A^{-1} as a product of elementary invertible matrices.	mananan kananan kanana
	So $A^{-1} = \Delta(2,-1) E(1,2,2) E(2,1,-2) P(1,2)$	
a a processor a a son a community of a comm	So $A = P(1,2)^{-1}E(2,1;-2)^{-1}E(1,2;2)^{-1}\Delta(2;-1)^{-1}$	**************************************
	Note: reversal of order	
· } ~~ ? ? . ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~	$A = P(1,2) E(2,1;2) E(1,2;-2) \Delta(2;-1)$	
	Check: $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	rminteminis (hisposivissis ed messionessenos)
	$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	honorestations has his his fall of the state
	$= \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	
	$= \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} = A$	industration described participal
	EXAMPLE $Q: A = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 3 & 1 \\ 0 & -2 & -1 \end{pmatrix}$	ruman en
	0 -2 -1/	
	V Find A-1	
***************************************	2) Express A ⁻¹ as a product of elementary matrices.	POSPARAMENTALINAS ANTONOS ANTO
	3) Thereby, express A as a product.	
AN III COMMON AND AN AND AN AND AN AND AN AND AN AND AN ANA AND AN ANA AND AN ANA AND AND	$\frac{1).(A I_0) = \begin{pmatrix} 1 & 2 & 0 & & 1 & 0 & 0 \\ 1 & 3 & 1 & & 0 & 1 & 0 \\ 0 & -2 & -1 & & 0 & 0 & 1 \end{pmatrix}}{0 - 2 - 1 & & 0 & 0 & 1}$	ananiamatesaanante Agemmeniam
-		elika eska eska eska eska eska eska eska es
	$\underbrace{\mathcal{E}(2,1;-1)}_{0} \to \begin{pmatrix} 1 & 2 & 0 & & 1 & 0 & 0 \\ 0 & 1 & 1 & & -1 & 1 & 0 \\ 0 & -2 & -1 & & 0 & 0 & 1 \end{pmatrix}$	The second of th
		Non-train the second of the se
	$\frac{\mathcal{E}(1,2;-2)}{\mathcal{E}(3,2;2)} \xrightarrow{\begin{pmatrix} 1 & 0 & -2 & & 3 & -2 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 2 & 1 \end{pmatrix}}$	commence of the second
		minint elliministi personer commence communication in communication in the communication in t
	$ \begin{array}{c c} & & & & & & \\ & & & & & \\ & & & & \\ & & & & $	······································
		and the second s
	$A^{-1} = \begin{pmatrix} -1 & 2 & 2 \\ 1 & -1 & -1 \\ -2 & 2 & 1 \end{pmatrix}$	- was a series of the series o
		almateria (1000 and 1000 and
	Check: $A^{-1}A = \begin{pmatrix} -1 & 2 & 2 \\ 1 & -1 & -1 \\ -2 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 1 & 3 & 1 \\ 0 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	eren eren er
	2.) $A^{-1} = E(2,3;-1)E(1,3;2)E(3,2;2)E(1,2;-2)E(2,1;-1)$	maj gi 11 to 18 to to 18 se signi de a provincia de primera e e e e e e e e e e e e e e e e e e

	3) $A = E(2,1,1) E(1,2,2) E(3,2,-2) E(1,3,-2) E(2,3,1)$	i i i i i i i i i i i i i i i i i i i
	2nd raw, 1st calumn is 1. Ist raw, 3rd calumn is -2.	
	Check:	mariha menenda sa masa sa
ganggan gangif da kina anja an anjana da a anim na anim na a	$ \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} $	anaanaanaanaanaanaanaanaanaanaanaanaana
	\(\(0 \ 0 \ 1 \\ \(\text{0 \ 0 \ 1 \\ \(\text{0 \ 0 \ 1 \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\	eessessa tarabasta tarabasta seessa ahaa ahaa ahaa ahaa ahaa ahaa a
	$X = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad Y = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$	
, , , , , , , , , , , , , , , , , , ,		
Control of the Contro	$ \begin{pmatrix} 1 & 2 & -2 \\ 1 & 3 & -2 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 3 & 1 \\ 0 & -2 & -1 \end{pmatrix} $	
	* If a matrix is not invertible, then after row operations, we'll get	
oeveressaan oo wenne oo wood (to the few de few de	(0 0 m m) Os row at the end i.e. we cannot set In	t magnetic family (specific and manife for a mignate a second and a second and a second and a second and a second as
	i.e. we cannot get In	
		reconstruction of the second second and the second
366546/46/48/48/48/48/48/48/48/48/48/48/48/48/48/		era en ara a un era a un ara en un un un un un un un un en
AND THE STREET OF THE STREET		* section to the section of the sect
ann an ann an ann an an ann ann ann ann		egan agan kengalan kenanakan adarah kengalan daran daran daran daran bermulan bermulan bermulan bermulan bermu
		gggpdfffffddfffggriffnasffffffffdariffffanai fffin ac ei filliaeth
and the state of the		· · · · · · · · · · · · · · · · · · ·
Armited Commission of the State State of the Commission of the Com		

primates a travalent personalismo y sello per del per del per del persona e travalent e travalent e travalent		
magnet e e e e e e e e e e e e e e e e e e		<u></u>
		· annum marinis - Significant from the most of the second control
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~		

*	l Eq.	21/10/16	(con	tinued)						
		•			[H140]: Algebra I					
	gammada arang ar kaning 12 kapa arang kanangan ang	elle la			f. Francis Johnson					
Chapter 2.	draka wa masana a a a a a a a a a a a a a a a a a		ر د در		positional Logic §					
2:1	Truth	Tables	· · · · · · · · · · · · · · · · · · ·	TO THE OF THE OWNER WAS AND						
. When the same it is a straight and it is a supplement of the straight of the same of the same of the same of	Φ <u>′νοτ</u>	Φ <u>´NOT´</u> · ´¬´ (negation)								
V		• p is a basic statement								
Var. d a stately immediately and the last an	* * * * * * * * * * * * * * * * * * *	e.g. $p = 'It$ is raining.' then $\neg p = 'It$ is not raining.'								
Manager everette and de and de and		<u>. р</u>	<u> </u>	ין ארך						
V 00033300 PP PP PP VI 1.11+1.11+1.11+1.11+1.11+1.11+1.11+1.1	**************************************	T			Called Trum Tables					
·	***************************************	F	<u>L</u> T							
A STATE OF THE STA				the same						
ran and a community of the same and a constant o	@ (AND	<u>': • '\'</u>	_		· · · · · · · · · · · · · · · · · · ·					
=b=e=eArririna araija(jiii;iiii)yyesisariine esista(siyiiii)yyesisyerie					'PΛ9'≡'P and 9'					
WWW.pipe.pa.nor.ce.emistan.pippippippippippippippippippippippippip			1	£ .	ndependent of each other					
**************************************		• ρ	1							
ermoningappe e errerere amméniquis pe present en amménique en est en amménique en est en amménique en est en a	***************************************	····	T   F							
***************************************		····	$\frac{\Gamma}{\Gamma}$	F						
19 st and delensationary or a series of a series delensation and a series of a series and a seri		·	F							
em en	ക'an'		***************************************	-1-1						
	<u> </u>	· EXClu	a   a	or'/La	tin aut'					
namentare e conservations.	er en det e en en entre franzische en		T	F	Johnson's notation : → We <u>DON'T</u> use it.					
r terre provide et et et ette ministra provide et	umakah fitu kelimilari da kerancu da da membangkan kepilipan da ke	T :	F	T	College Colleg					
	eccentral committee de la comm		T	Т	'either p or g. but not both'					
t termininterproporties et standa (p.p.p.) gregor te environ not et standard ((2) (2) (2) ), en nieste van	emineral control of the second	water of the control	F	F	WAL TIOL DVIII					
		· ¼′ (d	isiunc	tian)						
***************************************		<ul><li>· Λ΄ (disjunction)</li><li>· Inclusive ´or '/ Latin ´vel'</li></ul>								
r i a ariin ah annis sha a a a annis a r an an annis an	\$\\	ρ	i	ρ۷g						
Decession and the second and the sec		T.	T	T	'either p or g					
MANGELER EINE FEILER EINE FEILER EINE FEILER EINE GEREICH EINE GEREICH WERT AUS GEREICH WERT AUS GEREICH WERT EINE GEREICH WERT EIN EINE GEREICH WERT EINE E	oontended Allentation of the second of t	distance +	F	T	possibly both'					
<u> </u>		F	T	T						
· · · · · · · · · · · · · · · · · · ·	······································	mare i irrerovi urverskih krijevoj, jelom vo	F	F						
	<u> </u>	<u> FS' :•'=</u>	<b>⇒</b> ′ (	implicati	on)					
00000,6128-0-0,6				-						

and the second s	. ′p⇒q	' <b>E</b> ' j	f P, the	en <u>g '</u>	
	<u>.</u> P	,	P≑q		
unio de la recessión de completa per de la contra del la contr	(Scanner and American resignation) (Scanner and American	τ	l T		
		F	F		
		Τ	T		
		F	l T		amaa oo ahaa ahaa ahaa ahaa ahaa ahaa ah
	• Explan	ation			
	Р		(-1p) V g		
anne anne (m. 1864). Ann	annoceanaeus (Anadam cara rasarus) did a ministratura di sistema (sistema di sistema di	T	. T		graph and graph garden graph by the term and the armonic
		F	F	F	
engeng a generaliya dagara a sa a sa a sa anan ya gamanii bada ar a mar a a sa anan ar anan ar anan ar anan ar	and the second s	Т	T		and a a a comment of the second of the secon
		F	T	T	
en al 1, e a a a minimission de marco a marco a en empleo de marco de el a el el el en el entre el entre el en	So 7	p V q	has the	same truth table as P⇒9	e de la 1812 de 1813 de 1818 d
	P	•	(¬p) V p		and the second s
9,9,9,9,000,000,000,000,000,000,000,000		F	Т		
	F	Т	T		را مساور دو در دار و در در ۱۰ در و السناسيسي
_{recomm} ents of the section of the s	p⇒g	me	ans rauah	nly if p happens then g happens.	
and the second s				s always true,	
gang ya minanga yang gaggat kerendaha terunan menendagan dalam ana dalam menen				rue as long as 9 is true OR P is true.	
namente e comunicaçõe que mente e e a contrata de animações (se e fenerário e a comunica admitistra e e	Sau two statement			ent(´ミ') when they have the same truth table.	والمعاولة والمراجعة والمواجعة والمعادلة والمعادلة والمعادلة والمعادلة والمعادلة والمعادلة والمعادلة والمعادلة
2.2					and the second s
magang balangga at a yamunan a galigapid dalah da a a da masanamaninka andaramin k	dy5 ≡ 5yd·(I)			<u>commutativity</u>	
enemone (the second	.PVg = gVp				go 1. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2.
gagander mit en de se se unimpe). Se miner en el el el el se se se el	(I). p∧(g∧r) = (p∧	9) Ar	<b>!</b>	Lassociativity	
	• p V ( q v r ) = (p v g			J.,	
<u> </u>	(II) = (7VE) Aq · (II)		(pAr)		
	equivalent to			+pr distributivity	
aries, a e signi sumiti (më 1 6 6 6 6 7 1 °C s. 15 7 marie a pari    limate de aries 6 11 11 sie ani an anna m	. pV(qAr) = (pV	) N(P	pVr)		and the second s
and the second s	(1V) · P≡PAP			\ idempotent	
_{attention} ( ( of a dust	• p = pVp				
nga gang ang ang ang ang ang ang ang ang	Proof of PV(9)	\r) =	) N ( <u>p</u> Vq)	pvr) ;	many polyment are as a secure of the contract of the second of the secon
a a a a a a summitte de la come e a a vene a aquesti fina faste de en entre e e e acenta vene finance e e e a			rannik kafe din se ka 1 sa mengela kan kan ka 1 sa mengela kan ka 1 sa mengela kan ka 1 sa mengela kan ka 1 sa		
					12 A 2 A 2 A 2 A 2 A 2 A 2 A 2 A 2 A 2 A
માં સહિત કે જ જ જ જ તે કે	1 mm m m m m m m m m m m m m m m m m m				

					PN ₄			
		i 1		<del>_</del>			<	
-,>,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	Р	9	r	PV(g/n) =	(pvg) \(pvn)	PV9	ρVr	gAr
	and the second s	<b>T</b>	τ	T	T	T	T	7
		, r	F	F	Ţ	Т	T	F
·harhana)	**************************************	E.	Т	T		Т	T	F
	den de la		F	ļ F	T T	T	Τ	F
	www.wieleberlinum.www.kgllpeberlinum.eeerie		· T _	T	T	Τ	T	7
	·····E	,	F	F	F	Τ	F	F
	elsfellemererumetssammanubbesenmusumssammanubbssamman	*1=1atrev** **********************************	Т	F	F	F	Т	F
	en e	····	<u>ا</u> ج	F	F	F	F	F
	some!							

De Morgan's Laws

How does 'negation' behave?

 $\neg (p \land g) \equiv (\neg p) \lor (\neg g)$ 

duality

7 (pVg) = (¬p) (¬g)

					Y				
	Proof:	P	9	PN9	7(PAg)=	(¬p)V(¬g)		79	
		-T	Τ	Т	F	F	F	F	
~	and the figure are a construction of the state of the sta		F	F	T	T	F	Т	
		F	Τ	F	) T	T	Τ	F	
_	mad in none were the annual point and none of the contract and annual points and annual points and annual point		F	F	Τ	T	T*	Т	
-		20					···:-:::::::::::::::::::::::::::::::::		,

same!

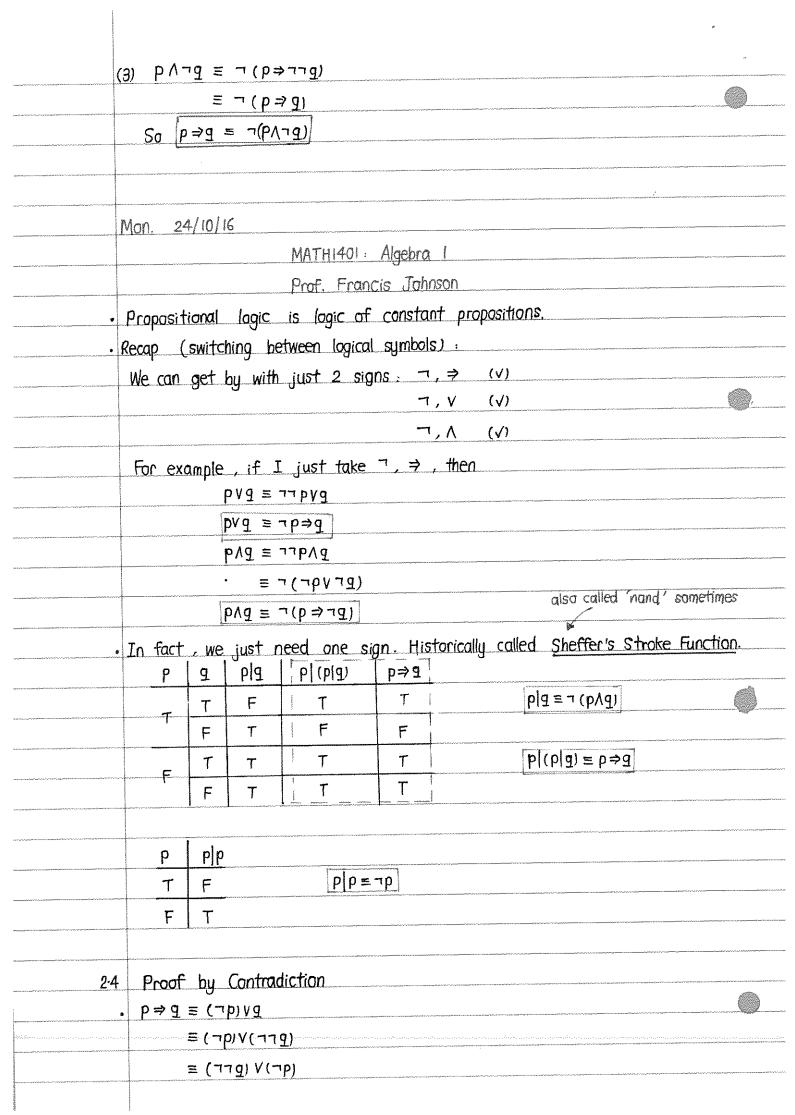
	T	15	( Inner many property and			٠.
P	9	ר(pVg)	(¬p)Λ(¬ <u>q</u> )	ľt.	79	
······	T	F	F	F	F	_
	F	F	F	F.	Т	****
F	T	F	F	Т	F	•••
er new e e e e e e e e e e e e e e e e e e	F	T	T	T	Т	_

same!

2:3 - How many signs do we need? Switching between logical symbols

$$(1) p \Rightarrow q \equiv (\pi p) Vq$$

$$(2) \neg (p \land q) \equiv (\neg p) \lor (\neg q)$$



 $p \Rightarrow g = (\neg g) \Rightarrow (\neg p)$  | 4 principle of proof by contradiction •  $\sqrt{(\neg 9)} \Rightarrow (\neg P)$  is the contrapositive of  $P \Rightarrow 9$ and they are equivalent (have the same truth table)  $\checkmark q \Rightarrow P$  is called converse of  $P \Rightarrow Q$  $p \Leftarrow q \neq q \Leftarrow p$ • Def.  $p \Leftrightarrow q = (p \Rightarrow q) \land (q \Rightarrow p)$   $\Leftrightarrow' = 'iff'$ • p,g,r,s ... are basic propositions that can be T or F. However, pAg ⇒r is called a <u>composite</u> proposition. <u>Def.</u> Composite propositions can be either (I) always T tautology (II) sometimes T, sometimes F contingent (II) always F contradiction eq. (I)  $(P \Rightarrow Q) \Rightarrow (\neg Q \Rightarrow \neg P)$  tautology (I) p∧g⇒r contingent since if P,9 are F, r is T, true if p, g are T, r is F, false (皿) pハ(コp) contradiction Note: A contradiction is the same as the negation of a tautology. e.g.  $PV(\neg p)$  is always T  $\neg (pV\neg p) \equiv \neg p \land \neg p$ = p/17p is always F 2.5 Valid Arguments e.g. a) If it is cold or raining , we shall stay inside. b) We are outside and it is not cold. c) Therefore, it is not raining. √ Is this argument valid? v Let P be 'it is cold' 9 be 'it is raining' r be 'We are inside', then √ a) is pvg ⇒r b) is (7r) 1(7p) The whole argument is then (pvg ⇒r) 1(¬r)1(¬p) ⇒ ¬g LHS

	√ Calcul	ate -	truth	table.				····	und gegebeben kantage proposasi sere erenes se kom er kom erene kom er kontage per erene erene erene erene kontage.	673+0+0+04941
W/AC	Р	р	r		(¬r)\(¬p)	LHS	-g	LHS ⇒ ¬9		
			Т	T	F	F	F	Т		
~,~;;;;	general mestimates for terms promote to particular to the terminal to the mestimate to the mestimate to the me		F	F		F	F	T		
(****** <u>*</u>	<b>1</b>	_	Т	Т	F	F	†	Т		100001110
dollare.		F	F	F	F	F	Т	T		
	-		T	T	F	F	F	T	Commission concrete playwhere Advised Anniel (IEEE/ANNIA AUTOSymphonys) are no newwent Africk for Early (IEEE/ANNIA AUTOSYMPHONYS)	NN:14/14.0m
þ.		· · · · · · · · · · · · · · · · · · ·	F	F	T	F	F	T.		
	<b>F</b>		Т	T	F	F	Т	T	en mikalomist damining kongressis engamma mikandas nomind ki mind kina nipind sesamana.	walamiani
	**************************************	***************************************	F	T	Т	Т	Т	Т		
cerres	e.g(2)	Language and and analysis of the second	<b>1</b>		Les sembles transfer de la president de marient de 1933 de la president de 1933 de 1933 de la president de 1933 de la president de 1933 de la president de 1933 de 193	engenn mynnes kenne	.1	Land Control of Contro	on and the second secon	*******
	· <u>e.y.</u>		mana aran di karan da karan d	en la mercela de seguina de CITTI La describito de marte de la mercela de la mercela de la mercela de la merce			***************************************	oversione-penningraphic percent reservoir 1990	erregennen menemen kontrologische von der der der der verbet der	

a) p∧g⇒r

b) 7 r 1 7 p

The w	hole	stater	nent is	(p/d ⇒ L) V(JL) V(Jb) ⇒ Jd				
ρ				(¬r)^(¬p)	LHS	79	TH2⇒14	
		T	T	F	F	F	T	
geografie og engenerer er en en en en en en et de de de de de de de 2000 for et en en en en	istoritame mineralistantist	F	F	f į	F	F	.00016	
<u>, , , , , , , , , , , , , , , , , , , </u>	E	Т	Т	F	F	T	T	
agig 27,77 ng manang na arang mananan anipulit a mat min	n kananan merupakan kelikika	F	T.	F.	F	Т	T	
		Т	Ţ	, F	F	F	T	
F	11 1 marity day for the man to a man and	F	T	Т	T	F	0	
		T	T	F	F	T	Τ	
	y a analyse of The measured	F	T	Т	T	T		

This argument is not valid (because at least one line gives a false)

Fri. 28/10/16

MATHI401: Algebra I

Prof. Johnson

· Textbook: Notes an Logics by PT Johnstone

We have done logic of constant propositions  $\neg$  ,  $\wedge$  ,  $\vee$  ,  $\Rightarrow$ 

2.6 Lagic of Variable Propositions (Predicate Logic)

```
• P(x) where x varies.
        \checkmarkSuppose x \in \{0,1\}, P(x) = `x^2 > 0'
         In this case, P(0) is false, but P(1) is true.
        ✓ Suppose x \in \{0,1,2\}, p(x) = (x^2 > 0)
         P(0) F P(1) T P(2) T
        ✓ Suppose x \in \{1,2\}, P(x) = x^2 > 0
                            P(2) T
    . The values over which x is allowed to vary is called the domain (of
      discussion). Denoted by D
        - Then P(x) is a statement about objects in {\mathcal D}
        - We are allowed to use \neg, \wedge, \vee, \Rightarrow
          Plus 2 new signs.
        √'∀' Universal : Quantifier (= for all')
          (\forall x) P(x) means that for any value of x in \mathcal{L}, P(x) is true.
                Existential Quantifier (= 'these exists')
          (\exists x)P(x) means that for at least one x in \mathcal{D}, P(x) is true.
    · EXAMPLE:
       V D= {0,1}
         Clearly, I can form P(0) and I can form P(1).
          (\forall x)P(x), in this case, means P(0) \land P(1).
          (\exists x) P(x), in this case, means P(0) \vee P(1).
                                                                      The Convention (for algebra)
       V D = {0,1,2}
                                                                      N= (0,1,2,...)
         (\forall x) P(x), in this case, means P(0) \land P(1) \land P(2)
         (\exists x) P(x), in this case, means P(0) \vee P(1) \vee P(2)
       ✓ Suppose \mathcal{D} = \mathbb{N},
         (\forall x) P(x) \sim P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge \cdots \wedge P(n) \wedge P(n+1) \wedge \cdots
                                                                  formulae of ∞ length
         (\exists x) P(x) \sim P(0) \vee P(1) \vee P(2) \vee P(3) \vee \dots \vee P(n) \vee P(n+1) \vee \dots
        We introduce 'V', '∃' predicator to avoid formulae of ∞ length.
       How to negate a quantifier?
2.6.1
     Start with D = \{0,1\}
        (\forall x)P(x) = P(0) \wedge P(1)
      \neg (\forall x) P(x) = \neg (P(0) \land P(1))
```

= 7P(0) V 7P(1)  $= (3x) \neg P(x)$  $\sqrt{D} = \{0,1,2\}$  $(\forall x) P(x) = P(0) \wedge P(1) \wedge P(2)$  $\neg (\forall x) P(x) = \neg P(0) \vee \neg P(1) \vee \neg P(2)$  $= (3x) \neg P(x)$ · First rule of negation  $\neg (\forall x) P(x) = (\exists x) \neg P(x)$ · Second rule of negation  $\nabla(x) = (x) = (x)$ e.g.  $(\exists x) P(x) = P(0) \vee P(1)$  $\neg (\exists x) P(x) = \neg (P(0) \lor P(1))$ = 7P(0) A 7P(1)  $\neg (\exists x) P(x) = (\forall x) \neg P(x)$ Interchange of Order of Quantifiers The order in which quantifiers come is extremely important. EXAMPLE: Suppose  $\mathcal{D} = \{0,1\}$ ,  $P(x,y) = 'x \neq y'$ Compare  $(\forall x)(\exists y)P(x,y)$  TRUE and  $(\exists y)(\forall x)P(x,y) \leftarrow FALSE$  $\sqrt{(\exists y) P(x,y)} = P(x,0) \sqrt{P(x,1)}$  $(\forall x)(\exists y) P(x,y) = (\forall x) [P(x,0) \vee P(x,1)]$  $= [P(0,0) \vee P(0,1)] \wedge [P(1,0) \vee P(1,1)]$  $= (F \vee T) \wedge (T \vee F)$  $= T \Lambda T$ = T $\sqrt{(\forall x)} P(x,y) = P(0,y) \wedge P(1,y)$  $(34)(4x) P(x,y) = [P(0,0) \wedge P(1,0)] \vee [P(0,1) \wedge P(1,1)]$  $= [F_{\Lambda} T] \vee [T_{\Lambda} F]$ = FVF. = F Axiom: (34)(4x) = (4x)(34)(6x)However, two quantifiers of same type commute

i.e. 
$$(\forall x) (\forall y) P(x, y) = (\forall y) (\forall x) P(x, y)$$
  
 $(\forall x) P(x \in Y) = (\forall x) P(x \in Y)$ 

#### MATH1201 Propositional Logic

Basic Propositions

1. Commutativity: P19 = 91P

PV9 = gvp

2. Associativity: PA(gAr) = (pAg)Ar

PV(QVr) = (PVQ)Vr

3. Distributivity:  $P \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ 

 $PV(q\Lambda r) \equiv (pVq) \Lambda(pVr)$ 

4. Idempotent : P≡P∧P

P= PVP

De Margan Laws

1. duality:  $\neg (p \land q) = (\neg p) \lor (\neg q)$ 

 $\neg (pr) \land (qr) \equiv (pvq) \neg$ 

2. PV9 = 7p ⇒9

 $P \wedge Q \equiv \neg (P \Rightarrow \neg Q)$ 

(Pr Nq) = P € q

Sheffers Stroke Function

 $P|q \equiv \neg (P \land q)$ 

 $p \neq q \equiv (p|q)|q$ 

p|p = 7p

Proof by Contradiction / iff

Contrapositive:  $P \Rightarrow q \equiv (\neg q) \Rightarrow (\neg p)$ 

 $p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$ 

Logic of Variable Propositions

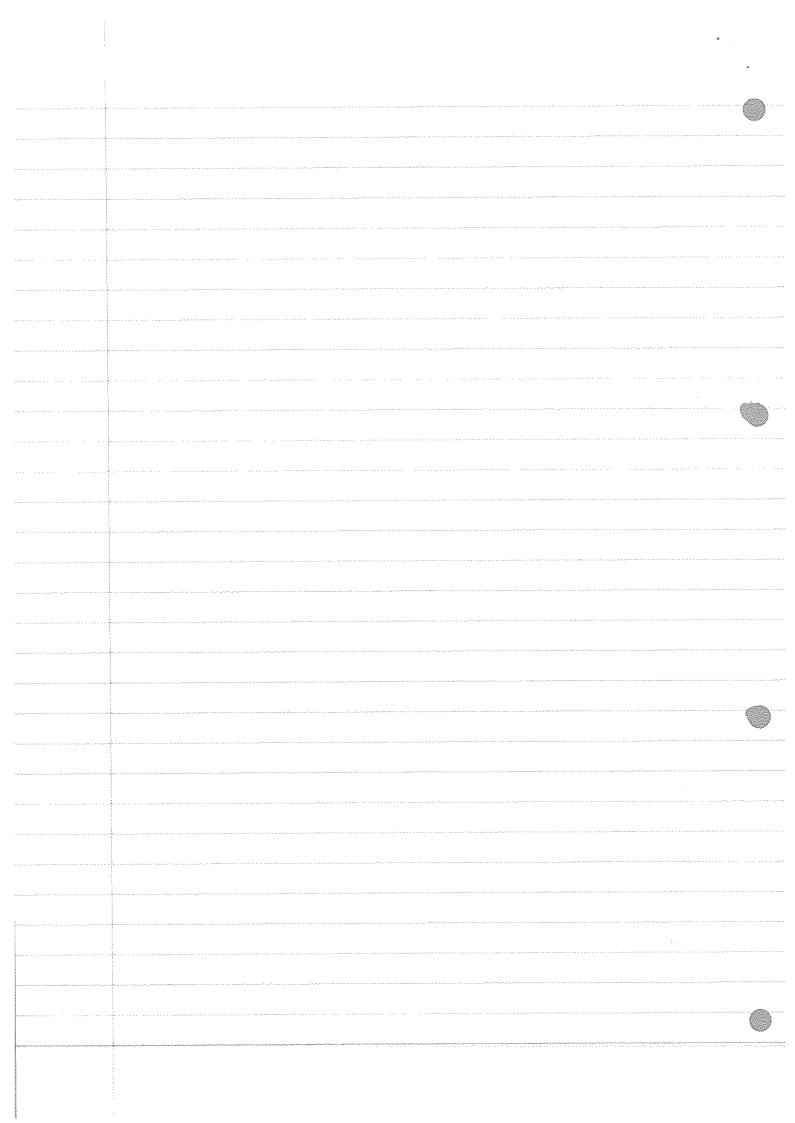
 $\neg (\forall x) P(x) = (\exists x) \neg P(x)$ 

 $(x)q^{\Gamma}(x\forall x) = (x)q(xE)$ 

 $(\exists y)(\forall x) P(x, y) \Rightarrow (\forall x)(\exists y) P(x, y)$ 

 $(\forall x)(\forall y)P(x,y) \equiv (\forall y)(\forall x)P(x,y)$ 

 $(y,x)^{2}(xE)(yE) = (y,x)^{2}(yE)(xE)$ 



## MATHI201 Propositional Lagic

#### Basic Propositions

p/g ≡g/p 1. commutativity:

qve = evq

 $\Lambda(D\Lambda q) = (\Lambda Q) \Lambda q$ 2. associativity :

nv(pvq) = (nvp)vr

 $(1 \wedge 4) \vee (1 \wedge 4) = (1 \wedge 4) \vee (1 \wedge 4)$ 3 distributivity :

PV(QNr) = (pVQ)∧(pVr)

P≡ pAp 4. idempotent :

 $p \equiv p \vee p$ 

#### De Margan Laws

 $I. \neg (p \land q) \equiv (\neg p) \lor (\neg q)$ 

duality

 $\neg (pr) \land (qr) \equiv (pVq) r$ 

2. pVg = ¬p ⇒ g

 $(Pr \Leftarrow q) r \equiv P \wedge q$ 

 $p \Rightarrow q \equiv \neg (p \land \neg q)$ 

Sheffer's Stroke Function

 $p|q \equiv \neg (p \land q)$ 

 $P(P|q) \equiv P \Rightarrow q$ 

p|p = 7p

Proof by Contradiction / iff

 $p \Rightarrow q = (\neg q) \Rightarrow (\neg p)$  contrapositive

(9¢P) V(E¢d) = B⇔d

Lagic of Variable Propositions

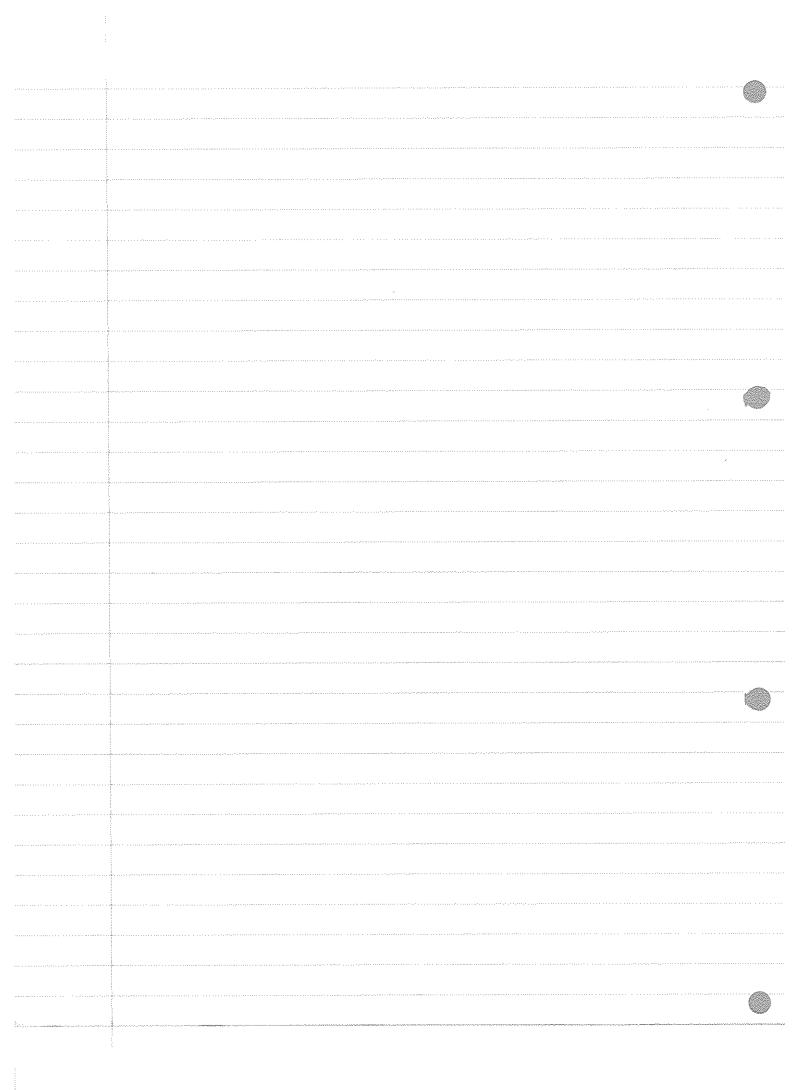
 $\neg (\forall x) P(x) = (\exists x) \neg P(x)$ 

 $\neg (\exists x) P(x) = (\forall x) \neg P(x)$ 

 $(\exists y)(\forall x)P \Rightarrow (\forall x)(\exists y)P$ 

 $(\forall x)(\forall y) P = (\forall y)(\forall x) P$ 

q(xE)(yE) = q(yE)(xE)



nementation for the first of the second seco	Fri: 28/10/16 (continued)								
emerojajajami jarkitusej opininine ereminek jasuus suses	MATH1401: Algebra L								
maioris (Nellare 13) Arabita (Arabita (	Prof. Johnson								
Chapter 3	§ Set Theory §								
1.6	Sets								
attributely graphic grap of the structure and the structure of the structu	$f(x) : f(x) \in A' = f(x)$ is a member of $A'$ belongs to $f(x)$ is a member of								
To the terminal of an all and a substitute of the substitute of th	✓ Every set starts & ends with a <u>curly bracket</u> .								
erretire en servició est allanta framedició de la propez fore en care est estado de la cale	e.g. [0,1,2,3,5,7] = [7,5,2,1,3,0]								
admining a filosom graffe state against a state against the design for a second state of the design for a second	□ In a set, the order in which elements are written is NOT significant.								
eenses vandees vandeerings (1950–1955) van de van van de versche van de versche van de versche van de versche v	Note: To stress order, use round brackets.								
industrial production and included in the contract of the cont	eg. $(0,1,2) \neq (2,1,0)$								
	$\oslash$ If A and B are sets, then $A=B$ iff $x \in A \Leftrightarrow x \in B$								
eneneneezemakilistiksepplikseppliksepitationistissessationistissessationistissessationistissessationistissessa	i.e. a set is determined by its members and nothing else.								
	Two sets are the same if all of their elements are the same.								
andelega et eminerat i este de este de descuelha e glorence e e e a que de de descela	'c':   * Do not confuse '∈' with '⊆'.  **								
	$\checkmark$ <u>Def.</u> $B \subseteq A$ iff $x \in B \Rightarrow x \in A$ ,								
alleger i n , c n , c n , c n , c n , c n , c n , c n , c n , c n , c n , c n , c n , c n , c n , c n , c n , c	so $A=8$ iff $(A \subset B) \land (B \subset A)$								
mandarum samura da manda da m	EXAMPLE: $A = \{0,1,\{0,1\},\{0,2\},\{2\}\}$								
erregering of the Anthony group and a second of the Anthony group (a) followed by the Anthony group (a) foll	Question. 0∈A T								
· · · · · · · · · · · · · · · · · · ·									
	$\{0\} \in A$ F $\{0\}$ is not an element of A $2 \in A$ F								
	[2] EA T (2) is an element of A								
nament enstance was colour enstance place	$\{2\} \subset A$ F because $\{2\} \subset A$ iff $2 \in A$								
	but $2 \notin A \Rightarrow \{2\} \notin A$								
and the second s	$\{2\}\subset A \qquad T \longleftarrow \text{because } \{2\}\in A \Rightarrow \{2\}\subseteq A$								
	Note, {0,1,1} = {0,1}								
*	To list elements of a set is quite naive.								
	More usually, we define sets by means of properties possessed by elements.								
	eg. $\{x \mid x \text{ is a TFL bus}\}$								
and the state of t	✓ In general , { typical   defined }								
	element property)								
VVV e profession de de de desamb									

e.g.  $\{x \in \mathbb{R} \mid x^2 > 3\}$  $\vee A = \{x \mid P_A(x)\}$  where Pa is the defining property of A. Note.  $\left\{\infty \in \mathbb{R} \mid \infty^2 < 0\right\}$  is an example of an empty set. Notation: Ø So if you're defining sets by properties, you will inevitably get an empty set. Nate: ØCA VA  $\vee$  |A| = number of elements in A Note:  $X = \{\emptyset\}$  is non-empty, since  $\emptyset \in X$ so  $0 = |\emptyset| \Rightarrow 0$  is natural 3.2 Predicates of Sets  $A = \{x | P_A(x)\}$  $B = \{x \mid P_{\theta}(x)\}$  $\sqrt{\text{Union}} \cdot \left[ A \cup B = \left\{ x \middle| P_A(x) \cdot P_B(x) \right\} \right]$ i.e. Auß consists of those elements  $P_{AUB}(x) = P_A(x) \vee P_B(x)$ that are in A or in 8 ✓ Intersection:  $A \cap B = \{x \mid P_A(x) \land P_B(x)\}$  $A-B = \{x \mid P_A(x) \land \neg P_B(x)\}$  i.e. A-B consists of those elements ✓ Difference:  $P_{A-B} = P_A(x) \wedge \neg P_B(x)$  which are in A but not in B. Note: Union / Intersection / Difference of sets = another set. Venn Diagram A (black) B(red) C (green) □ = A∩B∩C ()≡A∩B C-AUB Note: disadvantage = cannot represent > 4 sets because 4 sets requires 3D. · Direct Product of Sets  $\sqrt{\text{Formally}}$ , (a,b) = (a',b') iff a=a' AND b=b' called ordered pair'. so (a,b) = (b,a) iff a=b $\sqrt{A = \{x \mid P_A(x)\}}$  $B = \{x | P_{\theta}(x)\}$ 1st element from 1st set (A)  $A \times B = \{(x,y) \mid P_A(x) \wedge P_B(y)\}$ 2nd element from 2nd set (8) In general,  $[8 \times A \neq A \times B]$ e.g.  $A = \{0, 1, 2\}$ ,  $B = \{0, 1, 2, 3\}$ 

```
(0,1) - - - - - (1,1) - - - - (2,1)
            (0,0) ---- (1,0) ---- (2,0)
        \sqrt{|A|} = card of A = number of elements in A
           |A \times B| = |A| |B|
   · Which of the following is a function?
       f(x) = x + 1
       g(x) = \frac{1}{x+1}
                        These are formulae
       h(x) = \sqrt{x+1}
     Man. 31/10/16
                                MATHI201: Algebra 1
                                  Prof. Johnson
 3.3 Mappings (Functions)
3.3.1 What are mappings?
   · Informal Def
          Let A,B be sets.
          By a mapping / function
                                      f \colon A \to B
\text{domain of } f
\text{codomain of } f
          We mean a rule which assigns to each a \in A a single element f(a) \in B.
     EXAMPLE:
             A = B = \mathbb{Q} (rationals)
          ✓ could take as my rule f(a) = a+1
          ✓ whereas for g(a) = \overline{a+1} , this isn't going to work
          ✓ However, if I take A = Q - \{1\}, means A contains all rationals but 1
                   then 9: Q - \{i\} \rightarrow Q
                     g(a) = \frac{1}{a+1} is a mapping.
          \sqrt{\text{Take }} A = \left[x \in \mathbb{R} \mid x > 1\right] then
                    h: A \rightarrow \mathbb{R}
```

 $h(x) = +\sqrt{x+1}$ Note. h. This example is related to Riemann Surface. Camposition of mappings: Let A,B,C be sets. g. B→C  $f: A \rightarrow B$ Therefore,  $9 \cdot f \cdot A \rightarrow C$  is the mapping  $(g \circ f)(a) = g(f(a))$  Composition V EXAMPLE:  $g: \mathbb{R} \longrightarrow \mathbb{R}$  $f: \mathbb{R} \longrightarrow \mathbb{R}$  $f(x) = x^2 + 1$  $g(x) = \cos x$  $(9^{\circ}f)(x) = \cos(x^2+1)$ where  $(f \circ g)(x) = \cos^2 x + 1$ Are they the same?  $(9 \circ f)(0) = \cos 1 < 1$  $(f \circ g)(0) = 2$ fog # 9of BA 2 possible points define codomain, This isn't a mapping This is a mapping √ Composition is associative.  $A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{h} D$ g of = A → C hog : B→D Then  $(h \circ g) \circ f : A \rightarrow D$  $ho(gof) = A \rightarrow D$ Preposition: (ho(gof) = (hog)of Proof: Let a ∈ A  $(h \circ (g \circ f))(a) = h(g(f(a))) = h(g(f(a)))$  $((h \circ g) \circ f)(a) = (h \circ g)(f(a)) = h(g'(f(a)))$ **QED** · Identity mapping

Ida: A → A

Ida' = 'Ia' = 'Id' Ida(a) = a VaEA

Invertible mapping

Suppose A,B are sets and  $f: A \rightarrow B$  is the mapping,

We say that f is invertible where there exists mapping  $9: 8 \rightarrow A$ 

 $A \xrightarrow{f} B \xrightarrow{g} A \xrightarrow{f} B$ 

such that  $9 \circ f = I dA$ and fog = **I**ds

If this is the case, we say that 9 is inverse mapping to f.

Notation:  $9 = f^{-1}$ 

Note:  $f' \neq \frac{1}{f}$ 

√ EXAMPLE ①,

f:R>R

f(x) = 2x+1

Q. Is f invertible? If so, what is f'?

 $g(x) = \frac{x-1}{2}$ 

 $(f \circ g)(x) = f(\frac{x-1}{2})$ 

 $(9^{\circ}f)(x) = \frac{(2x+1)-1}{2}$ 

 $=2\left(\frac{x-1}{2}\right)+1$ 

 $=\chi$ 

= X

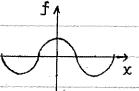
$$\Rightarrow f^{-1}(x) = \frac{x-1}{2}$$

### √ EXAMPLE ②:

 $f: \mathbb{R} \to \mathbb{R}$ 

 $f(x) = \cos x$ 

In this case, f is not invertible.



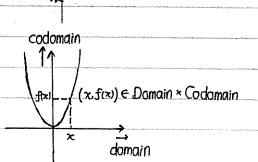
We'll deal with this formally very soon.

· Formal Def.

√ Graph of a mapping.

 $f: \mathbb{R} \to \mathbb{R}$ 

 $f(x) = x^2$ 



4 possible points

	✓ Formal Def.	listane estential politica del reducir es primi
	Let A,B be sets.	alum 1 mily 2011 o 2 chaire 2
	By a mapping $f: A \rightarrow B$ , I mean a subset	allinga pagagan sepiga paramaga i sa vara da ka vara da ka sa da sa d
sommers of the second	f ⊂ A×B	mmilliotationisty transport transport transport and an american angular and an exercise and an
syrramaniminin määritteittivivettuvin turtuvettuvi	with the following properties:	miller fragelædere folksiske frædenski kroossemen serenden månsem en semme
po especial communicación de constitución de constitución de constitución de constitución de constitución de c	D YOEA, BOOK S.t. (a, b) & f	e Com v may my pro-janjanjanjanjanjanjanjanjanjanjanjanjanj
menos compressoramo (esse son sensora son sensora son	(I then write b=f(a)) → It is defined.	poulges of the financial and an announced in principle of the set of
	2) $\forall a \in A$ , $(a,b) \in f$ and $(a,b') \in f$	enggyngangag pengagan pennenni kedian dalah dalah memol kehilikik
	⇒ b=b' There is only one point (one-	to-one)
	$\sqrt{f:A o B}$	mmenessed 1335/eds Stevender deledate venes triste vider men
	Suppose I have an inverse $f^{-1}: \theta \rightarrow A$ .	o an anticonocione de come antico de la come antico de la come antico de la come antico de la come antico de l
	f-'c8xA	and a name of a name of the first of a name of a state of a name of a state of a name of a state of a state of
****	I)' YBEB, BaeA s.t. baef-1	
1667 t 1 (1644 t ti th dinner recommende telebrahe de de casa, ed en decessaries de casa, est est est est est	I) $\forall b \in B, (b, a) \in f^{-1} \mid (b, a') \in f^{-1} \Rightarrow a = a'$	olde Modes for de see and de see see see somme se approvince modes productive approvince de se approvince de s
at and a find annual result in a find a superior of the Carlo	Question. Given a mapping $f: A \rightarrow B$	omengggenengg general Statistical Statisti
	What do I need to check before I can conclude that f is	invertible?
3:3:2		endigg opposite progresses en en en et semble de entitlemen el estreta i l'illet e
	D Injectivity:	
State et et la	Say that $f: A \rightarrow B$ is injective when given $a, a' \in A$ ,	namen a namentum siguiningan siguisiya siguisiya siguisiya si siguisiya siguisiya siguinin ka siguinin ka sigui
	$f(a) = f(a') \Rightarrow a = a'$ 'one-to-one'	mmengalangikan pagasasi sadalan sagasan pananan kanaman sanan manan sanan man
	✓ EXAMPLE 0:	ossoonaaniin oo filloogia
	$f: \mathcal{U}  ightarrow \mathcal{U}$	
1	f(x) = 2x+1	nemotal kajados (policidad de estado se asementos asementos de estado estado estado estado estado estado estad
	Then f is injective because	
	$f(x) = f(x') \Rightarrow 2x + 1 = 2x' + 1$	hadas kalalakas kakula sada sada saga sana saga saliman sama sa sama mulabada.
	$\chi$ = $\chi'$	
Which the month of the field of	✓ EXAMPLE Ø:	mgeesseeren mind meels televisegelisk stadis (1991) killes kinnin mee'r
	9	ngyngunundanundald bebelaktion tot sold sistem
in was no manuscripting to the first security of the first security and a 1500 first security and a 1500 first	$g(x) = x^2 + 1$	nggapangapananinnan kembank kembakka kambanahabahka sake berke
genes som kritis produkter i den betyr og prominen en ste fortiken het ble	Is 9 injective?	ogeneratives and the second
gaaret ammet i feet til fryst eft van men men et ar a anammen men til 1988 til 1995.	g(1)=2 = g(-1)=2	to the transfer on the second
	BUT 1≠-1 → not injective	
The state of the s		

eseines ( tabalista e e e e e e e e e e e e e e e e e e e	Fri. 04/11/16
	MATHI201: Algebra I
namman ampgant amagaan nammiiri s soomaa angaa,	Prof. Johnson
	② Surjectivity:
grigger var en ger var het verkommunet forbruik vogsen var var große.	We say that f is surjective when
	VbE8, BaeA 6.t. b=f(a)
inn in tra magaint is luminis lipolini is es la la tra la li	<ul> <li>Informally, surjectivity means every element of codomain is hit by something</li> </ul>
enellen i vere ann arampet promonent quant a	in the domain.
ražemnini s v mino s samo s samo s samo sako sako sako sako sako sako sako sak	$\vee$ EXAMPLE: $f: R \rightarrow R$
of fifther of terminal years are sure and an extract of constant o	f(x) = 2x + 1 is surjective.
annut at tannun et et en en et en	Let yer
ramadas arad dadd gafydydydyd y gyf y fyllyngh y gyngaeth y gannau y fannau g	Then, we need to write $y=2x+1$ $x = \frac{y-1}{2}$ $x = 0$ terms of $1$
eteriorius (em em exercisamente de disability de proprogram en pleas	$\frac{x=2}{x \text{ in terms of } y}$ $\therefore \forall y \in \mathbb{R}, \exists x = \frac{y-1}{2} \text{ s.t. } f(x) = y$
gajan na yarri nyarya manakanti dipalimya, yyan	Recall: f: A→B is invertible when
e de la companya de La companya de la co	$(  \exists \text{ mapping } 9: B \rightarrow A \text{ s.t. } 9 \circ f = Id_A \text{ and }$
entergene i i i i i i i i i i i i i i i i i i	fog = Id8
ΦΕΙΒΙΙΤΑΣΙΑΑΙΑΙΑΙΑΙΑΙΑΙΑΙΑΙΑΙΑΙΑΙΑΙΑΙΑΙΑΙΑΙΑ	Prop. Let f:A→8 be invertible, then
annada ja	1) f is injective and
distriction of the second of t	2) f is suggestive $A \stackrel{f}{\rightleftharpoons} B$
**************************************	Proof:
······································	1.) Let $g: B \rightarrow A$ s.t. $g \circ f = IdA$ and $f \circ g = IdB$
radiality o gotina ar anna ar anna an 1905 in an an an an	Suppose $f(a) = f(a')$ ,
and and the felicient and the second and the second annual and the second annual and the second annual annual and the second annual ann	apply 9 to both sides: $9(f(a)) = g(f(a))$
and the second s	$(g \circ f)(a) = (g \circ f)(a')$
annas farann farann san sannt 4 feil ei ei ann f féideacha bei bhíon	Ida(0)
kalan arma mambhal fi ya jamaha ya	Q = Q' QED
ٳڂڞۺۼڎڿڿڿڿٷڞڲڣڎڴڎڞڿۅڿۿڛۿٳۼۿۄڲۿڽۿۄڲۿڽٷڿٷؿڣڽڛ ۼڞڰۼۼؿۼؿۼؿۼؿۼۼۼۼؿۼۼۼۼۼۼۼۼۼۼۼۼۼۼۼۼۼۼۼۼۼۼ	2) Let $b \in B$ , put $a = 9(b)$
terlika firit ristind het til till til firit fire fra statiska fil statiska fil fire til statiska fil fire til	$f(a) = f(g(b)) = (f \cdot g)(b) = Id_B(b) = b$
(st. cred t konkret konkrije provinske material (m. s.)	⇒ f is surjective QED
<u>(3)</u>	Bijectivity
uninterrumonertuneethannaareappaaretuum	A mapping f is said to be bijective when

it is both injective and surjective. · Def. So by the previous example. I've shown that An invertible mapping is bijective. ρ⇒g converse' . In fact, the converse is also true.  $\checkmark$  Recall :  $f: A \rightarrow B$  is a mapping q ep ⇔ fcAxB st. I)  $\forall a \in A$ ,  $\exists b \in B$ ,  $(a,b) \in f$ II) If  $(a,b) \in f$  and  $(a,b') \in f$  $\Rightarrow b=b'$ V So if f CA×B,  $\Leftrightarrow f^{-1} = \{(b,a) \mid (a,b) \in f\}$ f is mapping. f CB XA  $fCA \times B$ I)  $\forall a \in A, \exists b \in B \text{ s.t. } (b, a) \in f^{-1}$ I) YaeA, BbEB st (a,b) ef implies f' is surjective  $\mathbb{I}$ )'  $(b,a) \in f^{-1}$  and  $(b,a) \in f^{-1}$  $\square$   $(a,b) \in f$  and  $(a,b') \in f$ ⇒ b=b' ← implies f-1 is injective  $\Rightarrow b=b'$  $\overline{\mathbb{I}}$  (b, a)  $\in f^{-1}$  and (b, a')  $\in f^{-1}$  $\mathbb{I}$ )  $(a,b) \in f$  and  $(a',b) \in f$ ⇒ a=a'  $\Rightarrow a = a'$ (IV)'  $\forall b \in B$ ,  $\exists a \in A$  s.t.  $(b,a) \in f^{-1}$ IV) Y b ∈ B, ∃a ∈ A s.t. (a.b) ∈ f Note: I), II) are conditions for f to be a mapping. II) is the condition for f to be injective. IV) is the condition for f to be surjective. WHILE II)', IV)' are conditions for  $f^{-1}$  to be a mapping. ✓ So if f satisfies I), II), II) & IV), then f' satisfies I)', II)', II)' & IV)'  $\Rightarrow$   $f^{-1}$  is a mapping and also  $f^{-1}$  is bijective • Th. D Let  $f: A \rightarrow B$  be a mapping, then f is invertible  $\Leftrightarrow$  f is bijective

```
I) If that is the case, then
                       f^{-1} is also a bijective mapping.
   \frac{Prop \quad Let \quad f: A \rightarrow B}{g: B \rightarrow C} \quad mappings
             If f, g are invertible / bijective, then
                   9°f is also invertible/bijective.
     Proof:
              If f, g are invertible with inverses f<sup>-1</sup>, g<sup>-1</sup>,
                           A \underset{f^{-1}}{\longleftarrow} B \underset{g^{-1}}{\longleftarrow} C
             (f^{-1} \circ g^{-1}) \circ (g \circ f) = f^{-1} \circ (g^{-1} \circ g) \circ f
                                 = f - Ide of
                                 = f - of
                                 = Ida
            (g \circ f) \circ (f^{-1} \circ g^{-1}) = g \circ (f \circ f^{-1}) \circ g^{-1}
                                = 90 Id8 0 9-1
                               = g . g -1
                               = Idc
                                                                                                           QED
           Notice that
                             (g \circ f)^{-1} = f^{-1} \circ g^{-1} reversal of order!
3.4 Permutations
  • f: \{1,2,...,n\} \to \{1,2,...,n\} mappings
                            Identity permutation
                                 can be written as (t^2, t^3)
                                can be written as \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}
 · Def. A permutation on n letters is a bijective mapping
    Convention:
```

e.g. 
$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 4 & 1 & 2 \end{pmatrix}$$

means 
$$f(1) = 5$$
  
 $f(2) = 3$   
 $f(3) = 4$   
 $f(4) = 1$ 

$$f(5)=2$$

• Composition of permutation:
$$f = \begin{pmatrix} 1 & 2 & \cdots & n \\ f(1) & f(2) & \cdots & f(n) \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 2 & \cdots & n \\ g(i) & g(2) & \cdots & g(n) \end{pmatrix}$$

e.g. 
$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 4 & 1 & 2 \end{pmatrix}$$

$$9 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 1 & 5 & 2 \end{pmatrix}$$

How to calculate 9°f?

$$\hat{g} \cdot \hat{f} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ \hline 16 & 3 & 4 & 1 & 2 \\ \hline 2 & 1 & 5 & 3 & 4 \end{pmatrix} \hat{f}$$

f come 1st, g comes 2nd Then cross out the middle line (row)

i.e. 
$$g \circ f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 5 & 3 & 4 \end{pmatrix}$$

How to calculate 
$$f^{-1}$$
?
$$f^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 2 & 3 & 1 \end{pmatrix}$$

'upside-down'

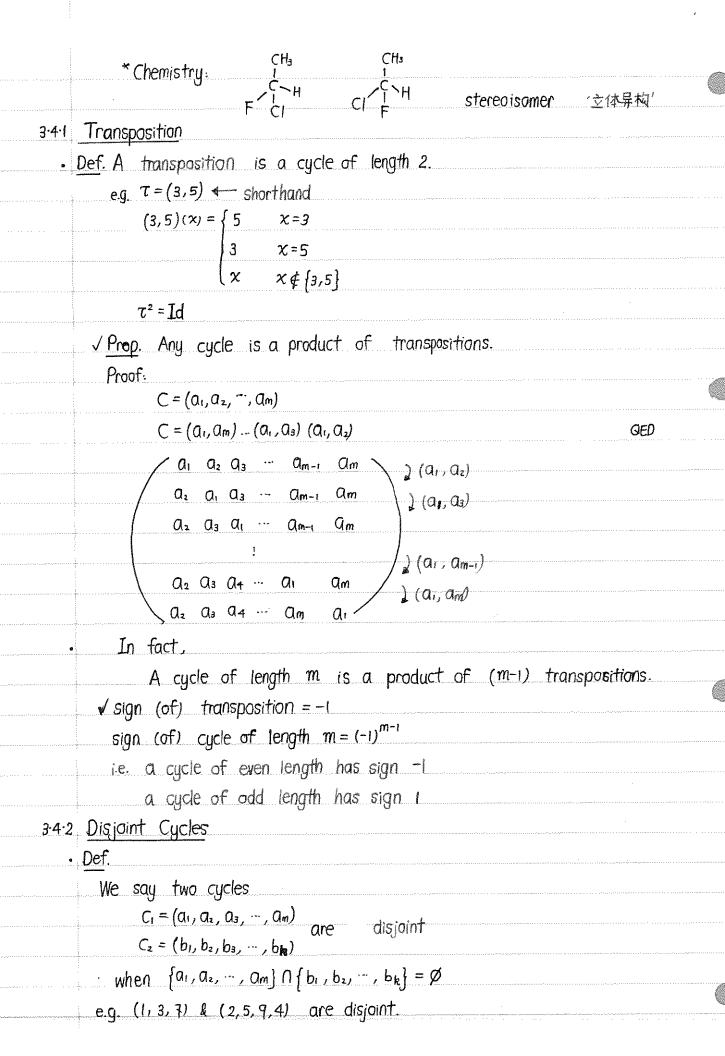
sigma

$$\sigma_n = \{f: \{1, \dots, n\} \rightarrow \{1, \dots, n\}\}\$$
, f is bijective

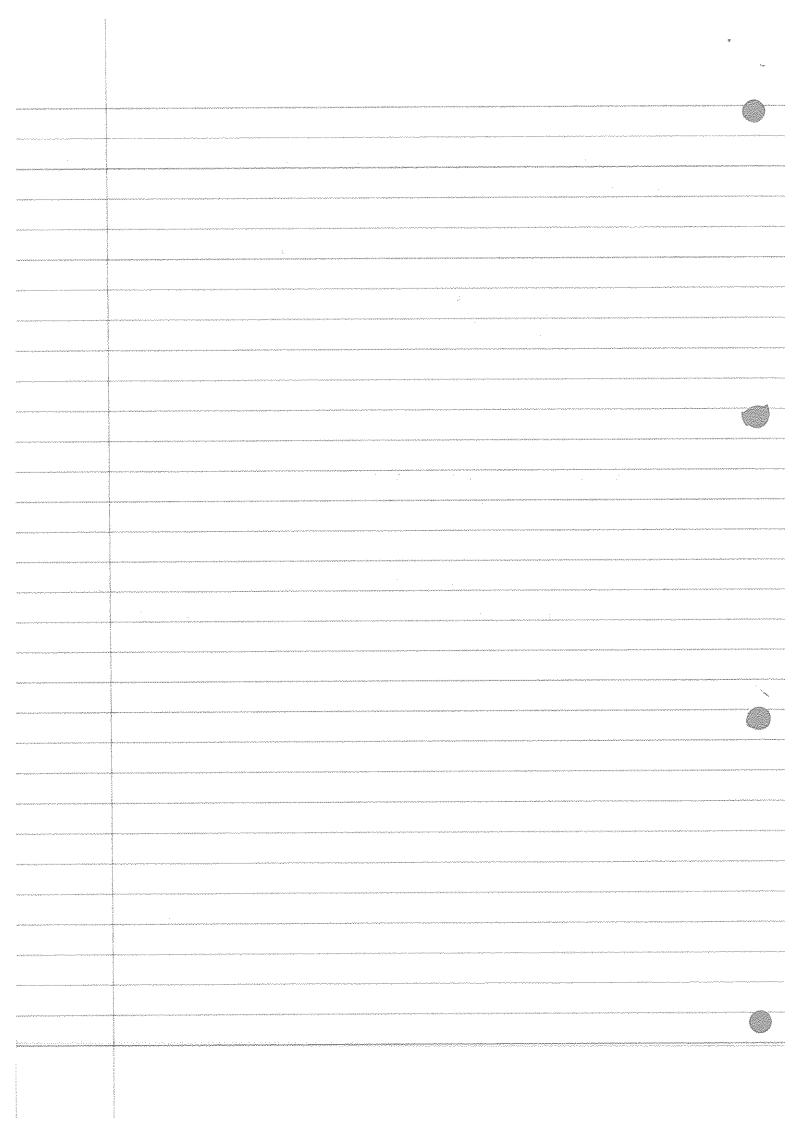
 $\sigma_n$  is the self of permutations on n letters | on |= n!

This is because f(n) has n possibilities

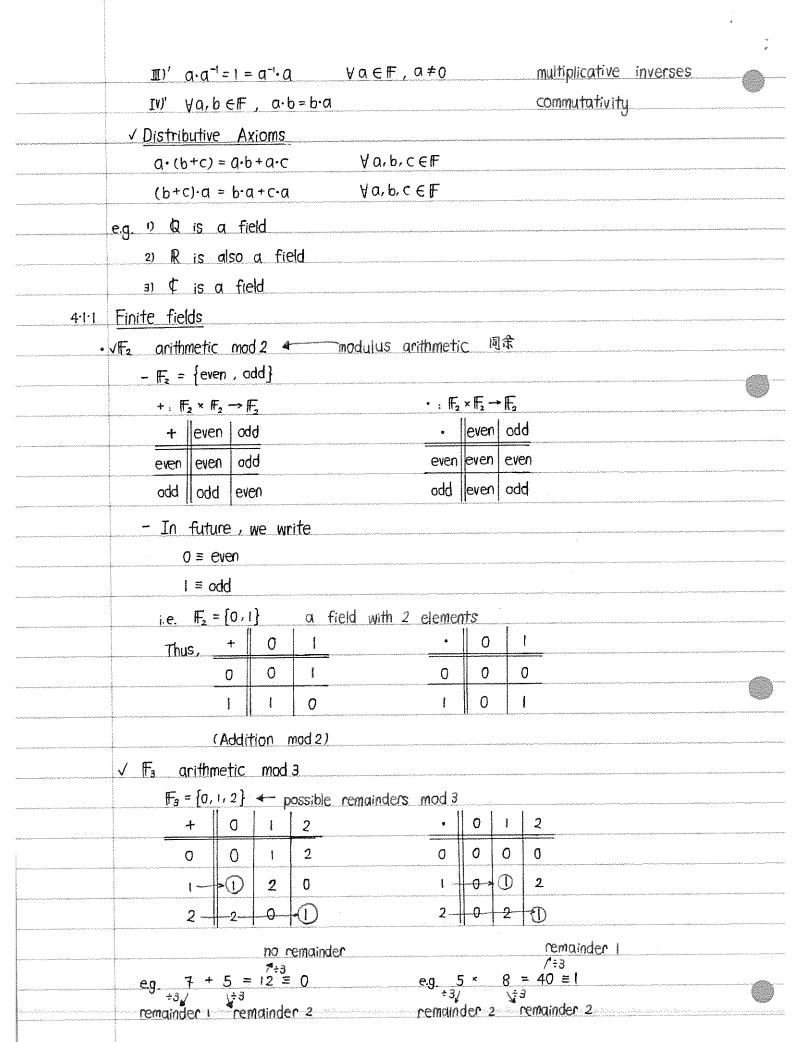
to promote of the state of the	e.q. f(1)=3 ¦ cań be written as ,	gaes to	leave out 2						
	f(3)=5 /12345) Sho	orthand: (1,3,5,4)							
and a construction of the least the least the construction of the least the	f(3)=5 (1 2 3 4 5) Sho f(5)=4 (3 2 5 1 4)		ermen vinder der der de konstellen Eugenbuck (dez 2000 gang de 2000 der de						
menmente transportuni per trata por la construencia de la construencia de la construencia de la construencia d	f(4)=		alla desente de la companya de la c						
	· Formal Def:								
	Let a ₁ , a ₂ ,, a _n ∈ {1,, n}		TOWNERS (SMOOTS (SMOOTS STATE OF STATE						
	By the cycle (a,,az,,am),	temberisten varian varia e e e e e e e e e e e e e e e e e e e	iko katala kumum wana wa wanaza ya katala katala katala kuma wa wa wa wa wa katala kuma katala kuma katala kum						
The second secon	$(a_{i}, a_{i},, a_{m})(a_{i}) = a_{2}$	11144444555555111444455555555555555555	annier 1900 2017 de 2007 en han de la companya de 2007 en 2007						
moveralmos (vessermos), vend v hammes ameri	$(a_1, a_2, \cdots, a_m)(a_2) = a_3$	Tommersenson (PCP) MODE (PCP) MODE (PCP) (	те те менен менен менен пред пред пред пред пред пред пред пред						
et tradition to the contract of the contract o		de-languagen and and and and and and and and an analysis of the analysis of the analysis of the analysis of the	Haratari kan atau maganata per-najangga pangga						
th had a thrown a great of his control of a factor of the	$(a_1, a_2, \dots, a_m)(a_{m-1}) = a_m$		therefore the commence of the						
	$(a_1, a_2, \dots, a_m)(a_m) = a_1$		2000 til statististe til 1000 och med Scholydes, kningskrip stille statistiske programme skrippingsgjeggjeggje						
ga pa Bargaran gara arawan da baran da da Baran da da Baran da da Baran da da Ba	$(a_1, a_2, \dots, a_m)(x) = x \text{ if } x \notin \{a_1, a_2, \dots, a_m\}$								
and the second seco	√ {a ₁ , a ₂ , ···, a _m } is called support of the cycle								
of the particular to	$C = (a_1, a_2, \dots, a_n)$ is called a cycle of length $n$								
(mAnnologie/mologie/syspectrum mm. p. pm + r. p.	<u>√ Def</u>								
	If $s \in S_n$ , then the order of $s$ is the smallest integer $n \ge 1$ $s.t.$								
Minimbili ellilli elektrone e en en	<u> </u>		en di dan matana mangka di magang gang gang gang kang kang kang dan kananan mana kang mana kang mana kang mana						
to Department of the second of	=								
errereereebeeseeseeseeseeseeseeseeseeseeseeseese	* EXAMPLE: $S = (1,3,7)$ a shorthand by def.								
755	Then $S(1) = 3$ , $S(3) = 7$ , $S(7) = 1$ , $S(x) = x$ otherwise								
ed a television del primi fina fina (de la fina fina de 11 d	$S^{2}(1)=7$ , $S^{2}(3)=1$ , $S^{2}(7)=3$ , $S^{2}(x)=x$ otherwise								
A A CONTRACTOR AND A CO	$S^{3}(1)=1$ , $S^{3}(3)=3$ , $S^{3}(7)=7$ , $S^{3}(\infty)=\infty$ otherwise								
[#[Ananiiniininanahmini	In this case, the order of $S$ is $3$ .								
S	*The order of cyclic permutation $(a_1, a_2, \cdots a_m)$ is simply m								
eren er en er	i.e. The order of a cyclic permutation is the length of the cycle.								
	J Applications:								
lid et antinet en felljelf i op allen kret sperioesf fri option	* Physics: Fleming's Right-hand Rule:	* Physics: Fleming's Right-hand Rule: Fleming's Left-hand Rule:							
to other on a comment to proper between the contract of	(electric generators)	(electric motors)							
		<b>(3</b> )							
(Mamueria intilizar neutoa gazarennazeana	2 	ermanespannischen Schwerzen werden werden werden werden werden der	million (1888) kan kakababa saara kan anda kan ang 15.55 milion (1884) kan kan kan ang 15.55 milion kan kan an						
PMAmana villa en anena y en en anana en an	9	0	(123)						



· <u>Prop</u>. If C₁ and C₂ are disjoint cycles, then  $C_1C_2 = C_2C_1$ - Th. Any permutation can be written (uniquely up to order) as a product of disjoint cycles. EXAMPLE O.  $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
4 & 5 & 9 & 7 & 3 & 10 & 8 & 1 & 2 & 12 & 6 & 11 \end{pmatrix}$   $\sigma = (1, 4, 7, 8)(2, 5, 3, 9)(6, 10, 12, 11)$  shorthand  $sign = (-1)(-1)(-1) = (-1)^3 = -1$ order = 4EXAMPLE @.  $\sigma = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\
4 & 5 & 9 & 7 & 3 & 10 & 8 & 1 & 2 & 13 & 6 & 11 & 14 & 13
\end{pmatrix}$   $\sigma = \begin{pmatrix}
1, 4, 7, 8 \\
4, 7, 8
\end{pmatrix} \begin{pmatrix}
2, 5, 3, 9
\end{pmatrix} \begin{pmatrix}
6, 10, 13, 14, 12, 11
\end{pmatrix}$  $sign = (-1)^3 = -1$ 最小公倍数' order = 12 - lowest common multiples of the order of disjoint cycles. In this case, 4 and 6 ⇒ lowest common multiple = 12 Note: {Laplace's Theorem Any transposition is a product of an odd no of adjacent transpositions. see Chapter 7. Basis Theorem "



s.	
± E	
· management of the first of the second state	Mon. 14/11/16
Suggestion and the state of the	MATHLEO! Algebra I
whytewares a committed the lateral as a recovery of a construction with the lateral as a construction of the lateral as a constructi	Prof. Johnson
Chapter 4	§ Fields and Vector Spaces §
4	L_Fields
And the state of t	• $N = \{0, 1, 2, \dots, n, n+1, \dots\}$
researce server over mountment ended	✓ cardinals of finite sets
emmeltet kreliniket et ennet um oost et oost en een ee	✓ can add , but cannot always { subtract
to the transfer of the contract of the contrac	
- the encrese arm over community to high	$Z = \{0, \pm 1, \pm 2, \cdots, \pm n, \pm (n + 1), \cdots\}$
And the second s	\( \sigma \) can add , subtract , multiply , but cannot always divide (by non-zero)
mannumaturana erana erana ayalagiliyiyi napa	$Q = \left\{ \frac{P}{q}, P, q \in \mathbb{Z}, q \neq 0 \right\}$ rationals
de the School has provided from the subject of School and the Scho	✓ rule of equality: $\frac{p}{g} = \frac{p'}{g'} \Rightarrow pg' = p'g$
ninggamnanaharingkaringkaringnin ann anasaninanaharindan	$\checkmark$ $\&$ is the first example of a field.
Material Control of the Control of t	• Def.
ţ <u>i</u>	By a field F, I mean a 5-tuple $F = (F, +, \cdot, 0, 1)$ where
seed to the control of the control o	(i). F is a set  An n-tuple is a (ordered)
*Паперетестван weimerätzeurewätzigigigladjag	(ii) 0,1 ∈ F , 0 +1 sequence of n elements
described a company of the company o	(iii) +: F×F→F (means + is a mapping)
*Promitorionical interdese decidad (mystery) (1999)	Write $a+b=+(a,b)$
material services	(W) •: F×F→F
removed executions abundance complete to an agree of place	Write a.b = (a,b)
**************************************	· These must satisfy the following rules
Mindispersion in comment or subject of the fill of the second	✓ <u>Additive Axioms</u>
take attak eleksiming sanggana yannagan yan e e e yangga	I) $a+(b+c)=(a+b)+c$ $\forall a,b,c \in \mathbb{F}$ associativity
Apophilisteriorale e e e e e e e e e e e e e e e e e e	I) a+0=a=0+a
Promotion substitution of the state of the s	ID) ∀a∈F,∃(-a)∈F st. additive inverse
14-mail An Astronomer en en en est en publica de entre en estado en entre en estado en entre en estado en entre en	Q+(-0)=0=(-0)+0
American en el el monto de la calabe el conserva el el conserva de la conserva el conserva de la conserva el c	IV) Ya, b EFF, a+b=b+a commutativity
	✓ <u>Multiplicative A×ioms</u>
	I)' a·(b·c) = (a·b)·c
	I)' a·1 = 1·a = a
	<b>Y</b>



	√ F ₅ ari	thmet	ic	mod	5		dende kantriklijk kantriklijk kantriklijk in stere kantriklijk in dende kantriklijk kantriklijk kantriklijk ka		e e promozene pose propez pos z p	time to a state of a state of a		and a second	rzwantywastranorowetwejw.	en til state til state skale ska I kale til skale ska	hannan (1115an i in 1114an in in in in in in in
	gg, e . e . e . e . e . e . e . e . e . e	{0,	1,2,	3,4	4}			,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,						en metermiljemeljensje sjeriemekom sjeljem merej i ne me je ne en en en men je ne je ne je ne je ne je ne je n	populari de la producció de la
spaggerman er kanne	+	σ	ı	2	3	4	er i 🍇	•	0	1	2	3	4	жий и метора по при	1858 or
And Combat and another and a superference of the superference of t	<b>0</b>	0	******************************	2	3	4	ded (m.) Zwiz Li Indogra (m.) diwani 1 111 Abran 111 mwa mwa	0	0	0	0	0	0	est i kankimontalikimotoo oo uudaa oo uudaa oo uudaa kankin kankin kankin kankin kankin kankin kankin kankin k	politica pooles constituto o com
	-	<b>-(</b> )	2	3	4	Ō		1-	0	0	2	3	4		ppm2mm / 24,mmm p. , mmm , co
Mydelynn i few fri i i i i i i i i i i i i i i i i i i	2	2-	3-	4-	-0-	1	the distribution of the desired and the desired distribution of the desired and the desired an	2 -	_0_	2		0	3	2 ⁻¹ = 3 mo	ıd 5
statelesteraniste (1910 e.	3 ~	3-	4	0-	<b>1</b>	2	and and and and a fine factor of the analysis of the same of the s	3	0	3	·D	4	2	4 ⁻¹ = 4 mod	d 5
Addition to the second commencer to the second management of	4 ~	4-	-0	<b>D</b>	2	3	րժժժժուսուժուհեռուսժությունը գուրֆվովալալ էլ էլ փորցիլ երևալուու	4	-0	4	3	<del>2</del>		igtet aufgliche krijst for her mog frei herbielijk forskrijk over britisk bris versammen til betom til freijsk for h	Universite de de la companya de la c
ta jagagan ya mini mu ya ya mini na ya mini ya ya mini ya ya mini ya wa	and the second and the second contract of the second contract of the second contract of the second contract of		_ 0	***************************************	n de		arran ar	didesti adam on aboo.			***************************************			tterktorius etterktorioonis (2000–2000–2000–2000).	the control of the particular section and a second
er kontralisk sterkeren kert konserrit til til sekt sektert skireren som er samme	e.g 4				***************************************	rtrettet tillsammet mont.	ter to the transfer of the tra	<u>e.g</u>	<u>3.</u>	4 =	12 =	2		Persentakan persentahan persentah kerapan pengangan pengangan pengan pengan pengan pengan pengan pengan pengan Persentahan pengan	wenneerengeranepophaguezzasza
halphalat farbum	F ₅					ganzeria pangegyeggi	att de entre de la companya de la c		***********	rumann a ann air de armen	······································	***************************************	lideks (delem for 11 for fortung er 1 en stjør	ээний тите от	1
/ARAWSAA/-000AAAAAAA/Y/50-5A/SA/SU/54/SSS/H/SA/W	√ Z₄ arit	nme⊤ {o,ı			<u>t</u>	errennen sor processor or processor of the second	aller State and the state a		4+0000x3v+0290v1vv		riekentren enemetek	ation of the state	***************************************	refore formal, at earth are use summers and the planting by the formal policy (Section 1984). Section 1984 198	**************************************
e kirkakus 1999 (1994) (1994) (1994) (1994) (1994) (1994) (1994) (1994) (1994) (1994) (1994) (1994) (1994) (19	o describido de construiros de construiros de construiros de construiros de construiros de construiros de const	0,1	, Z, 3	2	3	er	mar nazzoniar poznażamana pzinistaż probleż biolicz poznaża i		********************		zwawana i kadawa i k	entetikklistiken een	**************************************	ookkallasta 190-add 190-tiik ook ook ook ook ook oo 190-boor ah amaa ah amayaa ah aasay oo	vinità di suò si consuita con un u
	<u>ummonustation</u>	0	0	0	0			enementetetenen 188e il.			har e szargáhásssaszeztés fi	endere Herindonon	dridreeedrieendliriii.vuurusaaa	ан кантана ан кекендин гуранда Арубор Арубор Арубор үзүү жүү үзүү үзүү үзүү үзүү үзүү үзүү	terretester det terretes de desta de la comunidad de la comuni
At Straigs in Spilins I is all are essellen an earliern measure in mile	NVI UEIQ														
	2	0	2	0	2	***************************************	beco	use	2 h	as r	10 M	ultip	licativ	e inverse	N-1-1-1
Ahallağınağıklariyi (27 dağığı) (27 həsəsilər) əsəsəməsə) ahamasıysa azalla	3		3	*************	_ \rangle O	indrindrid Ladi aan brahii	indin e distribute se mandino i i rimene e e e e e e e e e e e e e e e e e e	#A3040{A1/273401,1703000?	el brotte belonderne	inner e en e		***************************************	niato Anterioto (desiliesto (Zessen	referenders er einstendurfennannhuttenfatteljän (VXIII) (n. 11.20/et) (d. 11.20/et) (d. 11.20/et) (d. 11.20/et	weenerger er er en en en en en en en egy en jost gener
Nationale perfection for the State of the St	√ When is	other Atambael and the	thme	<u> </u>			field ?		energe en	N			PA-10/MIDWINGSOMSWAYSOMS	эг гэх хэх хэх хэх хэх хэх хэх хэх хэх х	*Aleman ramon y angeley» (egye
Politica de la composição							(6	ouss)				*****************	en e	an der eine der eine der der der der der der der der der de	attituun kattan ja kustamin ja ja ja ja Kattan ja
reducerates and	et 17:0000 tissed on the tild to the tild tild tild tild tild tild tild tild	Americana, and significant	elementism senggapititi	nd diskriptivele enemene	eed ooming on an ord	remensusee (naket tee) and the second	ttadas ( timit på 1 folgas ( ) en villigliga plitiga på tima er sekard en en er	, , tomo 121, 121, 121, 121, 121, 121, 121, 121		-2227//4500-2442/-01	sed den automiter proprince de la den automiter de la den automiter de la den automiter de la den automiter de	ietoromenenidoren	norde contrate de la finalità (m. 13 estima	erry standard Nordon Assessment of the Standard Standard Standard Standard Standard Standard Standard Standard	della del de comitat de comitat de constitue de constitue de constitue de constitue de constitue de constitue d
	erricario de la companya de la comp	·	andarente en constantante	***************************************		******************	till state of the		**************************************	to the state of th		ersonesemmososo		Zelszereső, jaszerreszereszereszereszereszereszeresze	·w····································
eranieraanijariyyiniyiijiyyyiniiseereenii iriisaasee	Fri. 18/11/16	N Cambia Association Procedu	taatistiididea ayoont tota et toit		***************************************	haadinsa kiriinsa ka	energia per energia per a mantena de partir per de percentina per a constituir per de percentina per a constitu	maranina da salahan masasa	historia de la libraria de la compansión d	m.m.,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	en or distantial library	·	Olekelihiki (rirenaesusuma	rennersaanna na golgaanna parlan izinnii zaan izisi izin ya lisisin ya naraman na nagga [[=	MENTAN PERIOD I FORM AND AREA.
					MAT		*#					t to all observance		ad the letter before the section of	1800 (
erandii sebiimorasi erandahiyi jiyyafidalinii ilkaniyaa kabad	on prototic home we received to be received to the continue and the second to the continue and the second to the continue and the second to the continue and th	ezengerenenenekezzz	reconstruction and advantages	lderen laren erake		Pro	f. Johnson	. ####################################	a familika a ka filologia a fi	den en eksterek en eksterek	***************************************	er er end de er er end de foed fend	POLITING CL. 1445/494/4980	(APAPAPON) kansasa kakasakakkanyana mano entampenyanyan ata 1111 sebesa esebeste	
4.1.2	Quadratic		_	the and a contract the death and the State	?~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~		maa iiiinaa aa ka kiriinaa ka aa ka k				,		***************************************	the construction of the property of the state of the stat	
					W	nich	you unde	rstan	d.,	e.g	take	F:	= Q /	F=R	delikadestadadest et esterminadaksterriide
telestronnes del del del 25 de junio en communida de mario en conseguir en companyo en conseguir en conseguir	If α∈				n i na Niimi (na Niisaliae) a		erre en						29+11888#################################		
t annual account the fight player of a continue and find a fact for the first fact fact for the first fact fact for the first fact fact fact for the first fact fact fact fact fact fact fact fac			•				ave a solu								not through the following the formula is the sound to be a
1188 de la territoria de la companya de la company							no solutio	מו_מנ	R.	non-10-40-01/04/00-0	te de de la composition della		und de la description de la constantina de la constantina de la constantina de la constantina de la constantin	-tarket 1865-511 erick-bereich ill diese anmannerenschaffen gegegep j	detro <del>e</del> turaturiories ( · · · · · ·
kananan kan kining sa kananan k	√ Informal						nadd ddidd diddidd o ddiddid o y ach a'r ganhafagad ar manadd ar mae y cae		· costanole transcen	waarreen waara	ayamgaraezezepinas	olenien III i i innennen i	r o resident neveral trobusesse	rakepulak / / hijiminimi umikalidak kuligak kelaliya kepingi na yangan mendebakan kelalika ke	**************************************
Control of the Contro	- Constru	ct	a r	new_	elei	nent	· √α.	**************************************	terano de la comunida	en an	***************************************	15-44-14-15-15-4	ale al le distribution de la company	eritatriannete ennantipassiocinis immensiones inschisiospessiones inschisiospessiones in anni en inschisiospessiones inschisiospessiones in anni en inschisiospessiones inschisiospessiones inschisiospessiones in anni en inschisiospessiones in anni e	www.comes.comes.com/personal-comes.com

-Consider expressions x1+x2√\alpha: xi∈F - Add & multiply in usual way. 2)  $(\chi_1 + \chi_2 \sqrt{\alpha}) \cdot (y_1 + y_2 \sqrt{\alpha}) = (\chi_1 y_1 + \chi_2 y_2 \sqrt{\alpha}) + (\chi_1 y_2 + \chi_2 y_1) \sqrt{\alpha}$ @ zero: 0=0+0.1a  $1 = 1 + 0\sqrt{\alpha}$  $(1+0\sqrt{\alpha})(x_1+x_2\sqrt{\alpha})=x_1+x_2\sqrt{\alpha}$  This implies 1 is an identity √ Do inverses exist? - Trick:  $(\chi_1 + \chi_2 \sqrt{\alpha})(\chi_1 - \chi_2 \sqrt{\alpha}) = \chi_1^2 - \chi_2^2 \alpha + 0$ Need to know that  $x_1^2 - x_2^2 \alpha \neq 0$ . Otherwise,  $x_1^2 = x_2^2 \alpha \Rightarrow \left(\frac{x_1}{x_2}\right)^2 = \alpha$ Contradiction.  $(\chi_1 + \chi_2 \sqrt{\alpha}) \left( \frac{\chi_1}{\chi_1^2 - \chi_2^2 \alpha} + \frac{\chi_2}{\chi_1^2 - \chi_2^2 \alpha} \sqrt{\alpha} \right) = 1$  $x_1 - x_2 \sqrt{\alpha}$  is called the conjugate to  $x_1 + x_2 \sqrt{\alpha}$ . • If you guarantee that  $x_1^2 - x_2^2 \alpha \neq 0$ , when  $(x_1, x_2) \neq (0, 0)$ , then you get a field called F(Ja) Construct  $\sqrt{\alpha}$  as follows: Formal Def F(Q) = F×F  $\emptyset$  Addition  $(\chi_1, \chi_2) + (y_1, y_2) = (\chi_1 + y_1, \chi_2 + y_2)$ zero 0 = (0,0) Think  $x_1 + x_2 \sqrt{\alpha} = (x_1, x_2)$ so |= (1,0) 坂 = (O,1)  $(0,1),(0,1) = (\alpha,0) = \alpha(1,0) = \alpha.1$ ✓ EXAMPLE O:  $x^2 = \alpha$  has no solution in IF F=R ,  $\alpha=-1$ 「F(風) is a field'  $x^2 = -1$  has no solution in  $\mathbb{R}$ . So R(JFI) is a field.

```
VEXAMPLE Ø.
                       F = F_5 = \{0, 1, 2, 3, 4\}
            0^2 = 0
            12 = 1
                           This is because 4+1=0
            2^2 = 4 = -1
                                                       4=-1
            3*=4=-1
            42 = 1
                                         has no solution in F5.
           So F5 (12)
                           is a field with 25 elements.
                                          since F_5 = F_5 \times F_5
                                                5 elements 5 elements
4.2
    Vector Spaces
 • Def.
         Fix a field. (eq. F = Q)
         By a vector space V over FF, I mean a 4-tuple V = (V, +, 0, \cdot) where
              U V is a set
              2) 0 \in V
              3) +: V \times V \rightarrow V mapping
                 \underline{V} + \underline{W} = + (\underline{V}, \underline{W}) addition
              4) · : F×V → V mapping
                     \Lambda \cdot \underline{\vee} = \cdot (\Lambda, \underline{\vee}) scalar multiplication
        s.t. the following rules are held:
      I) \underline{u} + (\underline{v} + \underline{w}) = (\underline{u} + \underline{v}) + \underline{w}  \underline{v}, \underline{v}, \underline{w} \in V associative
                                        A ñ ∈ A
      additive identity
     .t. w→E , V∋r (I
              신+(-진) = 0 = (-진)+신
                                                               additive inverse
                                       A \tilde{n}, \tilde{\lambda} \in \Lambda
      IV) U + V = V + U
                                                               commutative
     I') \lambda \cdot (\mu \cdot \underline{\nu}) = (\lambda \mu) \cdot \underline{\nu}
                                        YN, MEF, AVEV
     11) 1・4 = 4
                                        V 4 EV
      亚) - 4= (-1). 4
     Plus two distributive laws.
      D1) λ.(Ϥ+Υ) = λϤ+λΥ ΥΛΕΕ, ΥϤ,νΕV
```

## i.e. multiplication of 71 distributes over addition YN, MEF, YUEV D2) (1+4). 4 = 14+44 √ Basic Example: (IF is fixed in advance) means vector space $\mathbb{F}^n = \left\{ \underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} : x_1 \in \mathbb{F} \right\}$ neans fields addition scalar multiplication ⇒ F° is a vector space over FF - an 1-direction EXAMPLE: vector space $= \left\{ \underline{x} \in \mathbb{R}^2 : x_1 + x_2 = 0 \right\}$ Note: 'VCFF' BUT V + F' in this example. means Every Fⁿ is a vector space. But not every vector space is an IF". V is a vector space. Any $\underline{x} \in V$ can be written as $\underline{x} = \begin{pmatrix} x_1 \\ -x_1 \end{pmatrix}$ . If $y = \begin{pmatrix} y_1 \\ -y_1 \end{pmatrix} \in V$ , then $\underline{x} + \underline{y} = \begin{pmatrix} x_i + y_i \\ -(x_i + y_i) \end{pmatrix}$ ⇒ $V \subset \mathbb{F}^2$ , but $V^{\neq} \mathbb{F}^2$ (counterexample: $\binom{1}{8} \notin V$ ) · IF is a fixed field. Let $A = (aij \mid 1 \le i \le m)$ m×n matrix aij EFF Consider the homogenous system S $S = (A \approx = 0)$

*		$\int_{\Gamma} a_{11} x_1 + a_{12} x_2 + + a_{10} x_0 = 0$						
the manage of the secretary and an angeled the second of t	S = -	$Q_{21}X_1 + Q_{22}X_2 + + Q_{2n}X_n = 0$	очения положения до должне не установания должно положения подобольно положения в доболь до положения родоболи					
rkanningster (Shift) (142 Merili (142 Meri			antaantaa ka ka maranantaan marana da da ka					
nemente de modelle e de semante e entre en en entre politique de participation de presentation de la constitució del constitució de la con		$(a_{m_1}x_1 + a_{m_2}x_2 + + a_{m_n}x_n = 0$	enement					
nt militar namadimul mulajakani ka ka ka mata ka kilika na ya ja ja juga ka ku ku ka ka ka ka ka ka ka ka ka k	✓ Temporary Not	The second secon	anaanan järjä, suurun aanaan ja jalan saaran saan ja lään ja					
*whiteless with the frequency of the section is a market budge, and a part of the section of the	• •	$F^n : A \times = \emptyset$ solution set	enter entered to the					
Adama]] sagagagagagamin sasta adalamin adalahaya galajaya ya ya ka	KACF		tarikhan monakatan)inga (Amunamintarian masamanan masaman apat Africana masa ayah) (inga memunan ang Anga Mana					
had distribution of the state of	If A≠0, th	en Ka‡F ⁿ .	er en					
e distinct was not the weather and the second of the second was a second of the second	V Prop.		тини менендиру до особорожника бай мененда болов на мененда болов до особорожно в особорожно в особорожно в осо					
Color Constitution of the annual temperature of the property of the second second	Ka is a v	ector space over F	and the second s					
and the second s	Proof: - If x ∈	Ka and YEKA, then X+YEK	A					
· · · · · · · · · · · · · · · · · · ·		$Q$ and $Ay = Q \Rightarrow Ax + Ay = Q + Q = Q$	addition (V)					
turnah Pharitzi (Prima) (**eemeesseeekin hahaaa ja ja ja		⇔ A (x+y) = Q	J dournor (Constitution of the Constitution of					
and the section of th	- <u>If</u> <u>x</u> ∈	- If $x \in KA$ and $x \in F$ , then						
oomeeseeeeeni (in) (?un) siipeeri eurooneeni eni ?va] (ee)	$A \cdot (\Lambda \underline{x}) = \lambda A \cdot \underline{x} = \lambda \cdot \underline{0} = \underline{0}$							
	So Az	scalar multiplication (V)						
highelpes securing to start a start a start and the security and selections of the start and the security of t	- Zero:							
hiddelygg aggada yw mae'r chaddlyng glycgog y gangag ar yn y	So	zero (V)						
و مرور در	- The ren	- The remaining axioms are automatically satisfied.						
etti kannin kalangan pendikan pendikan pengan pendikan pendikan pendikan pendikan pendikan pendikan pendikan p	(satisfied already in F")							
erena muunga sa	Hence, K	a is a vector space over 1F.	·					
4-2-1	Linear Independe	nce						
eren 2000 tales e francisco de la compansión de la compan	• Def.	enter de la companya del la companya de la companya del la companya de la company	The more and residence of the state of the s					
entrigget had t sid till till for edland til at 150 km/s för för en sig et til sid till till til sid till till	Let V be a v	ector space over F						
t the second and analysis of the second analysis of the second analysis of the second analysis of the second and analysis of the second ana	Suppose v. v.	V ₃ , Vn E V	t out o our cumulum page, process to change digram from the state and makes discharace and change and constitution of the state and change and					
httimikkan jagan sakuda makukun kepatan sangan sakun kan sakuda sakun sakuda sakuda sakuda sakuda sakuda sakud	An expression of the form							
	$\underline{V} = \lambda_1 \underline{V}_1 + \lambda_2 \underline{V}_2 + \lambda_3 \underline{V}_3 + + \lambda_n \underline{V}_n \qquad \lambda_i \in \mathbb{F}$							
threeder 130 th three extrements throughous security and the security throughous security and the security throughous security through security throughous security through the security through security through the security th	is called a	is called a linear combination in V1,, V0						
t the Section Control of Section 1985 and 1985 a	✓ Notice that I can always get 0 as a linear combination.							
	ž	$0\underline{V}_1 + 0\underline{V}_2 + 0\underline{V}_3 + + 0\underline{V}_n$						
Semijoto je ili ili ili ili ili ili ili ili ili il	✓ Informal Def.							
	{ <u>Y</u> , , <u>Y</u> n} is	s said to be linearly independent	(L.I.) when the ONLY way to get					

0 is with all coefficients = 0. Formal Def. vi, ..., vo? is linearly independent (LI) when  $\lambda_1 V_1 + \lambda_2 V_2 + ... + \lambda_n V_0 = 0$ ⇒ λ1 = λ2 = ... = λn = 0 ✓ EXAMPLE : V= F4  $\underline{\mathbf{e}}_{1} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \underline{\mathbf{e}}_{2} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \underline{\mathbf{e}}_{3} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad \underline{\mathbf{e}}_{4} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ Then {e, e, e, e, e} are LI. Proof. 7101 + 7202 + 7303 + 7404 (X) So, if (*) = 0, then  $\Rightarrow \Lambda_1 = \Lambda_2 = \Lambda_3 = \Lambda_4 = 0$  $\Rightarrow \{e_1, e_2, e_3, e_4\}$  is LI. Generalisation. Then {e, e, ..., en} is LI. ✓ EXAMPLE ①:  $\underline{\varphi}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$   $\underline{\varphi}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  $y_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Then,  $\{\underline{\Psi}_1, \underline{\Psi}_2, \underline{\Psi}_3\}$  is LI. Proof:  $\lambda_1 \underline{\Psi}_1 + \lambda_2 \underline{\Psi}_2 + \lambda_3 \underline{\Psi}_3 = \begin{pmatrix} \lambda_1 \\ \lambda_1 \\ \lambda_1 \end{pmatrix} + \begin{pmatrix} \lambda_2 \\ \lambda_2 \\ 0 \end{pmatrix} + \begin{pmatrix} \lambda_3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda_1 + \lambda_2 + \lambda_3 \\ \lambda_1 + \lambda_2 \\ \lambda_1 \end{pmatrix}$ So if  $\lambda_1 \Psi_1 + \lambda_2 \Psi_2 + \lambda_3 \Psi_3 = 0$ , then

	So λ1 = 0   Λ1 + λ2 =	$0 \qquad \lambda_1 + \lambda_2 + \lambda_3 = 0$		
,-	But 71:	=0 But $\lambda_1 + \lambda_2 = 0$		e e e e e e e e e e e e e e e e e e e
	So 1/2 =	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	tti 1880-raanaa espani aantiinaa espanjaksi periorise enim-kammi asimaan kahisele en-moonisele en-	erri elerrikkakarijakirjakirjiji. Sajirji jelikirji kerdine magga sungajuka pen
	. ⇒ (Ψ, Ψ, Ψ) is i		der er er und versus er eine versus er eine er	OPERIOREN FORMANDE PROPERTY P
	✓ EXAMPLE ②.			M. Sent School St. Sent School was a sent and make his to dead on the Sent School (Sent School (Sent School (Se
****		$\psi_{3} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$ \sqrt{4} = \begin{pmatrix} -\frac{1}{3} \\ \frac{3}{3} \end{pmatrix} $	h Ministrik kalistiden kalesses sida kalistasses sa
,	Then $\{\underline{\psi}_1,\underline{\psi}_2,\underline{\psi}_3,\underline{\psi}_3,\underline{\psi}_4\}$	14) is not LI.		
/-	(It is linearly dep		and the second of the second o	ndddigiol y rilligyd felirionarus arminin maethaleid felirioldigil fenyddyr myd fel
.,,	Work over A	eesses attalikuutta kaja kaja ja		and the dead to the control of the c
	Proof: Solve Λιψι+λιψι+	15 V3 + 14 V4 = 0	000 km oo aan oo aan aan oo aan ay qayay kaasaa 11 goodhahaad oo ah aan ah	т. Селетен и интерператория (Селетория (Селетория) и селетория и селетория (Селетория (Селетория) (Селетория) (Сел
,,,,	+2			et eksikeri ibissaeri erritatik (SSE)
	$(\lambda_1 - \lambda_2 + \lambda_3 - \lambda_3)$	1 1 1		ды жүүрүү катаматын тарын тарын байда жайын жайын Тары
r.A.	$\sum_{n=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{n} + n_2 + n_3 + 3n_3$	magaman nagam (a polymera a sociole p para a social la communa de constitue de polymera para de para de para de polymera de po	ant tyrman magnet photos (1960-1964) distributed bahari an anaman magnetim py 10 pmp photos (1960-1964) distributed baharian	erekantera jairini, kwadisiai ilikuwa wa maranzanji namija kwa ilikuwa ilikuwa wa wasa wa wa wa wa wa wa wa wa
	/1102/0		taka animat atau ka jima anggota 1988 si ilong palikasa anima a tauka kamay panjayong panjayong panjayong panj	м тотперательного добрабора добрабора добрабора по терева потперва потперва у подрага добрабора добрабора добра
	(1 -1 1 -1 0		likkas dieses an anterior an analy spanjah kanipak pikki pisyly senis et alkona anamana asy engenasie dee sityah et an isi senan	sadanananananakkakkikkilaisissassadakanananananakkilassassassa
	111310		150 M. St. 1448 M.	erritariente uterritariente des prints filosofistes filosofistes de financialis anno acua una greganam
		v echelon form:	1988-cultura artikalari ilmahari ganakari gan 1986 da	dreessaareessaananaasterkiristä krissikkireessaanaasteraaaasteraaaa
	& (2,1,-1)	1 0 2 1 0		See are not a common participation of the contract of the cont
	€ (3,1;-1) ( <u>-</u>	-2 1 -3 0		til eksternet melririnst AfrikalifAhjiniyi []]]jimmayan yernetirininin turket (ji) turkjessa
	$ \begin{array}{c c} \hline E(3,1;-1) & 0 \\ \hline D(2;\frac{1}{2}) & 0 \end{array} $	0	ann an talan an a	dermining fragetis ASS and a contract of management of the contract of the con
-	$\xrightarrow{\mathcal{L}}(2;\overline{z}) \to 0$	-1 1/2 -3/2 0	aarumaaa kalisiiseettä kiisekää käänisten on muuta muuta muuta kalisiksi ja kalisia ja joinnin on on makatatutaksia	\$\$\$\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
	The second control of		the state of the s	11000000 commencentum taken paking taken paga paga paga paga paga paga paga pag
-		0 1/2 1/2 0	er e e e e e e e e e e e e e e e e e e	PHILIPPE PARTER SETTING AND STREET AND
		-1 ½ -3½ 0 0 1 1 0		ara aranda magani a masani ana ana ana ana ana ana ana ana ana
-14	(0	0 0 0 0 0	and person and interest and in the contract of	120-oldhol oehiiseeeliisee24ghqbbbhaclaebabolaeeeeliinn oseesidhabababab
- :	. <b>I</b> 1	1/ U 1/ (1 \		

$$\frac{\underbrace{\&(1,3;-\frac{1}{2})}}{\&(2,3;-\frac{1}{2})} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

	(A) = - 1/4	Λ ₃ = -1	
	$\Lambda_4 = \Lambda_4$	λ4 =1	
	Then <u>0</u> - 2 \ 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2 - 2 \ 2	$-\psi_9 + \psi_4 = 0$	
		≠0,1≠0,	
	We have a linea	or combination in which not all $n_i = 0$ .	
•	Def		
		dependent (LD) when 3 linear combination	
	7, <u>1</u> + 12 <u>12</u> + 16 <u>13</u> + +	$\lambda_n V_n = 0$	
	in which at least one 1	1i ≠0	
	✓ Such an expression in whic	h some 7i≠0 is called a dependence	
	relation.		
	✓ In the previous example,		
	01/1-21/2-1/3+1/4:	= 0 is a dependence relation.	
			,,.
	Mon. 21/11/16		
	MATH120	0): Algebra I	
	Prof	Johnson	
	Recap:		
	Def 1. linearly independent		
	V is a vector space	over F.	***************************************
	{⊻,, ¼,,√n} ⊂ V		
	$\{\underline{V}_1,\underline{V}_2,,\underline{V}_D\}$ is LI		
	$\sum_{i=1}^{n} \lambda_i \forall i = 0 \Rightarrow \Lambda$	$_{1}=\Lambda_{2}==\Lambda_{n}=0$	
4-2-2	Spanning		
•	<u>Def.</u>		
	V is a vector space over	F.	
	$\{\underline{V}_1,\underline{V}_2,\cdots,\underline{V}_0\}\subsetV_1$	ł	
	We say (⊻, ½, ½) span		
**************************************	$\forall \underline{W} \in V, \exists \lambda_1, \lambda_2,, \lambda_n$	$ \mathcal{E} F  \mathbf{S.t.}  \underline{\mathbf{W}} = \sum_{i=1}^{n} \lambda_i \mathbf{V}_i $	

Take 1 = 0

1/2 =-2

**(1)** = 0

	✓ In English (informal)	
urturd mysgygggg f felestrat i det artikus (felussifet e lestifet lestifet e lestifet e lestifet e lestifet e		sed as a linear combination $\{v_1, \dots, v_n\}$ .
salist eller de tradition de tradition de la constitution de la constitution de la constitution de la constitu	V EXAMPLE O:	
umnerenninkasjärtyntynjärtyttyvaerenninkarenninkarenninkarenninkarenninkarenninkarenninkarenninkarenninkarenni	Let $e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$	En = ( ; )
and design and a second design and a second assessment as the second assessment as the second assessment as the	be the standard vectors in F ⁿ	, then [e₁, e₂,, eʰ] spans F°.
mmelel kallilinnel karlilinnel et artikunlarlara sassijih seja sassijih sassijih sassijih sassijih sassijih s	_ <b>_</b>	
and the second s	Proof: $(x_1, x_2, x_3) \in \mathbb{F}^n$	
alaska aranan kan dilameka kan dingga Zija ya Jimaya arana, ya	$\underline{x} = x_1 \underline{e}_1 + x_2 \underline{e}_2 + + x_n \underline{e}_n$	
	$= \begin{pmatrix} x_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ x_2 \\ \vdots \\ 0 \end{pmatrix} + \dots + \begin{pmatrix} 0 \\ \vdots \\ x_n \end{pmatrix}$	
	√ EXAMPLE @.	
сти к ден теренер мененик карануу (д. ) дүүнү ж	$V = \left\{ \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} \in \mathbb{F}^3 \; ; \; \chi_1 + \chi_2 + \chi_3 = 0 \right\}$	
	$\frac{\varphi_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}}{0} \qquad \frac{\varphi_2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}}{1}$	
tadd d this international and a state of the	Then $\{\underline{Y_1},\underline{Y_2}\}$ spans $V$	
anterior de l'acciondrate d'accionne de l'accionne de l'accionne de l'accionne de l'accionne de l'accionne de	Proof: $\textcircled{0}+x_2+x_3=0 \rightarrow \text{could substit}$	the either & 1/2 or xa - was a common
tin de le	$x_1 = -x_2 - x_3$	top and the conflicions
and the constitute of the configuration of the special part of the	X2 = X2	i stant stant
enerre en neutra de desta por la differencia de la consecución e en invento	\[ \times_{\chi_3} = \chi_3 \]	standard basis comm
**	Then <u>x = x y , + x y .</u>	To show $\Phi$ spans $V$
trad at transfer for some or extrement of the manages extended	$= \begin{pmatrix} -x_2 \\ x_2 \\ 0 \end{pmatrix} + \begin{pmatrix} -x_3 \\ 0 \\ x_3 \end{pmatrix}$	o express & in terms of $\Phi$
arradonados er como contrato a como persona en proper de proper de proper de proper de proper de proper de prop	Company to the contract of the	eg. $\epsilon_n = \lambda_1 \mu_1 + + \lambda_n \mu_n$
	$=\begin{pmatrix} -\chi_2 - \chi_3 \\ \chi_2 \\ \chi_3 \end{pmatrix} = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}$	
and a more or an accordance of the contract of	( X3 / (X3/	$\underline{x} = x_1 \underline{e}_1 + \dots + x_n \underline{e}_n$
n 1-да ¹¹ Мийгуу учин уууун түүдүү түү байгуу учин уууу түүдү	$\Rightarrow \{\Psi_1, \Psi_2\} \text{ spans } V$	= x1 (714++2n4n)+
		= <u>4</u> ( ) + + <u>4</u> n ( )
4.2.3	Basis and Dimensions	and the state of t
er temperatura est	<u>Def.</u> ´/F' ∈ ´o₁	ELF'
notari i sili isahinga ya fiya iyo mumundiyayinga i	Let V be a vector space over IF	
	Let (E, E,, En) CV.	- THE CONTROL OF THE
	We say that $\{E_1, E_2,, E_n\}$ is a basis	for V when
	\$ Control of the cont	

$(E_1,E_2E_n)$ is LI.	
and 2) $\{E_1, E_2, \dots, E_n\}$ spans $V$	on error among to hand de trattale (and file file areas franches a
Prop.  Let $e_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ , $e_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ ,, $e_n = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ be the standard	
vectors in $\mathbb{F}^n$ . Then $\{e_1,e_2,\dots,e_n\}$ is the basis for $\mathbb{F}^n$ .	ge Ny Jengur e Rhugury e Urzydd o Euronod de Niddiddoe Eurob
Proof:	
(e1, e2,, en) is LI. (proved in last lecture)	and harmon har so see show the special
{e₁,,en} spans F". (just proved) □	gggggaaaaaa
✓ EXAMPLE (②):	ttinggeriiiiiiiiiiiiiiiiiiiiiiiiiiiiiiiiii
$V = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \frac{\varepsilon \mathbb{F}^3}{1 + x_2 + x_3} = 0 \right\}$	44.0022.000.000.000.000.000.000.000.000.
,	
$ \varphi_1 = \begin{pmatrix} \overline{1} \\ \overline{0} \end{pmatrix} \qquad \varphi_2 = \begin{pmatrix} \overline{0} \\ \overline{0} \end{pmatrix} $	en kriegenieg kan einem eine kriegen eine einem eine einem eine eine eine
Then $\{\Psi_1, \Psi_2\}$ is the basis for $\mathbb{E}^3$ .	a gangangan ganamaa ammamamid amad 11 ah 1464 b
Proof:	egangemen aa aa amman ahki emintel kritissis ki ki kirileet kirile
{Ψ ₁ ,Ψ ₂ } spans V (shown in the previous pg)	
$\lambda_1 \Psi_1 + \lambda_2 \Psi_2 = 0$	nd 113 denne 144 stylles fra grann ann amainn a 123 hann d'Ar
$\begin{pmatrix} -\lambda_1 \\ \lambda_2 \end{pmatrix} + \begin{pmatrix} -\lambda_2 \\ 0 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	e e egy zigy e gyggenen e e e e e e e e e e e e e e e e e
$\Rightarrow \lambda_1 = \lambda_2 = 0$	
$\Rightarrow$ $\{\underline{y},\underline{y}\}$ is LI.	Anterior of the second
. Main Theorem. The Basis Theorem.	
Let V be a (non-zero) vector space over F, then	કે મુદ્દે ફેલ્લુલ્લે કરા (તે આફ્લોન ) માન્ય જ્ <u>લ્</u> લા માન્ય કર્યા હતા.
(i) V has at least one basis.	sawiisinga ganga gan
(ii) Any two basis for V have the same number of elements.	en f ennightenning de sind finden er ennighe den de sind finden finden finden finden finden finden finden find
• DEF	een pooleen een een een een een een een een een
The number of elements in a basis for V is called the dimension.	ndadinandin namangganggapangganggangg
of V written	aangaaaan qaannaad ashiriinaan ballista ka
- dim(V) (or dim=(V))	er transporter generalist fragisk med generalist fra en de generalist fr
so dim (F") = n because it has a basis e,, en with n elemen	its.
VEXAMPLE O:	

$$\underline{\varphi}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad \underline{\varphi}_2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad \text{is a basis for } V.$$

Then dim(v) = 2.

VEXAMPLE @:

$$W = \left\{ \underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{F}^2 : x_1 + x_2 = 0 \right\}$$

Then dim(W)=1, since

$$W = \left\{ \chi_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} : \chi_2 \in F \right\}$$

$$\Rightarrow \{(-1)\}$$
 is a basis for W.

√EXAMPLE ③:

$$A = (a_{ij}) \quad |si \leq m|$$

$$|sj \leq n|$$

m × n matrix over F

$$K_A = \{ \underline{x} \in \mathbb{F}^n : A\underline{x} = \underline{0} \}$$

Compute dim(Ka), so first

find a basis for Ka.

e.g. Easy Case.

Assume 
$$A = \begin{pmatrix} 1 & -1 & 0 & 3 & 0 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$
 3×6

Take F = Q.

A = 0 Write out general solution.

$$(x_1) = x_2 - 3x_4 + x_6$$

$$\chi_2 = \chi_2$$

$$\bigcirc = -2\chi_4 - \chi_6$$

$$\chi_{4} = \chi_{4}$$

$$\chi_6 = \chi_6$$

$$\underline{X} = \begin{pmatrix} x_2 - 3x_4 + x_6 \\ x_2 \\ -2x_4 - x_6 \\ x_4 \\ -x_6 \\ x_6 \end{pmatrix}$$

Therefore, the obvious basis for KA: Ist choice:

$$x_1=1$$
  $x_4=0$   $x_6=0$ 

$$\underline{\varphi}_{z} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\chi_2 = 0 \qquad \chi_4 = 1 \qquad \chi_6 = 0$$

$$\psi_2 = \begin{pmatrix} -3 \\ 0 \\ -2 \\ 1 \\ 0 \end{pmatrix}$$

3rd choice:

$$\chi_{2} = 0 \qquad \chi_{4} = 0 \qquad \chi_{6} = 1$$

$$\underline{\psi}_{6} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \\ -1 \end{pmatrix}$$

 $\therefore \underline{x} = x_2 \underline{\psi}_1 + x_4 \underline{\psi}_2 + x_6 \underline{\psi}_6$   $\Rightarrow \underline{\{\underline{\psi}_1, \underline{\psi}_4, \underline{\psi}_6\}} \text{ spans } K_A.$ 

We know that

$$\underline{x} = x_2 \underline{\psi}_r + x_4 \underline{\psi}_2 + x_6 \underline{\psi}_3$$

$$= \begin{pmatrix} x_2 \\ \vdots \\ x_4 \\ \vdots \\ x_6 \end{pmatrix} - = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

So,  $x_1 = x_4 = x_6 = 0$ Hence,  $\{\Psi_1, \Psi_2, \Psi_3\}$  is LI.

Fri. 25/11/16

MATHI201, Algebra 1

Prof. Johnson

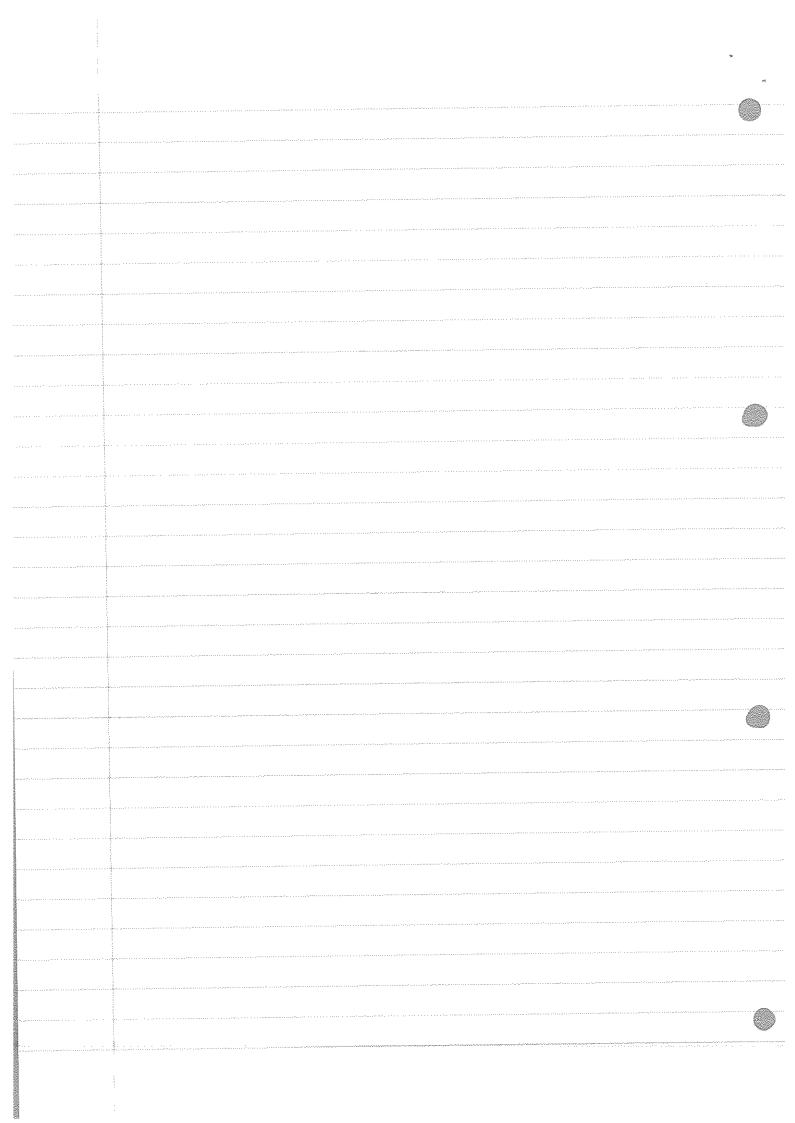
· Basis Theorem (Coming Soon)

- 1) V has at least one basis for V
- 2) Any two basis for V have the same number of elements (dimV)  $\{E_1, ..., E_n\}$  is a basis for V when
  - (i) {E₁,...,E₃} is LI.
  - (ii) {E1,..., En} spans V.

If  $2 \in V$  , we can write

 $x = x_1E_1 + x_2E_2 + x_3E_3 + ... + x_nE_n$  where  $x_1, ..., x_n \in F$ .

Linear independence tells us that this expression is unique. i.e. if  $x = y_1 E_1 + y_2 E_2 + y_3 E_3 + ... + y_n E_n$ , then ··· ②  $x_1 = y_1, x_2 = y_2, ..., x_n = y_n$ Proof. 0-0:  $0=\sum_{i=1}^{n}(x_i-y_i)E_i$  $\Rightarrow x_i - y_i = 0 \quad \forall i$  $\Rightarrow \chi_i = y_i$ ∀i · Prop. If  $\{E_1, \dots, E_n\}$  is a basis for V , then for each  $x_i \in V$  , there is a unique expression. x = x, E, + ... + x, En



Fri. 25/11/16 (cont.) MATHI201 : Algebra I Prof. Johnson Chapter 5. § Linear Mapping § 5-1 Linear Mappings and Connection to Matrices 5·1·1 Def. Let V, W be vector spaces over F.  $T: V \to W$  is a mapping. We say that T is linear when (i)  $T(\underline{x}+\underline{y}) = T(\underline{x}) + T(\underline{y})$   $\forall \underline{x}, \underline{y} \in V$ (ii)  $T(\Lambda \Sigma) = \Lambda T(\Sigma)$ √ Standard Example:  $W = \mathbb{F}^m$ V = #F"  $A = (Q_{i}) \frac{1 \le j \le m}{1 \le i \le n}$ aji € F m×n matrix Define  $T_A: \mathbb{F}^n \to \mathbb{F}^m$ by  $T_A(x) = Ax$  where x =matrix provided Ta is obviously linear. Proof:  $T_A(x+y) = A(x+y)$ = Ax + Ay  $= T_A(\underline{x}) + T_A(\underline{y})$  $T_A(\lambda x) = A(\lambda x)$  $= \Lambda A \times$  $= 7 T_A(3)$ · To what extent does an arbitrary linear map look like a standard example? ✓ EXAMPLE ②: (Differentiation) Let  $V = \{a_0 \cdot l + a_1 x + a_2 x^2 + a_3 x^2 : a_0, a_1, a_2, a_3 \in F_1\}$ Define  $D: V \rightarrow V$  to be differentiation. Denote  $a(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ , then differentiate  $\rightarrow D(a) = a_1 + 2a_2 x + 3a_3 x^2$ D(a+b) = D(a) + D(b)Then we can prove that  $D(\Lambda a) = \Lambda D(a)$ 

· Take V be a vector space with basis {E1, ..., En}, W be some other vector space. What do I need to do to specify a linear map  $T: V \rightarrow W$ ?  $\checkmark$  Ans. It is enough to specify the values  $T(E_1), ..., T(E_n)$ . √ Proof: Let Wi, ..., Wo EW. Consider the following mapping  $T: V \rightarrow W$  $T(\underline{x}) = x_1 \underline{W}_1 + x_2 \underline{W}_2 + ... + x_n \underline{W}_n$  where  $\underline{x} = x_1 \underline{E}_1 + x_2 \underline{E}_2 + x_3 \underline{E}_3 + ... + x_n \underline{E}_n$ is the unique expression for x. Claim: (1) T is linear. (2)  $T(E_i) = W_i$ Proof of (U:  $\underline{x} = \sum_{i=1}^{n} x_i \underline{E}_i$  ,  $\underline{y} = \sum_{i=1}^{n} \underline{y}_i \underline{E}_i$ Then,  $\underline{x} + \underline{y} = \sum_{i=1}^{n} x_i \underline{E}_i + \sum_{i=1}^{n} y_i \underline{E}_i$  $= \sum_{i=1}^{n} (x_i + y_i) \underline{E}_i$ Since  $T(\underline{x}) = \sum_{i=1}^{n} x_i \underline{w}_i$ ,  $T(\underline{y}) = \sum_{i=1}^{n} y_i \underline{w}_i$  $T(x+y) = \sum_{i=1}^{n} (x_i+y_i) \underline{W}_i$ = T(x) + T(y)⇒ T is additive.  $\lambda \Sigma = \sum_{i=1}^{n} (\lambda x_i) E_i$ So,  $T(\Lambda X) = \sum_{i=1}^{n} (\Lambda X_i) W_i$  $= \lambda \sum_{i=1}^{n} \chi_i \underline{W}_i$ = 71 T(x)网(1) Therefore, T is linear. Proof of (2): effectively replace j with i  $E:=\sum_{j=1}^{n} S_{ij}E_{j}$  is the unique expression of  $E_{i}$  in terms of 巨,巨,二、5  $T(E_i) = \sum_{j=1}^{n} \delta_{ij} \underline{W}_j = \underline{W}_i$ (2) So we have shown the following.  $\sqrt{1}h$ 

#20Military and a second secon	A linear map T is determined (uniquely) by the values	T(E ₁ ), T(E ₂ ),,
*Antigip Assumption Apolinaries in 40 feetiles on recession man	T(En), where (E1,, En) is a basis for damain.	katharinkonkonat intersettettetetetakan johtooloonin johtooloonin ja
	- Property 1:	er nett er skilleder skilleder stil er skilleder skilleder skilleder skilleder skilleder skilleder skilleder s De nett er skilleder
	If T: V→W is linear, then T(0)=0	
and the state of t	$Proof: O\underline{v} = O\underline{v} + O\underline{v}$	
a thail thair of for the construction of a second of the construction of the theory of the construction of	$T(0\underline{v}) = T(0\underline{v} + 0\underline{v}) = T(0\underline{v}) + T(0\underline{v})$ by def.	et en commontant et till det pellomeljon frejskje promine en communication om et til pay opgograf og gr
t delemant deleterate de deleteratura establish de semant per promone, describer	Add -T(Ov) to both sides:	
Hamilataris Pamilla Mahada Aramila Artanida Aramila Aramila Aramila Aramila Aramila Aramila Aramila Aramila Ar	0 = T(0y) + T(0y) - T(0y)	erak-userangenahannggapa pagagapangapangapangapangapanga
desirable desi	<u>O₩</u> = T( <u>O</u> ⊻) + <u>O</u>	
	So, $T(0 \lor) = 0 \lor \Rightarrow T(0) = 0$	
the literature from the state of the state o	- Property 2: Matrix Multiplication	
Ministration of the control of the c	Let T: U→V; S: V→W be linear, then	manyaisten vii juuraanaanaanaanaanaanaanaanaanaanaanaanaan
htt ppilitud paraserriudka Verilium Verilium vekininska kaladig e (nejud	S°T: U→W is also linear	20-00-00-00-00-00-00-00-00-00-00-00-00-0
	Proof: Let ¾,y ∈ U	
Protect Manufagin [confe] of Manufactures a simple consequence of the conference of	$(S \circ T)(\underline{x} + \underline{y}) = S(T(\underline{x} + \underline{y}))$	
el el l'hand d'hill and g'un agige d'hannour a para ann a ann an ann an ann an	= S (T(x)+T(y)) since T is linear	
Affiliad Ashfronous Mayorilly on a copylyte by copylyt	= S(T(x)) + S(T(y)) since S is linear	
Manuscontrate contrate and participate or contrate and produce of the contrate and	= (s•T)( <u>x</u> )+(s•T)( <u>y</u> )	
**************************************	$(S \circ T)(\Lambda \underline{x}) = S(T(\Lambda \underline{x}))$	
**************************************	= S(NT(X))	
namanananii fi saa saa saa saa saa saa saa saa saa sa	= ns(T(x))	t transfer (1 To 1995) 1998 - A Polisido Polisido Polisido A Polisido Polis
t the title the time to the	= ∧ (s∘T)( <u>×</u> )	
5.1.2	Associating a Matrix with a Linear Map	
	Let V be a vector space with basis $\mathscr{L} = \{E_1, E_2,, E_n\}$ ,	
straniga y su mono s su mono s susceptivos p sum una s su projectivos susceptivos su su projectivos susceptivos	W be a vector space with basis $\emptyset = \{\Psi_1, \Psi_2, \dots, \Psi_m\}$ .	
t o de trade de servicio como como como por tento de servicio de s	Then $dim(V) = n$ , $dim(W) = m$ .	
and mindges to minute from any or the first of the first	Let T: V→W be linear	- The second and the second
**************************************	Let W1, W2,, Wn ∈ W be the vectors s.t.	er til med er en
The second of th	$T(E_i) = W_i$	etteettita ette ette ette ette ette ette
talagajan i samasa isigan isi isi ya samuunii iyo gabayaa ya ja	Then Wi has a unique expression in terms of 4,4,,4m	recommense million of 1 (1) we had a like in the second commense and the second second second second second second
······································	$\underline{\mathbf{w}}_{i} = \sum_{i=1}^{m} a_{ji}  \underline{\mathbf{q}}_{j}$	
	— Jai v 12	1960-1968 (1970) - Welstad Bernards and Park engine in the service and activities and field (1970). A benard i

So,  $T(E_i) = \sum_{i=1}^{n} a_{ji} \varphi_j$ · Def. |5| ≤m |≤i≤n  $M(T) = (Q_{ji})$ where  $T: V \rightarrow W$  is linear  $\mathcal{E} = \{E_1, E_2, \dots, E_m\}$  is a basis for Vcalled "matrix of T wrt & on the left and  $\mathcal{Q}$  on the right" T(E) = Egit determined bu ✓ Convention chosen so as to coincide with standard example. i.e. Let A= (aji) | sj sm an ef Take {E, E, ..., En} to be standard basis for IF" Take  $\{\Psi_1, \Psi_2, \dots, \Psi_m\}$  to be standard basis for  $\mathbb{F}^m$ .  $T_A: \mathbb{F}^n \! \to \! \mathbb{F}^m$  $T_A(X) = AX$ so that M(TA) = A Proof: Calculate TA(Ei)  $T_{\Delta}(E_i) = \begin{cases} Q_{ii} & \cdots & Q_{1i} & \cdots & Q_{1n} \\ \vdots & \ddots & \ddots & \vdots \\ Q_{ji} & \cdots & Q_{ji} & \cdots & Q_{jn} \end{cases}$ 1 → ith position (row) = E aji yj That is M(Ta) = A 77/ √ EXAMPLE: 2×2 matrix over Q. Take  $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$  $\Psi_1 = e_1 + e_2$ ,  $\Psi_2 = e_1 - e_2$  $T_{A}(\underline{e}) = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2\underline{e}_{1} + \underline{e}_{2}$  $T_A(\underline{e_2}) = \underline{e_1} + 2\underline{e_2}$ ¥ e, -e2

Since T is linear,

$$T_{A}(\underline{\Psi}_{i}) = T_{A}(\underline{e}_{i} + \underline{e}_{i})$$

$$= 3\underline{e}_{i} + 3\underline{e}_{j}$$

$$= 3\underline{\Psi}_{i}$$

$$T_{A}(\underline{\Psi}_{i}) = T_{A}(\underline{e}_{i} - \underline{e}_{i})$$

$$= \underline{e}_{i} - \underline{e}_{i}$$

$$= \underline{e}_{i} - \underline$$

What's the relation between  $M(S \circ T)_{\mathcal{S}}^{\Psi}$ ,  $M(S)_{\overline{\mathcal{E}}}^{\Psi}$  and  $M(T)_{\overline{\mathcal{S}}}^{\overline{\mathcal{E}}}$ 

 $\{\Psi_1, \dots, \Psi_m\} = \mathcal{D}$ 

 $\{\psi_1,\ldots,\psi_p\}=\Psi$ 

Put 
$$q_{y}$$
 by  $q_{y}$  in  $f$   $\leftarrow$  commutativity.

So  $(s \circ T)(f_{x}) = \int_{T}^{T} \int_{T$ 

Mgn. 28/11/16

```
MATHI201: Algebra 1
                                      Prof. Johnson
 Recap:
  T: V \rightarrow W is linear.
  \mathcal{E} = \{E_1, \dots, E_n\} basis for V
  M(T)_{\mathcal{E}}^{\mathcal{I}} = (Q_{ic})_{1 \le i \le n}^{1 \le j \le m} m \times n matrix \rightarrow transformation matrix
  T(\underline{E}_i) = \sum_{i=1}^{m} \underline{Q}_{i} \underline{\psi}_i
EXAMPLE 1:
  P_n = \{a_0 + a_1x + a_2x^2 + ... + a_nx^n : a_i \in \mathbb{F}\} This means P_n is the set of polynomials of
                                                   degree ≤ n with coefficients in F
       basis: {1, x, x2, ..., xn}
       dim(P_n) = n+1
 D: P_3 \rightarrow P_3
                                                   Note: derivative of sum = sum of derivative
 D = differentiation = \frac{4}{3}
We want M(D) \in \{ \{ \} \} a matrix D with \{ \} on the left and \{ \} on the right
                                E_1 = 1
              D(1) = 0
              D(x) = 1
                               E_2 = X
 derivative
           \rightarrow D(x^2) = 2X
              D(X^3) = 3X^2
                   D(Ei) = faji Ej
             Since D(1) = 0E_1 + 0E_2 + 0E_3 + 0E_4
                          (a11 E1 + a21 E2 + a31 E3 + a41 E4) + 1st column
                                                                 put the coefficients in the 1st column
                                                                (NOT the 1st row)
              D(E_3) = D(x) = 1E_1 + 0E_2 + 0E_3 + 0E_4 \leftarrow 2^{nd}
                                                                  column
              D(E_3) = D(X^2) = 0E_1 + 2E_2 + 0E_3 + 0E_4 \leftarrow 3^{rd}
                                                                 column
              D(E_4) = D(x^3) = 0E_1 + 0E_2 + 3E_3 + 0E_4 \leftarrow 4^{th}
                                                                 column
\checkmark How about D^2 = D.D
                                                2<sup>nd</sup> derivative
```

$$M(D^3)_{\xi}^{\xi} = M(D.D)_{\xi}^{\xi}$$

$$= M(D)_{\xi}^{\xi} \cdot M(D)_{\xi}^{\xi}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Check using the same method.

$$D^{2}(1) = 0$$

$$D^{2}(x) = 0$$

$$D^{2}(x^{2}) = 2$$

$$D^{2}(x^{3}) = 6x$$

$$V = \left\{ (a_1 + a_2 x + a_3 x^2) \exp(x) : a_1, a_2, a_3 \in \mathbb{Q} \right\}$$

$$(\underline{E_1}) \qquad (\underline{E_2})$$
Then 
$$\left\{ \text{basis} : \left\{ \exp(x), x \exp(x), x^2 \exp(x) \right\} \right\}$$

$$\dim_{\mathbb{Q}}(V) = 3$$

Again, take  $D: V \rightarrow V$  differentiation is always linear

Then,

$$D(\underline{E}_1) = \exp(x) = \underline{E}_1$$

$$D(\underline{E}_2) = \exp(x) (1+x) = \underline{E}_1 + \underline{E}_2$$

$$D(\underline{E}_3) = \exp(x)(x^2+2x) = 2\underline{E}_2 + \underline{E}_3$$

$$M(D)_{\underline{E}_3} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

So,
$$M(D^{2})^{\frac{6}{6}} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$
means
$$2^{\text{nd}} \text{ derivative} = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

Similarly,  

$$M(D^{3})_{\delta}^{\delta} = M(D^{2})_{\delta}^{\delta} \cdot M(D)_{\delta}^{\delta}$$
  
 $= M(D)_{\delta}^{\delta} \cdot M(D^{2})_{\delta}^{\delta}$   
 $= \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$   
 $= \begin{pmatrix} 1 & 3 & 6 \\ 0 & 1 & 6 \end{pmatrix}$ 

Calculate 
$$\frac{d^3}{dx^3} \left( \frac{2 \exp(x) - 3x \exp(x) + 5x^2 \exp(x)}{a(x)} \right)$$

Soln: Represent 
$$a(x) = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$$
  
Then,  $D^3(a(x)) = \begin{pmatrix} 1 & 3 & 6 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$ 

$$= \begin{pmatrix} 23 \\ 27 \\ 5 \end{pmatrix}$$

This means  $23\exp(x) + 27x\exp(x) + 5x^2\exp(x)$ .

## Integration

$$D \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{differentiation}$$

$$\begin{pmatrix}
1 & 1 & 0 & | & 1 & 0 & 0 \\
0 & 1 & 2 & | & 0 & | & 0 \\
0 & 0 & | & | & 0 & 0 & |
\end{pmatrix}
\xrightarrow{\&(2,3;-2)}
\begin{pmatrix}
1 & 1 & 0 & | & | & 0 & 0 \\
0 & 1 & 0 & | & | & | & | & | & |
\\
0 & 0 & | & | & | & | & | & | & |
\end{pmatrix}$$

$$\xrightarrow{\&(1,2;-1)}
\begin{pmatrix}
1 & 0 & 0 & | & | & | & | & | & | & |
\\
0 & 1 & 0 & | & | & | & | & | & |
\\
0 & 1 & 0 & | & | & | & | & |
\\
0 & 0 & | & | & | & | & |
\end{pmatrix}$$

$$\begin{array}{c}
\underbrace{E(1,2,-1)} \\
0 & 1 & 0 & 0 & 1 & -2 \\
0 & 0 & 1 & 0 & 0 & 1
\end{array}$$

Therefore, 
$$D^{-1} = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$
 integration

## √ EXAMPLE:

$$\int \left\{ 2\exp(x) - 3x \exp(x) + 5x^2 \exp(x) \right\} dx$$

Soln: 
$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 15 \\ -13 \\ 5 \end{pmatrix}$$

$$\Rightarrow \int \left\{ 2\exp(x) - 3x \exp(x) + 5x^2 \exp(x) \right\} dx$$

$$\sim$$
 15 exp(x) -13 x exp(x) +5  $\chi^2$  exp(x) + c

## 5.3 Change of Basis Formula

· Let V be a vector space with basis

$$\mathcal{E} = \{\underline{E}_1, \underline{E}_2, \dots, \underline{E}_n\}$$
 both are basis for  $V$ .

भ = द्विग्राह्य Likewise , Ψ = Σ Qj= Ej In general,  $\varphi_i = \sum_{j=1}^n a_{ji} E_j$ So we get a matrix
(aji) | sisn (aji) is simply (aji) = M(U) \$\varphi\$  $\underline{\varphi_i} = \operatorname{Id}(\underline{\varphi_i}) = \sum_{i=1}^n Q_{ii} \underline{E_i}$ Now, express E: in terms of  $\{\Psi_1, \dots, \Psi_n\}$ 트=취향탈 Then ,  $\underline{E}_i = \sum_{j=1}^{n} b_{ji} \underline{E}_j$ Then,  $b_{ji} = M(Id)_{\delta}^{2}$ Prop.  $M(Id)^{\cancel{E}}_{\cancel{E}} = [M(Id)^{\cancel{E}}_{\cancel{E}}]^{-1}$ (use the composition formula) Proof:  $M(Id)_{\mathcal{E}}^{\mathcal{E}} = M(Id)_{\mathcal{E}}^{\mathcal{E}}$ .  $M(Id)_{\mathcal{E}}^{\mathcal{E}}$  by composition formula.  $M(Id)^{\frac{\pi}{\Phi}} = M(Id)^{\frac{\pi}{\Phi}}$ .  $M(Id)^{\frac{\pi}{\Phi}}$  $\Rightarrow M(Id)_{\overline{E}} = I_n / M(Id)_{\overline{E}}$ Fri 02/12/16 MATH1201: Algebra 1 Prof. Johnson ✓V is a vector space. dim(V) = n $\mathcal{E} = \{\underline{E}_1, \dots, \underline{E}_n\}$  both basis for V $\vec{\Phi} = \{ \underline{\Psi}_1, \dots, \underline{\Psi}_n \}$ 

Express  $\Psi$  as a linear combination in  $\{E_1, \dots, E_n\}$ .

```
T: V \rightarrow V [inear
       \checkmark Suppose we have a linear mapping represented by matrix M(T)^{\mathcal{E}}_{\mathcal{E}} of T wrt \mathcal{E}.
        ie. T(E_i) = \int_{-\infty}^{\infty} Q_{ii}E_{j}
              M(T)_{\varepsilon}^{\varepsilon} = (Q_{ji}) is known.
       √ T has a matrix M(T) ₩rt Φ.
                          T(\underline{\Psi}_i) = \sum_{j=1}^{n} b_{ji} \underline{\Psi}_j
              M(T)^{\frac{\Phi}{\Phi}} = (b_{ji})
5.3 Change of Basis
   - Q. What is the relationship between M(T)_{\mathfrak{E}}^{\Phi} and M(T)_{\mathfrak{E}}^{\mathfrak{E}}?
              We first consider
                     Id: V \rightarrow V It effectively replaces j with i Id(\underline{E}_i) = \underline{E}_i = \sum_{j=1}^{n} S_{ji} \underline{E}_j^{j}
                     So, M(Id)_{\xi}^{\varepsilon} = I_{n} = (\delta_{fc})
                     Similarly,
                                 M(Iq)^{\frac{2}{4}} = I^{u}
       \checkmark However, we can express {\mathscr Z} in terms of {\mathscr E}
                              \frac{\varphi_{i} = \sum_{j=1}^{n} C_{ji} E_{j}}{M(Id) \frac{g}{2} = (C_{ji})} \xrightarrow{1 \leq j \leq n} \frac{1}{1 \leq i \leq n}
|M(Id) \frac{g}{2} = C_{ji} = C_{ji} = C_{ji}
                Likewise, we can express \mathcal E in terms of oldsymbolarPhi

\underbrace{F_i = \sum_{j=1}^{n} f_{ji} \, f_j}_{1 \in j \leq n}

i.e. M(Id)_{\delta}^{\underline{\Phi}} = (f_{ji}) 1 \leq i \leq n

    Prop.

                           M(Id)_{\overline{\Phi}}^{\overline{\Phi}} = \left[M(Id)_{\overline{\Phi}}^{\overline{\Phi}}\right]^{-1}
                          W(Iq)^{\frac{\pi}{2}} = W(Iq \cdot Iq)^{\frac{\pi}{4}}
     √ Proof:
                                        = M(Id) M(Id) by composition formula
                          I_n = M(Id)_g^g
                                = M(Id · Id) &
                                = M(Id)_{\overline{\Phi}}^{\underline{\Phi}}, M(Id)_{\overline{E}}^{\underline{\Phi}} by composition formula
                          Therefore,
                              M(Id)_{\mathcal{E}}^{\mathcal{E}} = \left[ M(Id)_{\mathcal{E}}^{\mathcal{E}} \right]^{-1} by def.
     √Carallary. Change of Basis Formula
```

$$T: V \rightarrow V$$
 linear  $\mathcal{E} = \{ \underline{E}, \dots, \underline{E}_n \}$  both basis for  $V$   $\underline{\Phi} = \{ \underline{V}, \dots, \underline{V}_n \}$   $M(T) \underline{\Phi} = M(Id) \underline{\Phi}$   $M(T) \underline{\Phi} = M(Id) \underline{\Phi}$   $M(T) \underline{\Phi} = M(Id) \underline{\Phi}$ 

Proof: Use composition formula.

$$= M(Id)^{\frac{\pi}{2}} \cdot M(T^{\bullet}Id)^{\frac{\pi}{2}}$$

$$= M(Id)^{\frac{\pi}{2}} \cdot M(T^{\bullet}Id)^{\frac{\pi}{2}}$$

$$= M(Id)^{\frac{\pi}{2}} \cdot M(T^{\bullet}Id)^{\frac{\pi}{2}}$$

## V EXAMPLE O.

Consider 
$$T: \mathbb{Q}^3 \to \mathbb{Q}^3$$
.

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4x_1 - 2x_1 + 2x_3 \\ 3x_2 - 2x_3 \\ -x_1 + 2x_2 - x_3 \end{pmatrix}$$

T is linear

- Take standard basis

$$\mathcal{E} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$\mathbf{E}_{\mathbf{I}} \quad \mathbf{E}_{\mathbf{L}} \quad \mathbf{E}_{\mathbf{J}}$$

If this is the standard basis, just take the coefficients out.

$$M(T)_{\xi}^{\xi} = \begin{pmatrix} 4 & -2 & 2 \\ 0 & 3 & -2 \\ -1 & +2 & -1 \end{pmatrix}$$

 $M(T)_{\overline{\Phi}}^{\overline{\Phi}}$  is 'Nice'? -Can we find a basis ⊈ s.t.

Let's try 
$$\underline{\Phi} = \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$$

$$\underline{\varphi}_{1} \quad \underline{\varphi}_{2} \quad \underline{\psi}_{3} \quad \text{from standard basis to } \underline{\Psi}_{1}.$$

$$\underline{\varphi}_{1} \quad \underline{\varphi}_{2} \quad \underline{\psi}_{3} \quad \text{just take the coefficients in } \underline{\Psi}_{2}.$$

So,  $M(Id) \stackrel{\varepsilon}{\underline{\phi}} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ 

Therefore, 
$$M(Id)_{\overline{E}}^{\underline{\Phi}} = \left[M(Id)_{\overline{\Phi}}^{\underline{\Phi}}\right]^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 2 & -2 \\ 1 & -2 & 3 \end{pmatrix}$$

By change of basis formula,

$$M(T)^{\frac{\pi}{2}} = M(Id)^{\frac{\pi}{6}}M(T)^{\frac{6}{6}}M(Id)^{\frac{6}{4}}$$

$$= \begin{pmatrix} 1 & -1 & 1 \\ -1 & 2 & -2 \\ 1 & -2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 4 & -2 & 2 \\ 0 & 3 & -2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -3 & 3 \\ -2 & 4 & -4 \\ 1 & -2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

✓ EXAMPLE ②:

Consider 
$$T: \mathbb{Q}^3 \to \mathbb{Q}^3$$

$$T = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} = \begin{pmatrix} 4\chi_1 - 2\chi_1 + \chi_3 \\ 2\chi_2 \\ 3\chi_3 \end{pmatrix}$$

T is linear.

- Take standard basis.

$$\mathcal{E} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Then, 
$$M(T)^{\frac{6}{6}} = \begin{pmatrix} 4 & -2 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

- Take another basis

$$\Phi = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$

So, 
$$M(Id)^{\frac{6}{5}} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Therefore, 
$$M(Id)_{\overline{\underline{a}}}^{\underline{\underline{F}}} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix}$$

So, 
$$M(T)^{\frac{\pi}{\Phi}} = M(Id)^{\frac{\pi}{\Phi}} M(T)^{\frac{\pi}{\Phi}} M(Id)^{\frac{\pi}{\Phi}}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 4 & -2 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ 4 & -4 & 4 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \end{pmatrix}$$

mm-vertischer der der der der der der der der der d	$T(\Psi) = T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = 2\Psi_1$	, , , , , , , , , , , , , , , , , , ,
	$T(\underline{\Psi_{1}}) = T\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix} = 3\underline{\Psi_{1}}$	
	$T(\Psi_3) = T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = 4\Psi_3$	
designation would problem to the contract of t	$\Rightarrow M(T) \frac{\underline{\sigma}}{\underline{\sigma}} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$	alawa kalanda da d
oodarabeettistoora vargaarii varaa varabeetti 1866eta saad		e poes progres segment is secured invisionistic right \$55 for a protest program as a militare in human
1119 gan vermeise mis mis 150 kalent 111 mis en menenist de		sentaministrusti mitaresse og sessen gressen met sensen set etnimistret mitret til til 1555 i 1770 til 1565 ti
ygessen de de de la company		recomplication of the second
		medienisseers kritiskiskiskiskiskist. Het kritiske kisse er en ste dienet vidervisielist skiller het vidervisi
gyn, armstå en 22 villakkirk propriet er vikenes en ennem en en krik (1982).		most hammer of produce to the second to the second substitute to the second second second second second second
yyyyyyydd a gyfydd a gyllygaeth a gyllygaeth a gymraeth y dodd y chaf y		333 gammins jasseng ersenskilerinismi lapit i kultigisti di idapit qilatin 1944 mesembi milat
in province and a superior and a su		gas pumoon gaarenna elektris et 1222 e La stanta et 1222 et 1
en e		nd and de distribution of \$250 days for several and assume a self-distribution of the defendance of the second
		ologistiski sammataniski kiristindi idanialiski riskir politikati eta kament
SSEN BANTING HAN STANKS ST		my nymon historica (nh. 1865). 1865 ann an Aireann ann am Aireann an Aireann an Aireann an Aireann an Aireann a
		ne en in montre de la respectación de la regue en la menanta trada en entre de la contractiva de la contractiva
		omming mil 1777 kg klomer poor mye mark marekarek indonési (1875 kg kate ar
igassenanatzmennai suullised inkopassanaamakkokuuli dukk		and designed 1982 (Sept a method of September 2) and a member and designed a state of September 2).
esse a summining, in insufficient page agos as a season as a sum to the first to the first the terminal and the		nd 2 parl 2 pd 4 raijim 22 pa o o gara a primipinimi da parli et in 122 lank talente (1 parle et in 122 parl 1
lama di senera de ser em ente entre a membra de la costa de se desta de la costa de la costa de la costa de la		essande son en
oossaassaassa saassa ee maassa saababa saassa s		and the second
ng NEED-25-25 or 5 met on 5 memberahannang September 15 5 5 5 5 5 5 met banah		
ak ti karasan la samban samanan ti maman sayas lan kamanan ti paga		
andekalettisel limister trepumi karismusekaletistä lisisteri kansen		
meren in der stelle der der stelle		

	Fri. 02/12/16 (cont)
	MATHI201: Algebra I
***************************************	Prof. Johnson
Chapter 6.	
•	• Def
	T: V → W linear
	$\operatorname{Ker}(T) = \{ \underline{\vee} \in V, T(\underline{\vee}) = \underline{Q} \}$ kernel of $T$
	$I_{M}(T) = \{ \underline{W} \in W : \exists \underline{V} \in V, \ T(\underline{V}) = \underline{W} \}  \text{in age of } T$
	(i.e. the set of all elements in W by T when
thankalas yann an a san a	Kernel applied to V
	of T \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \
The second secon	
Annual de la constitución de la	Th. Kernel-Rank Theorem
	dim [Ker(T)] + dim [Im(T)] = dim(V) = dim(domain)
41-23-4-4	Def:
	V is a vector space over IF
	UCV. We say that U is a vector subspace of V when
A thinking for the agree of a comment of the field of the forgonium comment out fail	1) Q ∈ U
	2) YX,4EU X+4EU
Adjophysisessississannahahgimsini imadaabababababababas	3) YZEU, YNEF, NZEU
	Prop.
10000011111111111hhamman oo amayayayayaa ii oo	If UCV is a vector subspace, then U is a vector space on its own right.
al milat to the companion of the company of the com	v Proof: - U has addition.
	#: U×U → U
	$(x,y) \rightarrow x + y$ by 2)
moderni feni forskum saksjum menskum same feni se	– U has scalar multiplication.
	• : F×U→U
	$(\lambda, \underline{x}) \to \lambda \underline{x} \qquad \text{by 3}$
	- Q∈U by ?)
ennegljungen en er e jenegljung jest jest e je ammeteljungen jest som	- U has additive inverses.
	$\underline{x} \in U$ then $-\underline{x} = (-1), \underline{x} \in U$ by $3$ )
	$\frac{1}{10000000000000000000000000000000000$
:	

```
- All remaining axioms are already satisfied because
                       it is satisfied in V.
                                                                                      6-1 The Kernel is a Vector Space
     · Prop.
         T: V→W linear.
              Ker(T) = \{ \underline{V} \in V ; T(\underline{V}) = \underline{Q} \} is a vector subspace of V.
      Proof: - O_V \in Ker(T) Ker(T) has a zero.
      (since T(Q_V) = Q_V)
               - If x, y \in Ker(T), then
                         T(\underline{x}+\underline{y}) = T(\underline{x}) + T(\underline{y})
                                                   since T is linear
                                 = Q + Q
                                  = 0
                 So, x+y \in Ker(T) Ker(T) has addition.
              - If Σ∈ Ker(T), Λ∈ F, then
                           T(\lambda Z) = \lambda T(Z) since T is linear
                                  = 7.0
                                  = 0
                  So, 1≥∈ Ker(T) Ker(T) has scalar multiplication.
    6.2 The Image is a Vector Space
       · Prop
              T: V → W linear
              Im(T) = \{ w \in W : \exists v \in V, T(v) = w \} is a vector subspace of W.
        Proof: -Q_w \in I_m(T) because T(Q_v) = Q_w I_m(T) has a zero.
                  - If \underline{w}_1, \underline{w}_2 \in Im(T), choose \underline{V}_1, \underline{V}_2 \in V s.t. \underline{T}(\underline{V}_1) = \underline{W}_1, \underline{T}(\underline{V}_2) = \underline{W}_2
                          T(\underline{V_1} + \underline{V_2}) = T(\underline{V_1}) + T(\underline{V_2})
                                    = \underline{W}_1 + \underline{W}_2
                                                         I_{m}(T) has addition.
                      So, W_1 + W_2 \in Im(T)
                      If W \in Im(T), \Lambda \in \mathbb{F}, choose Y \in V s.t. T(Y) = W
                      Then
                              T(\Lambda \vee) = \Lambda T(\vee) = \Lambda \Psi
```

aparante (se anima) a se	So, $\wedge \underline{w} \in Im(T)$ . $Im(T)$ has scalar multiplication.
6.	The Kernel-Rank Thearem
**************************************	• Standard Example:
- production has been a consistent of the constitution of the cons	$ \sqrt{A = (Q_{ji})} \underset{1 \le i \le n}{ s \ne m } m \times n  \text{matrix} $
والمراجعة	(9ji) ∈ F
and the second s	$T_A \colon \mathbb{F}^n \to \mathbb{F}^m$ linear
ar se en	$\Leftrightarrow Ax = b \qquad x \in \mathbb{F}^n$
	$\Leftrightarrow T_{A}(\underline{x}) = \underline{b}$
Anhalis manayayan yangan ya famakaya manayayakan hali yayayayayayayay	- Suppose we have two solns, ≥ and y, then
harhijama karanan karani k	Ax = b $Ay = b$ $Ay = b$ $A (x-y) = b-b=0$ i.e. $Az=0$
Manager of process of	AU = b
Indiana mara a sa a sa a sa a sa a sa a sa a s	$- \operatorname{Ker}(T_A) = \{ \underline{x} \in \mathbb{F}^n, A\underline{x} = \underline{0} \}$
	This is what we have previously called Ka.
trapolity of proceed to define and includence or appearance includence.	i.e. In this case, (system)
· mellem ombjem objektimps prompting and dalaka pylykyyyy oppy	(system) (system) (system) (system)
eddddinaa gaglayaa y y y y y y y y y y y y y y y y y	- How about Im(Ta)?
for the second s	General egn: Ax=b x∈F", b∈F"
telakum kuleunthhhushkelessamurtessanist statikukleiki	$I_m(T_A) = \{b \in \mathbb{F}^m \text{ s.t. } A \underline{\times} = \underline{b} \text{ has a soln} \}$
at the second	Th. Kernel - Rank Theorem
uneende hett glock exprose humbuses highlijk kresses her uit	$T: V \rightarrow W$ linear then
-training of the state of the s	
***************************************	Proof: The typical case is where
al all the state of	$\operatorname{Ker}(T) \neq \{Q\}$ and $\operatorname{Im}(T) \neq \{Q\}$
A destructed a physiciana a sum und endendende y la specify para proper process	Plus two special cases basis: Ø
and the second	(a) $Ker(T) = \{0\}$ In these cases $\{0\}$ is the 0-dimensional
ekkilojoja pipomora viinojakoj vidagajoj pipomora viinojaja vajor	(b) $Im(T) = \{0\}$ we define dim(zero vector space over if we define dim(zero vector space) = 0
	O Typical Case.
III Nobel and The Secretary and control of the Secretary and control of the Secretary and the Secretar	Ker(T) ≠ {0} so Ker(T) has a basis
	and $Im(T) \neq \{Q\}$ so $Im(T)$ also has a basis.
	- Let (E1,, Ek) be a basis for Ker(T).
	Let {\(\text{\tensure}\),,\(\text{\tensure}\)m\) be a basis for Im(T).
The state of the s	Choose $\{E_{k+1}, \dots, E_{k+m}\} \subset V$ so that $T(F_{k+1}) = \emptyset$ . $\  \ $
	Choose $\{\underline{E}_{k+1}, \dots, \underline{E}_{k+m}\} \subset V$ so that $T(\underline{E}_{k+1}) = \underline{\varphi}_1 $ $T(\underline{E}_{k+j}) = \underline{\varphi}_j$ $T(\underline{E}_{k+m}) = \underline{\varphi}_m$

```
- Claim: \{E_1, E_2, ..., E_k, E_{k+1}, ..., E_{k+m}\} is a basis for V
              So we need to show that
                                    a) \{\underline{E}_1, \dots, \underline{E}_{k+m}\} is LI.
                       and b) {E1, ..., Extm} spans V
          - Proof of a):

Suppose \sum_{i=1}^{k+m} \lambda_i \underline{E}_i = Q (*)
                  Apply T, we get T((\Lambda_i E_i) \cup (\Lambda_i E_i) \cup ... \cup (\Lambda_{k+m} E_{k+m})) = T(Q)
\sum_{i=1}^{k+m} \Lambda_i T(E_i) = Q
Since T is linear,
as it is a
basis for Ker(T) Since [E1, ..., ER] C Ker(T), A,T(E) + A,T(E) + ... + AR+mT(ER+m)= Q
                   T(\underline{E}_{i}) = T(\underline{E}_{2}) = \dots = T(\underline{E}_{k}) = \underline{0} \qquad \text{by def. of Ker(T)}
So \underline{\mathbb{F}}_{i} \Lambda_{k+i} T(\underline{E}_{k+i}) = \underline{0} \qquad \text{since } \underline{\mathbb{F}}_{i} \Lambda_{i} T(\underline{E}_{i}) - \underline{\mathbb{F}}_{i} \Lambda_{i} T(\underline{E}_{i}) = \underline{0} - \underline{0} = \underline{0}
Since T(\underline{E}_{k+i}) = \underline{Y}_{i}
                                  M λk+1 4c = 0
                   Since \{\Psi_1, \Psi_2, \dots, \Psi_m\} is a basis for Im(T),
                               (4, 4, ..., em) is LI.
                   So \lambda_{k+1} = \lambda_{k+2} = \dots = \lambda_{k+m} = 0.
                    Substitute back into (*).
                                  <u>Σ</u>λί<u>Εί = 0</u>
                   But [E_1, E_2, ..., E_k] is LI since it is a basis for Ker(T).
                   So \lambda_1 = \lambda_2 = \dots = \lambda_k = 0
                    Therefore,
                              IniE:= 0 ⇒ Vi n=0
                                                                                                                    M LI
           - Proof of b):
                      let YEV.
                      We need to show that \underline{V} = \sum_{i=1}^{R-1} \lambda_i \underline{E}_i
                      Apply T , we get
                              W = T(\underline{V}) \in Im(T) by def. of Im(T)
                      Then, since
                                   \{\Psi, \dots, \Psi_m\} is a basis for Im(T),
                               we have
                              T(\underline{V}) = \sum_{i=1}^{m} \mu_i \underline{Y}_i \qquad \mu_i \in F
                      Put v'= EHIERT
```

```
T(V') = \sum_{i=1}^{m} \mu_i \overline{\uparrow} (E_{k+i})
T(V') = \sum_{i=1}^{m} \mu_i \overline{\uparrow} (E_{k+i})
R_{i+1}
                                                                      But we have chosen { Ext., ..., Ext., ..., Ext., ...
                                                                     so that T(Ek+j) = Yj
                       T(Y-\underline{V}')=T(\underline{V})-T(\underline{V}') since T is linear
                 Therefore.
                          \underline{V} - \underline{V}' \in \text{Ker}(T)
                                                         by def. of Ker(T)
                 Sa, write
                 V - V' = \sum_{i=1}^{k} \lambda_i E_i
\Rightarrow V = \sum_{i=1}^{k} \lambda_i E_i + V'
= \sum_{i=1}^{k} \lambda_i E_i + \sum_{j=1}^{m} \mathcal{U}_j E_{k+j}
                  So, \vee is a linear combination in \{E_1,...,E_{R+m}\}
                  Finally, put
                     Ne-j= Mj
                  So, (Er, ..., Ek+m) spans V
          - Hence , dim(V) = k+m = dim[ker(T)] + dim[Im(T)]
  Mon. 05/12/16
                               MATH1201: Algebra 1
                                      Prof. Johnson
              Kernel-Rank Theorem
• Ih.
            T:V Inear
            dim[Ker(T)] + dim[Im(T)] = dim(V)
  Note: Rank(T) = dim[Im(T)]
  ✓ proved when both Ker(T) \neq \{Q\} and Im(T) \neq \{Q\}
  Two special cases:
     1). \operatorname{Ker}(T) = \{Q\} i.e. \dim[\operatorname{Ker}(T)] = 0
           So it is sufficient to show that \dim(V) = \dim[\operatorname{Im}(T)]
       Proof:
```

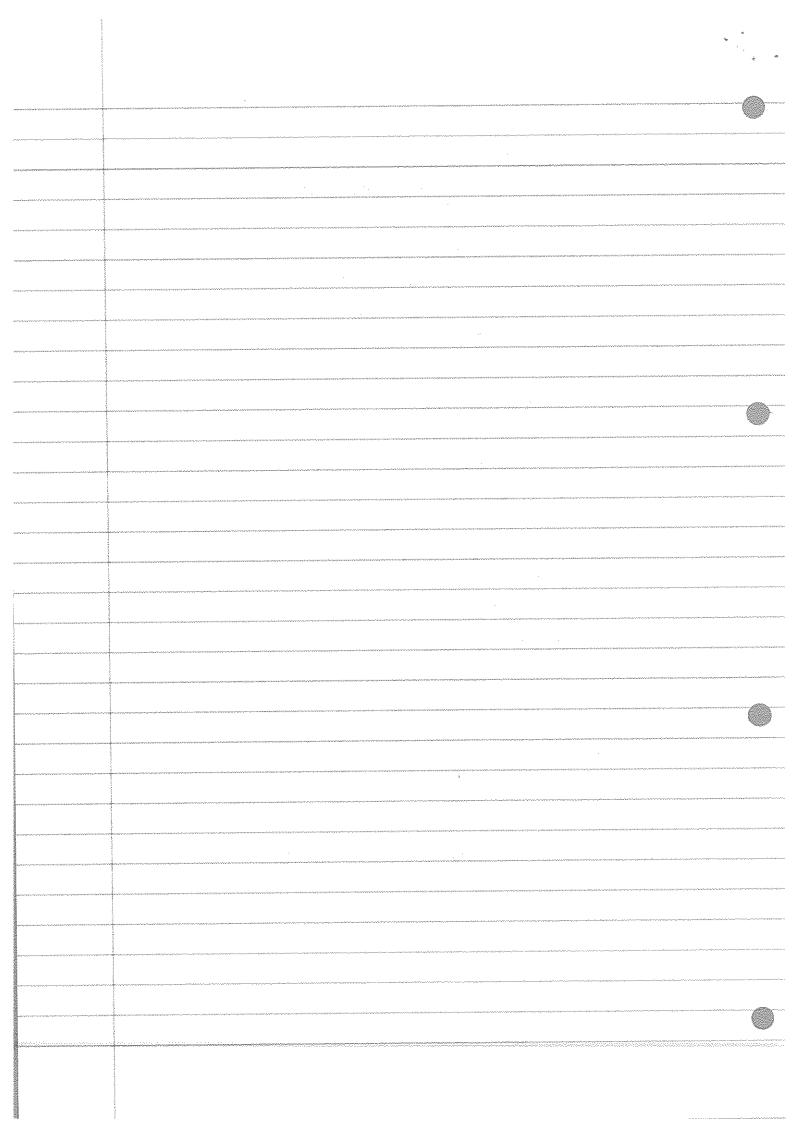
```
Take basis \{\mathcal{Q}, \dots, \mathcal{Q}_m\} for Im(T).
       Choose \{\underline{E}_i, \dots, \underline{E}_m\} \subset V s.t. T(\underline{E}_i) = \underline{\varphi}_i
        Claim: (E,..., Em) is a basis for V.
                0 (E1, ..., Em) is LI
                   Suppose we have a linear combination
                                     \sum_{i=0}^{m} \Lambda_{i} E_{i} = 0
                   Since T(E_i) = \Psi_i,
                                    Σ Ni Yo = O
                    But because 14, ... 4m is a basis for Im(T),
                              (4, ..., 4m) is LI.
                    So, \Lambda_4 = \Lambda_2 = \dots = \Lambda_m = 0
                     Therefore,
                               \sum_{i=0}^{m} \Lambda_{i} \underline{E}_{i} = \underline{0} \Rightarrow \Lambda_{i} = \Lambda_{2} = ... = \Lambda_{m} = 0
                                                                                           ØLI.
                 @ Spanning.
                      We need to find n_1, \dots, n_m \in \mathbb{F} s.t. Y = \sum_{i=1}^m n_i E_i
                      Apply T to ¥:
                                     T(Y) = Im(T)
                      Write T(v) = \sum_{i=1}^{m} N_i \Psi_i
                      Put \underline{v}' = \sum_{i=1}^{m} \lambda_i \underline{E}_i, then
                                     T(y') = \sum_{i=1}^{m} \lambda_i T(E_i) = \sum_{i=1}^{m} \lambda_i \Psi_i = T(y)
                      So , we have
                                     T(\underline{Y} - \underline{Y}') = T(\underline{Y}) - T(\underline{Y}') since T is linear
                      So, ⊻-⊻'= Ker(T)= {Q}
                      \Rightarrow \underline{V} - \underline{V}' = 0
                      i.e. v = \sum_{i=1}^{m} \lambda_i E_i
                                                                                             spanning
            Therefore, dim(V) = dim[Im(T)]
Special Case 2) Im(T) = Q def. of Im(T) = \{ w \in W, \exists y \in V \text{ s.t. } T(y) = w \}
         So Y Y € V , T(Y) = Q *
```

and a silver of a second and	So $V = Ker(T)$ by def. of $Ker(T)$	
	Then, $dim(V) = dim[ker(T)]$ i.e. $dim(V) = dim[ker(T)] + dim[Im(T)]$	<u> </u>
6.4	Connections with Injectivity, Surjectivity & Bijectivity	
	<u>Injectivity</u>	obeh (ushnore)messaassa
,	<u>Prop.</u>	
	Let T: V→W be linear.	Injective:
aran na matan kana kana kana matan na m	Then $\ker(T) = \{0\} \Leftrightarrow T$ is injective.	$f(a_i) = f(a_2)$
	i.e. dim [ker(T)]=0 ⇔ T is injective.	$G_1 = G_2$
	V Proof:	ers i mere den komunikaj projemin serse i se en elektromin milijajam yar es sem i se eminimin seljejem yapes merem
	(⇒) Assume Ker(T) = {0}	ann o an ann amhailt a 1979 (1981). Suime ann an an ann amhaideach (1984) agus ann ann ann an ann agus ann an a
	Then $T(V) = 0 \rightarrow V = 0$	
	$T(\underline{V}) = \underline{0} \Rightarrow \underline{V} = \underline{0}$ by def. of Ker (	<u> </u>
antidetetthilathis (1988), opinia i i i i opi i i i opini	Suppose $\underline{V}' \& \underline{V}'' \in V$ satisfy $\underline{T}(\underline{V}') = \underline{T}(\underline{V}'')$	defined a distribution of the continuous definition of the continuous de
t til et i til ett utstandlikken jaljaljan jangajajan en ja	Then $T(\underline{y'}-\underline{y''}) = T(\underline{y'}) - T(\underline{y''}) = \underline{Q}$	
······································		makatana kikangan di didunun seruman dalah dimindi di dimindu di menera seruma dalah gi digunun mujumun seruma dalah dinindi di dibundu di menera seruma dalah di dibundu di d
ariin da 19a daldahda aannayda ar fannayida fyraynaya aannayda	$S_0 \qquad \underline{\vee}' - \underline{\vee}'' = 0$ $\Leftrightarrow \qquad \underline{\vee}' = \underline{\vee}''$	and the second s
e y e e e e e e e e e e e e e e e e e e		kernetunduktuman para sa ammatunduktun palapat 11. 11 matumiyaktum 11. 11 matumiyaktum 11. 11 matumiyaktum 11.
t delet ad delet del playera y y conservament amendad	Therefore, T is injective.    (⇒)	tert tradition that the commence and the commence of the comme
	(⇐) Suppose $T(\underline{\vee}) = \underline{0}$ and $T(\underline{0}) = \underline{0}$ , then $\underline{\vee} = \underline{0}$	oortpunserververuutuluust televatel Heinipen proprinteeris muutatuut talankipin pipameen eessa saamuut kiipin pipameen ori muutatuusta saamuut talankipin s
žinini krimi i vinima kazijama žimija, ži viji		
6.4.2	The second secon	om 10 miles in the contract of
	Prop.	
er en	Let T: V → W be linear.	Correction of the transfer of
e de la communicación de la com	Then T is surjective $\Leftrightarrow$ Im(T) = W	Surjective: V b ∈ B , ∃ a ∈ A
	i.e. T is surjective $\Leftrightarrow$ dim[Im(T)] = dim(W).	s.t. f(a) = b
and a second confusion of the second control	✓ Praof: trivial	3.000
	Look at def. of Im(T).	e etnettittittittitti kirilinin kunin kastipani minni jungumme e e e einimminkkentaleessi kirilinin massassa saji (ja
	✓ Corollary ©:	
Ō	T: V → W linear	rterretuurt aanamassaan pirkeiste saan massaan massaan kan kan kan kan kan kan kan kan kan
enter en	J. J	

- ·

```
Then T is bijective iff Ker(T) = \{0\} & Im(T) = W
 ✓ Corollany ②:
                   T: V→W linear.
               Then T is invertible \Leftrightarrow Ker(T)= \{Q\} & Im(T) = W
• Prop.
  Let T:V→W be linear.
         If T is bijective, then T^{-1} exists and T^{-1}: W \rightarrow V is also linear.
  ✓ Proof: Let w, w ∈ W , then
                     T\left(T^{-1}\left(\underline{W}_1+\underline{W}_2\right)-T^{-1}\left(\underline{W}_1\right)-T^{-1}\left(\underline{W}_2\right)\right)
                   = TT^{-1}(\underline{W}_1 + \underline{W}_2) - TT^{-1}(\underline{W}_1) - TT^{-1}(\underline{W}_2) since T is linear
                   = \underline{W}_1 + \underline{W}_2 - \underline{W}_1 - \underline{W}_2
                   = 0
               But T is injective.
                   \Rightarrow T^{-1}(\underline{W}_1 + \underline{W}_2) - T^{-1}(\underline{W}_1) - T^{-1}(\underline{W}_2) = T(\underline{0}) = \underline{0}
                   \Rightarrow T^{-1}(\underline{W}_1 + \underline{W}_2) = T^{-1}(\underline{W}_1) + T^{-1}(\underline{W}_2)
                     i.e. T' is additive.
                Similarly,
                       T\left(T^{-1}(\Lambda \underline{W}) - \Lambda T^{-1}(\underline{W})\right)
                     = TT-1(ΛW) - ΛTT-1(W)
                     = NW-NW
                     = 0
                 But T is injective.
                     \Rightarrow T - (\(\Delta\W\) - \(\Delta\T^{\delta}(\W)\) = T(\(\Q\)\) = 0
                     \Rightarrow T^{-1}(\Lambda \Psi) = \Lambda T^{-1}(\Psi)
                Therefore, T<sup>-1</sup> is also linear
        Let A/8 be n×n matrices over F.
         If AB=In , then BA=In.
   ✓ Proof: Let Ta. F"→ F"
                      T_8: \mathbb{F}^n \to \mathbb{F}^n be linear map T_A(\underline{x}) = A\underline{x}
                                                        T_8(\underline{x}) = 8\underline{x}
               Therefore,
```

```
TAB(X) = ABX
                  = T_A T_B(x)
              AB = I_h
       Since
              TaTe = Id ← Identity matrix over F<sup>n</sup>
       So Ta surjective.
       If W \in \mathbb{F}^n, then T_A(T_B(\underline{W})) = \underline{W}
      Since \dim[\ker(T_A)] + \dim[I_m(T_A)] = n = \dim(\mathbb{F}^n),
          and Ta surjective \Rightarrow dim[Im(Ta)] = dim(F<sup>n</sup>),
        \Rightarrow dim[Ker(Ta)] = 0
       So.
           Ta is injective.
       → Ta is invertible.
       → Ta' is linear.
       So we can write
            T_A^{-1} = T_C where C = M(T_A^{-1})_{\varepsilon}^{\varepsilon} \varepsilon is the standard basis.
       So To Ta = Iden
       > TcA = IdFn
       ⇒ CA = Id<sub>E</sub>r
      But since we have A8=In
                           CA=In ,
            (CA)B = I_n.B = B
           C(AB) = C \cdot I_n = C
            ⇒ B = C
          i.e. AB = I_n
              BA = I_n
                                                               (von Neumann Property for n×n matrices)
```



Fri. 09/12/16
MATHIZOI: Algebra I
Prof. Johnson
§ The Basis Theorem §
<u>Th</u> . Basis Theorem
Let V be a non-zero vector space over F, then
(I) V has at least one basis. → existence
(II) Any two basis for V have the same number of elements = $dim_{F}(V)$
L- uniqueness
Exchange Lemma
V is a non-zero vector space over F.
(U,, VR) CV is II
[W,, Wm] CV spans
Then (i) k ≤ m and
(ii) 3 spanning set (W',, Wm') for V s.t.
Wi'= Vi for  ≤i≤k
AND $\underline{W}' \in [\underline{W},, \underline{W}_m]$ for $k \leq i$
Exchange Lemma
Baby Exchange Lemma dummy variable, so subscript obes not matte
Let $v \in V$ , $v \neq 0$ , and let $\{w_1, w_2, \dots, w_m\}$ be a spanning set for $V$ .
Write Y= 1/2 W1 + 1/2 W2 + + 2m Wm 2 EF
If $n_i \neq 0$ , then
(Wi,, We-1, V; Wir1, Wm) also spans V
here we swapped we for $\underline{V}$ as long as $\lambda_i \neq 0$
Proof: Since $V = \sum_{j=1}^{n} \gamma_j W_j$
write $v = \lambda_i \underbrace{W_i}_{j \neq i} + \sum_{j \neq i} \lambda_j \underbrace{W_j}_{j \neq i}$ where $\lambda_i \neq 0$ take out the special term
Then $\lambda_i \underline{W}_i = \underline{V} - \underline{\Sigma} \lambda_j \underline{W}_j$ ( $\lambda_i \neq 0$ so $\lambda_i \in F$ )
divide through by $\lambda_i$
$W_{i} = \left(\frac{1}{\lambda_{i}}\right) \vee + \sum_{j \neq i} \left(-\frac{\lambda_{i}}{\lambda_{i}}\right) \psi_{j} \qquad (*)$
We claim that $\{\underline{w}, \underline{w}_2, \dots, \underline{w}_{i-1}, \underline{v}, \underline{w}_{i+1}, \dots, \underline{w}_m\}$ spans $V$ .
Let ≼ ∈ V, then we can write
$\simeq = \int_{-1}^{\infty} \xi_j  \underline{W}_j$ since $\{\underline{W},, \underline{W}_m\}$ spans $V$

Then  $x = \xi_i \underline{W}_i + \sum_{i \neq j} \underline{W}_j$ substitute  $\chi = \left(\frac{\xi_i}{\lambda_i}\right) V + \xi_i \sum_{i \neq j} \left(-\frac{\lambda_i}{\lambda_i}\right) W_j + \sum_{j \neq j} \xi_j W_j$  $= \left(\frac{5i}{2i}\right) V + \sum_{j \neq i} \left(\frac{5}{5} - \frac{5}{5}i \cdot \frac{7i}{7i}\right) W_j$ Therefore,  $\times$  is a linear combination in  $\{W_1, W_2, \dots, W_{k-1}, \dots, W_m\}$ . i.e.  $\{W_1, \dots, W_{i-1}, | V_i, | W_{i+1}, \dots | W_m \}$  spans V. . We observe that any set which contains of cannot be LI.  $\checkmark$  proof: Suppose  $\{0, \underline{u}_1, \dots, \underline{u}_n\}$ , then we have 1.0 + 0. 42 + 0. 43 + ... + 0 40 = 0 with at least one coefficient #0. 7.1.2 Full Exchange Lemma: V is a non-zero vector space over F.  $\{V_1, \dots, V_R\} \subseteq V$  is LI,  $\{\underline{W}_1, \dots, \underline{W}_m\} \subset V$  spans VThen 1) k s m and 2) 3 spanning set (W, Wm') for V s.t.  $W_i' = V_i$  for  $1 \le i \le k$ AND W:' € {W, ..., Wm} for k ≤ i  $\checkmark$  Proof: (By induction on k) - The case k=1 is precisely the Baby Exchange Lemma. We put ⊻=Vi. Induction Base ✓. Then we get a spanning set  $\{\underline{w}_1, \dots, \underline{w}_{i-1}, \underline{v}_i, \underline{w}_{i+1}, \dots, \underline{w}_m\}$ We can re-index the terms s.t. Ui+1 = Wi+1 4) = N  $\underline{u}_i = \underline{v}_i$ ,  $\underline{u}_j = \underline{w}_{j-1}$  for  $2 \le j \le i$  This means  $\underline{u}_i = \underline{w}_i$ U1+2 = W1+2 (la = W2 AND Uj = Wj for j≥r+1 Um=1 = Wm-1 Ui-1 = Wi-2 Then, we have Um = Wm Ui = Wi-1 {u, u≥, ..., um} spans V Induction Step: Suppose true for k-1, so k-1≤m. And we have a spanning set {u, u2, ..., um} for V in which u = v for  $1 \le i \le k-1$  &  $u \in \{W_1, \dots, W_m\}$  for i > k-1

	If k-1=m, we would have
thille and a thirties of the thirties of the second and the second and the second and the second and the second	$\{\underline{V}_1, \dots, \underline{V}_{k-1}\}$ spans $V_{k-1}$
	Then, we could write $V_k = \sum_{j=1}^{k-1} \mu_j V_j$ (as a linear combination of the
raenarenas (glubo) (glubo) en en enege	So, $\sum_{k=1}^{k-1} \mu_k y_k + (-1)$ . $V_k = Q$ new spanning set)
Manufacturian and the same of	J=1,
	But this is a dependence relation in $\{\underline{v},\underline{v}_{\epsilon},,\underline{v}_{\epsilon}\}$ since $-1\neq 0$ .
	Contradiction, as [vi,, ve] is LI.
	So we must have k≤m. Z(1)
	2) Now we can write
and and the state of the state	2) Now we can write $\underline{V}_{k} = \sum_{j=1}^{k} \eta_{j} \underline{V}_{j} + \sum_{j=k}^{m} \eta_{j} \underline{u}_{j}  (\underline{V}_{k} \neq \underline{0} \text{ as } [\underline{V}_{1},, \underline{V}_{k}] \text{ is } \underline{I})$
	We claim that
	$\eta_{j} \neq 0$ for some j $(k \leq j)$ since $\{V_1, \dots, V_k\}$ is LI
garan aranan areen aran aran garee for aran ee be	Otherwice we get
ann ann an	$V_{k} = \sum_{j=1}^{k-1} \lambda_{j} Y_{j} \text{ which is a dependence rel! in } \{v_{1}, \dots, v_{k}\}$
	Contradiction.
	So $\eta_j \neq 0$ for some $j$ $(k \leq j \leq m)$
· · · · · · · · · · · · · · · · · · ·	WLOG, we can re-index s.t. $\eta_k \neq 0$
maanaa aa a	$V_{k} = \sum_{j=1}^{k-1} \eta_{j} V_{j} + \eta_{k} U_{k} + \sum_{k \in j} \eta_{j} U_{j}$
omora antonos fra gram ya a rayininin arkingayên	(η k ≠ 0)
milioter democratic and the second	Use Baby Exchange Lemma again.
	Swap Uk for Vk
made aminotale transfer and extremely a series of the seri	Now we have a spanning set
oreste annima persona asserbe sina pigay	$\{\underline{V}_1, \dots, \underline{V}_k, \underline{U}_{k+1}, \dots, \underline{U}_m\}$ where $\underline{U}_{k+1}, \dots, \underline{U}_m \in \{\underline{W}_1, \dots, \underline{W}_m\}$
ndirent enantes de d'inspire de colònica de destada de la colònica del colònica de la colònica de la colònica del colònica de la colònica del colònica de la colònica de la colònica del colònica de la colònica del colò	
7-2	
aliren er e er er er en er en er	Corollary: Uniqueness Part of Basis Theorem
and description and the second state of the se	Let V be a non-zero vector space.
trasa de terrestrato de la miserta e de terrestrato de la materia de la miserta de la compansión de la companda	Suppose (E,, Em) and (Q,, Yb) are basis for V.
	Then m=1.
The state of the s	Proof: Since $\{E_1, \dots, E_m\}$ is LI and $\{\Psi_1, \dots, \Psi_m\}$ spans $V$ ,
The second secon	by the exchange lemma, $m \le n$ .
	Since [4,, 4n] is LI and [E,, Em] spans V,

by the exchange lemma , n≤m.	
Then, m≤n≤m	e gengengan (1994) (1944) (1944) and and a gengenan (1944) (1944) (1944) (1944)
So we have $m = n$ .	of parallel graphic services for programs processing and classical conditions of the
· Carollary: Existence of Basis	
V 15 a non-zero vector space.	nganggang ngganaanaanaa and and and and and and and a
Then V has at least one basis	Nelsonomano/supple/Allistiffs a sequest y ancrey a transcent a
Proof:	varansamsamsineri iliteri ili iliteri
- V has at least one spanning set, namely V itself. (i.e. V spans	t projekt skriveterne er
$V = \{\underline{v}_1, \dots, \underline{v}_n\}$ is the maximal LI set of $V$	A Power of the Contract of the
If $\{v_1,, v_n\} \subset \{w_1,, w_n\}$ and $n < N$ , then $\{w_1,, w_n\}$ is not	LI.
- We need to search through <u>all</u> possible spanning sets and	anamata saturinata ina sa na galifata da manana ana a
choose one with the smallest # of elements.	magger
Write your chosen minimal spanning set (५,, ५०)	essennes et serininst triping et e energiant art mæhabblisk inn
-Claim: [4, 9]	m.jm,m.f.tyngehfft?m.mru-vianderediddrividini
If not, choose a dependence rel".	raggigue que que pare a enera esta esta esta esta esta esta esta est
$\lambda_1 \Psi_1 + \lambda_2 \Psi_2 + \dots + \lambda_n \Psi_n = 0$ where $\lambda_i \neq 0$ for some	t
We claim that $\{\Psi_1,, \Psi_{i-1}, \frac{1}{2}, \Psi_{i+1},, \Psi_m\}$ still spans.	proportion from the frequency starting determinations of the frequency from the frequency
empty space	o gangasannagas gang ang gang penembah di dibili melikulan kanda di dibili melikulan kanda di dibili melikulan
We can write $ \varphi_{i} = \sum_{j \neq i} \left( -\frac{n_{i}}{n_{j}} \right) \varphi_{j} + 0 \qquad (\#) $	respendent generalen militario (17 energi 19 militario)
	<u>ettiminen promoverioristista til et et propriet et enemiste et e</u> l di invisit e
So if $x \in V$ , we have $x = \xi_i \psi_i + \sum_{i=1}^{n} \xi_i y_i$	erglannskrumer (d. 1920 met 1947)
	y pyrymaeth i fairfichaigh aight a e ga amhaid a maeadh ardinnai an
Substitute (#): $x = \xi_i \sum_{i} \left( -\frac{2\pi}{N_i} \right) y_i + \xi_i \xi_i y_i$	ogrammene i finistrad fråmmaler ennementer framskrif som me
The contract of the contract o	n generale en en feligie begelden generale de en fille de la frie termen
$= \sum_{j \neq i} \left( \xi_j - \frac{\xi_i \lambda_i}{\lambda_j} \right) \ell_j \qquad (a linear combination)$	\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$
So, $\{\Psi_1, \dots, \Psi_{i-1}, \dots, \Psi_m\}$ still spans, and is	erest selvet (1 selten i semigemennististist virilet (1 mil 1 m I (1 papaga perpension selvet i timbe provincia di Grandia (1 mil 1 m
smaller than any supposedly smallest spanning set	gan ag ganama amanimi amanimi manami manama amana a
Contradiction.	professional services of the s
Hence, $\{\Psi_1, \dots, \Psi_n\}$ both spans and is LI.	a karalanda karaland
Hence, it is a basis for V.	

i.e.  $\{T(E_1), T(E_2), ..., T(E_n)\}$  is LI.

```
Let ₩ € W
         Put x = T^{-1}(\underline{W}) \in V
         Since {E1, E2,..., En} spans V,
                  y = \sum_{i} \lambda_i E_i
         So we have \underline{W} = T(T^{-1}(\underline{W})) = T(\underline{V}) = T(\underline{\Sigma}^{n} \wedge i\underline{E}i)
         Therefore, \{T(\underline{E}_1), T(\underline{E}_2), ..., T(\underline{E}_n)\} spans.
         So, \{T(\underline{E}_1), T(\underline{E}_1), ..., T(\underline{E}_n)\} is a basis for V.

✓ Carollary:

          V \cong W \Rightarrow dim(V) = dim(W)
  Note: converse is also true
✓ Prop.
          If dim(V) = dim(W), then V≅W.
  Proof: If dim(V) = dim(W) = 0, then
                 V = \{Q\}, W = \{Q\}
                                                 ~iso"≡~isomorphic"
          So 0→0 is iso.
     Suppose dim(V) = dim(W) = n \ge 1
          Let F" be a standard vector space with standard basis {e,...,en}.
         Let (E, E, ..., E) be a basis for V.
               \{\Psi_1, \Psi_2, \dots, \Psi_n\} be a basis for W, then
                  n = dim(v) = dim(w)
         Let S: \mathbb{F}^n \to V be a linear map, then
                S\left(\frac{x_i}{x_i}\right) = \sum_{i=1}^{n} x_i \underline{E}_i
          So, S is linear & bijective. (S invertible)
          Note: S(ei) = Ei
          Let T: \mathbb{F}^n \to W be a linear map, then
          So, T is linear & bijective. (T inverible)
          \mathbb{F}^n \cong V and so V \cong \mathbb{F}^n.
```

Since V=F"&F"=W,

	and and the contract of the co	tin di dilatan da di distanti ya aki piliki ya sana aki di
	$T \circ S^{-1} : V \to W$ is isomorphic.	
entant d'Andréanna, y projection — 19 mily de y new paper y manana.		norrannalalalarisassassannan missaplawassassassassassassassassassassassassas
n namman na na na na na garaga na garaga na	Mon. 12/12/16	erne ennigende deside er ennimente de grippe est de enverendente.
nterplant in the control of the cont	MATH(20]: Algebra I	anderes aprijes a erre aprimente kilosophisme er a aprijes trad anteknissiona y konziljenske
PodPinad Park and and and delegated delegated as a supplementary of the	Prof. Johnson	Caraman ann an Amhailt agus an an ann agus taige, an an agus taige, an an an agus taige, an an an agus taige,
and the second s	· Compute basis for Ker(Ta) and Im(Ta)	### Ast**/***** [[111115/6/67] \$11 \$#################################
enterministerheite (im jene jene jene jene jene jene jene jen	Suppose A: $m \times n$ matrix / F, so $T_A : F^n \to F^m$	d de la comunidad de de la lacción de la companya d
	TA(X) = AX	und et milledig op person in serviced to file france per per und et sich hinde pr
enem	We can compute Ker(Ta) by reducing A to row echelan form.	ere en
	Then dim[Ker(Ta)] = no. of uncircled variables	anderen (1965) er
eemanees ja ja ja ja jaa jaa ja ja ja ja ja ja j	And we also know that	The second s
	$dim[Ker(Ta)] + dim[Im(Ta)] = dim[F^{\circ}]$ Kernal-Rank Theorem	- The state of the
*************************************	$\Leftrightarrow$ dim[Im(Ta)] = dim F ⁿ - dim[Ker(Ta)]	
the_ran	of A" = n - no. of uncircled variables	interpretation and the Secretary and the Secreta
a securing a garage place gas a comment of garage place of a company of a company of a comment.	⇔ dim[Im(Ta)] = no. of circled variables	oblicero mercificare mercifica
and a suite of the	How do we compute a basis for Im(TA)?	the Art and the state of the st
and the second section of the s	$\sqrt{a_{ii}}$ $a_{i2}$ $a_{in}$ $\sqrt{x_i}$	and the state of the
eterritative e escata recorrence e tenentative e e e e e e e e e e e e e e e e e e	$A_{\mathcal{X}} = \begin{pmatrix} a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$	and the second second and second
and the second of the second o		and the second s
	(a _{m1} a _{m2} a _{m3} ··· a _{mn} \x _n / the n th column of A	er i varande et komune et e varant en kombiske kan kombiske et varant en kombiske et e kombiske et e kombiske
t millet her mar entdisse piserram maras e eeuw	$T_A(\underline{x}) = x_i Col_i(A) + x_2 Col_i(A) + + x_n Col_n(A)$ where $Col_j(A) = j^{th}$ column	of A
ы Раймын Алынай Ангер үчүү тоон айын айын үчүү түүнүү түү	$= \begin{pmatrix} q_{ij} \\ q_{ij} \end{pmatrix}$	THE EXPERIENCE STREET S
Annulus on a sure of a 2 2 2 2 2 2 2 4 2 2 2 2 2 2 4 2 2 2 2	Q <b>ii</b>	
ermandele hangsburg og grunning skriverska av det avgrund skriverska av det av grunde skriverska sk	So we get	ورسيدورو و دوالسند در سندور بهدار در استان و سندور بهدار در سندور و سن
denough of the state of the sta	V Prop.	an established adamasyon and a second purple growing and an engage of the second purple growing and a second a
All and the state of the state	$(Col_{j}(A))$ $1 \le j \le n$ is a spanning set for $Im(T_{A})$	opa (diaperini - reserva plykerini - reserva plykerini - reserva a februarijski reserva
	$\vee$ How to find a maximal LI subset of $\{Col_j(A)\}$ , $1 \le j \le n$ ?	the Mariembal Construction of the Paris of American American Construction of the Const
	- To see how to do this, consider the special case where A is in re	educed
***************************************	row echelon form.	
***************************************		

In this case, a basis is given the columns which lie above the circled variables.

i.e. a basis for Im(TA) is

$$[\Psi_1, \Psi_2, \Psi_3, \Psi_4] = \begin{cases} \int_{0}^{1} f(x) & Gold &$$

✓ In general , if A is  $m \times n$  and reduced to A' (also  $m \times n$ ), we do this by left multiplication by an invertible matrix P.

A' = PA

 $T_{A'} = T_{P} \circ T_{A}$ 

Since Tp is invertible, we get

V Prop.

 $T_P: I_m(T_A) \xrightarrow{2} I_m(T_{A'})$  is an isomorphism  $T_{P'}: I_m(T_{A'}) \xrightarrow{2} I_m(T_{A})$ 

√Cacallary:

To obtain a basis for In(Ta)

- 1) reduce A to A'
- 2) take the columns in A which lie above circled variables in A'

VEXAMPLE:

Wark over Q

Take  $T_A: \mathbb{Q}^7 \to \mathbb{Q}^3$ 

Find 1) basis for Ker(TA).

2) basis for Im(Ta).

Soln: 
$$A \rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 & 0 & -2 & 0 \\ 0 & 0 & 2 & 2 & 0 & 2 & 0 \end{pmatrix}$$

```
adjacent transposition: (1.171)
        k k+1
                                 S_{\sigma} = \{(i,j) \in S : \{i,j\} \cap \{k,k+1\} = \emptyset\} \leftarrow i \neq k, i \neq k+1, j \neq k, j \neq k+1
                                S_1 = \{(i,k): i < k\} \leftarrow j = k \Rightarrow (i,j) = (i,k)
                                S_2 = \{(i, k+1) : i < k\} \leftarrow j = k+1 \Rightarrow (i,j) = (i,k+1)
                                S_3 = \{(k, k+1)\} \leftarrow i=k, j=k+1 \Rightarrow (i,j)=(k,k+1)
                                S_4 = \{(k,j): k+1 < j\} \leftarrow i = k \Rightarrow (i,j) = (k,j).
                               S_5 = \{(k+1,j): k+1 < j\} \leftarrow i=k+1 \Rightarrow ...
                              Therefore, for any permutation \vec{e}, we clearly have
        (fixed in advance)
                                 L(\rho) = L_0(\rho) L_1(\rho) L_2(\rho) L_3(\rho) L_4(\rho) L_5(\rho)
                                            where \angle i(P) = \prod_{(i,j) \in S_t} (P(j) - P(i))
                        - With T=(k,k+1), we have
                                                                                                    şwap k&k+i
                                                      stay the same eg. \mathcal{L}_{i}(\sigma\tau) = \{(i, k+1), i < k\}
                              \mathcal{L}_{o}(\sigma_{\tau}) = \mathcal{L}_{o}(\sigma)
                            \mathcal{L}_{1}(\sigma\tau) = \mathcal{L}_{2}(\sigma)
                                                        > ewap around
                            \mathcal{L}_{2}(\sigma\tau) = \mathcal{L}_{1}(\sigma)
                                                                                                       =\mathcal{L}_{s}(\sigma)
                                                            change the sign \mathcal{L}_3(\sigma \tau) = \{(k+1,k)\}
                           L_3(\sigma \gamma) = -L_3(\sigma)
                                                       \Rightarrow swap around = - \{(k, k+1)\}
                           \mathcal{L}_4(\sigma\tau) = \mathcal{L}_5(\sigma)
                           \mathcal{L}_5(\sigma\tau) = \mathcal{L}_4(\sigma)
                                                                                                    = - \mathcal{L}_{a}(\sigma)

√ Carollary:

                                \mathcal{L}(\sigma\tau) = -\mathcal{L}(\sigma) when \tau is an adj trans
              √ Corollary:
                            If T, T; ... Tm are all adj trans,
                                  \mathcal{L}(\sigma\tau_{i}\tau_{i},\tau_{m})=(-1)^{m}\mathcal{L}(\sigma)
                  special case: \sigma = Id
                                    We cannot write Id as a product of an odd no. of adj trans.
                                    This will give \mathcal{L}(Id) = -1
                                   However, \mathcal{L}(\mathrm{Id})=1.
                                   Contradiction.

√ Corollary. Laplace's Theorem

                               If Ti .... To are all adj trans, then
                                    L(t, t2 t3 ... Tm) = (-1) mL(Id)
            - Laplace's Def. of sign(σ):
```

```
16/12/16
          Eri
                                        MATH1201. Algebra 1
                                         Prof. Johnson
          σ. [1, ..., n] bijective
          sign (\sigma) = \frac{\prod}{\sigma(j) - \sigma(i)}
                                             where L(\sigma) = \prod_{1 \le i \le j \le n}
          If \sigma = \tau_1 \tau_2 ... \tau_m where each \tau_i is adj trans,
                              sign(\sigma) = (-1)^m
   7.4.2. Prop.
                Any transposition is a product of an odd number of adj trans
           Proof: - (i.j) is any transposition.
                     Define gap(i,j) = |j-i|
                     We'll show that a transposition with gap = k is a product of (2k-1)
                  adjacent transposition.
                  - Proof (by Induction on k). __odd
                     k=1: nothing to prove since an adj trans has gap(i,j)=1
 This means
  (i+1,j) with gap=k €1Assume
                              true for k-1, and suppose 9^{ap}(i,j)=k.
is a product of
                    Then
  an odd number of
                                                    ) (i, +1) [adj trans]
                                                   ) (i+1,j) \longrightarrow according to assumption | j - (i+1) = |(J-i)-j| = |k-1| is true has gap = k-1
 adi trans.
                       i.e. (i,j) = (i,i+1)(i+1,i)(i,i+1)
                    By induction, since gap(i,j)=k, gap(i+1,j)=k-1
                     * (i+1,j) is a product
                                                    2(k-1)-1=2k-3 \text{ adj trans.}
s.t. it's odd
                    So, (i,j) is a product of 1+(2k-3)+1=2k-1 adj trans
                                                                                          I from above I
         ✓ Corollaru:
                                                 (i,i+1) (i,i+1)
                  If P is a transposition, then
                             \operatorname{Sign}(\rho) = (-1)
```

Proof:  $P = \tau_1 \tau_2 - \tau_{2k-1}$  where  $\tau_i$  is adjacent. Therefore,  $sign(e) = (-1)^{2k-1} = (-1)$ • Prop. Let  $C = \{a_1, a_2, \dots, a_n\}$  be a cycle of length n. Then C is a product of (n-1) transpositions. ( i : : : : )
( az az ... an a, )  $(a_1, ..., a_n) = (a_1, a_n)(a_1, a_{n-1})...(a_1, a_3), (a_1, a_2)$ √ Carollary: A cycle of length (n-1) is a product of adjacent transpositions. But every permutation is a product of (adjacent) transpositions, so Any permutation is a product of adjacent transpositions. - Suppose  $\sigma: \{1, 2, ..., n\}$  is bijective. Then  $\sigma = \tau_1 \tau_2 - \tau_N$  where each  $\tau_i$  is adj trans.  $sign(\sigma) = (-1)^{N}. \qquad 0$ -Suppose  $\rho:\{1,...,n\}$  is also bijective. Then  $\rho = \theta_1 \theta_2 \dots \theta_M$  where each  $\theta_i$  is adj trans. So  $sign(e) = (-1)^{M}$ - If we compose  $\sigma \& \rho$  , we get σ. ρ = τ, τ, - τ, θ, θz... θm where τi & θj adjacent Then sign  $(\sigma \rho) = (-1)^{N+M}$  $= (-1)^{N} \cdot (-1)^{M}$ By O and O , we get  $sign(\sigma \rho) = sign(\sigma)sign(\rho)$ Therefore, we have proved  $\sqrt{Pmp}$ If σ, ρ. {ν..., n} bijective, then 

```
If C = \{a_1, \dots, a_n\} is a cycle of length n, then
                sign(C) = (-1)^{n-1}
    Proof: C = (a_1, a_n) \dots (a_r, a_d)(a_1, a_d)
              and sign (a_i, a_j) = (-1)
             ⇒ sign(C) = (-1)(-1) ... (-1)(-1) = (-1)<sup>n-1</sup>

(n-1)
• Def.
         We say a permutation \sigma is even when sign(\sigma) = (+1)
                                       odd sign(o) = (-1)
 √Sa, a cycle of even length is odd.
         a cycle of odd length is even.
 \checkmark In general, if \sigma is a permutation, write
                 \sigma = C_1C_2...C_M where C_i are disjoint cycles.
              sign(\sigma) = sign(C_i)... sign(C_M)
 ✓ EXAMPLE :
              3 4 5 6 7 8 9 10 11 12 13
                                                                    15
        6 9 13 8 12 14 11 2 15 1 3 4
    = (1.5,8,11)(2,6,12,3,9)(4,13)(7,14,10,15)
  sign = (-1) . (+1) . (-1) . (-1) = (-1)
 arder = 20 (lawest common multiple of 4,5 & 2)
 \sqrt{\sigma_n} = \{f: \{1, \dots, n\} \rightarrow \{1, \dots, n\}\} where f is bijective.
     Then |\sigma_n| = n!
     \sigma_n^{\text{even}} = \{ f \in \sigma_n : sign(f) = +1 \}
     \sigma_n^{\text{odd}} = \{ f \in \sigma_n : \text{sign}(f) = -1 \}
 √ Prop.
         |\sigma_n^{\text{even}}| = |\sigma_n^{\text{odd}}| = \frac{n!}{2}
  Proof: Let 7 be your favourite transposition.
           Consider T*: On → On.
                 T_*(f) = Tf
```

√ Corollary: