1401 Mathematical Methods 1 Notes

Based on the 2016 autumn lectures by Prof R Halburd

The Author(s) has made every effort to copy down all the content on the board during lectures. The Author(s) accepts no responsibility whatsoever for mistakes on the notes nor changes to the syllabus for the current year. The Author(s) highly recommends that the reader attends all lectures, making their own notes and to use this document as a reference only.

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	Wed. 28/09/16	
~~//	MATH1401 Calculus (G	imin I)
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	Prof. Rod Halburd	
	r. halburd @ ucl. ac. uk	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
Menden at 2 ¹ and 12 are	1 Room 703	στη τη τη στη στη στη στη στη στη στη στ
1964 per et a constant e constant a constant	CHAPTER O. Differential Calculus	
	Recall that differentiation is about ra	ites of change : speeds, acceleration, et
	With distance-time graphs, question abou	•
	about gradient.	
\$	0.1 Defining differentiation	
	fixin / gradient of the	curve at P
	$f(x+h) = \frac{Q}{P}$ \approx gradient of PQ	
*****	$f(x) \qquad \qquad$	
	$\approx \frac{f(x+h) - f(x)}{h}$	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
		E Rigorous Treatment in 1101 }
	If h is small enough , df [f(x+h)-	f(x)]
a de la companya de l	gradient at $P = dx = \lim_{h \to 0} \lim_{h \to 0} \frac{h}{h}$	
	so long as the limit exists independently	of how $h \rightarrow 0$.
$-\bigcirc$	i.e. not if	fun.
	f(*)/ !xl	f(x) { sign_function"}
	f'(x) = -1	Lisgn(x)
	K ×	
		$\frac{1}{h \to 0} \left[\frac{1}{h} \right]$
	<u>Not smooth</u>	аланан талан та Талан талан тала
	Differentiable a fair barren e Casting	NOT continuous
	Differentiable everywhere ⇒ Continuous everyw	1
		MMM [MATH 7102]
	Weierstrass equation: continuous everywhere	VV, XV
	BUT differentiable nowher	$re : \qquad W \rightarrow$
······		,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,

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Use for approximation If we don't go (all the way) $h \rightarrow 0$, then $f'(x) \approx \frac{f(x+h) - f(x)}{h}$ $f(x+h) \approx f(x) + hf(x)$ so long as h is small Find \$25. EXAMPLE : so $f(x) = \sqrt[3]{x}$ and we know $\sqrt[3]{27} = 3$ If x = 27, h = -2 $\Rightarrow f(25) = f(27) + (-2) \cdot f'(27) \approx \frac{79}{27} \ (\approx 2.926)$ Geometric Application If you have two curves crossing at right angles, then locally their gradient are normal to -f(x)each other , i.e. they're negative reciprocals <- " one over " $f'(x_0) \cdot g'(x_0) = -1$ χ_{o} 9(x) Implicit Differentiation ⇒ 0.2 Sometimes you can't write y as a function of x directly, but you can still differentiate $x^2 + y^2 = r^2$ Ecircle } EXAMPLE 0: Chain Rule: f = df dydy dx rDifferentiate: $2x + 2y \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$ EXAMPLE Q: $y = x^x$ $\ln y = \ln(x^{x}) = x \ln x$ normal to 클 Natural Loa Differentiate: $\frac{1}{y} \frac{dy}{dx} = \ln x + x \frac{1}{x}$ $= \ln \chi + I$ ⇒ dy dx = ylnx+y $\frac{dy}{dx} = x^{*}(\ln x + 1)$

⇒ 0.3 Curve Sketching

Rather than being precise, curve sketching is about capturing the essential features of a graph. Consider U odd / even / neither 2) poles / singularities or crossing axes 3.) behaviour : $\rightarrow +\infty, -\infty,$ poles LAST! \neq (4.) stationary points ("extremal") \Rightarrow f'(x) = 0 5.) points of inflection \Rightarrow f"(x) = 0 Recall: odd : f(-x) = -f(x)180° symmetry in origin even: f(-x) = f(x)reflection in Y-axis _____ for all functions" "Most" functions are neither, but VF we have $\frac{f(x) = \frac{1}{2} [f(x) + f(-x)] + \frac{1}{2} [f(x) - f(-x)]}{even}$ odd Note: odd × odd = even function { more like $even \times even = even$ function $odd \times even = odd function$ EXAMPLE Q: Sketch $y = \sqrt{x^2 + x^4}$ for $x \ge 0, y \ge 0$ 1.) No symmetries 2.) No pole 3) $\chi \rightarrow +\infty$, $\chi^2 \gg \chi^{\frac{1}{4}} \Rightarrow \gamma \rightarrow \sqrt{\chi^2} = \chi$ $\chi \rightarrow -\infty$: $\chi^{\frac{1}{4}} >> \chi^2 \Rightarrow \eta \rightarrow \sqrt{\chi^4} = \chi^{\frac{1}{8}}$ 4.) $\frac{dy}{dx} = \frac{1}{2}(x^2 + x^{\frac{1}{4}})^{-\frac{1}{2}}(2x + \frac{1}{4}x^{-\frac{3}{4}}) = 0$ $2\chi = -\frac{1}{4}\chi^{-\frac{3}{4}}$ $\chi^{\frac{1}{4}} = -\frac{1}{8}$ i no solution ∴ no stationary point ⇒ It just joins up. EXAMPLE @: Sketch $y = \frac{\sqrt{x^2+1}}{(x+1)^2}$ 1.) No symmetry

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2) poles at
$$x = -1$$

 $y(x) \neq 0$, $y(0) = 1$
a) $x \Rightarrow +\infty$, $[x^{2+1}] \rightarrow [x^{2}] = [x]$
 $(x+1)^{2} \rightarrow x^{2}$, $y \rightarrow [x^{2}] = [x]$
 $(x+1)^{2} \rightarrow x^{2}$
 $(x+1)^{2} = x^{2}$
 $(x+1)^{2} = x^{2}$
 $(x+1)^{2} = x^{2}(x+1) - x^{2}$ from both sides
 $41 \frac{1}{2x} = \frac{1}{2}(x+1)^{2} - 2(x^{2}+1)^{\frac{1}{2}}(x+1)$
 $(x+1)^{2} = x(x+1)^{\frac{1}{2}}(x+1)^{-\frac{1}{2}}(x+1)$
 $= \frac{x(x^{2}+1)^{\frac{1}{2}}[x(x+1)-2(x^{2}+1)]^{\frac{1}{2}}(x+1)}{(x+1)^{2}} = 0$
 $(x^{2}+1)^{-\frac{1}{2}}[x(x+1)-2(x^{2}+1)] = 0$
 $x(x+1) = 2(x^{2}+1)$
 $x^{2} - x + 2 = 0$
No real roots
 \therefore No stationary points $\Rightarrow It$ just joins up.
Thurs $2q/0q/16$
 0 Exponential Function.
 $x(x+1) = 2(x^{2}+1)$
 $x^{2} - x + 2 = 0$
No real roots
 \therefore No stationary points $\Rightarrow It$ just joins up.
 (10) Definition, $e^{x} = 1 + x + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots$
 $= \frac{p}{1} - \frac{1}{1} - \frac{1}{1} - \frac{p}{1} - \frac{1}{1} - \frac{1}$

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• Exponential growth / decay is faster it an algebraic powers

$$\lim_{k \to 0} \left(\frac{e^{\alpha x}}{e^{\alpha y}}\right) \rightarrow 0 \quad \forall n. a \ge 0$$

$$\lim_{k \to 0} \left(\frac{e^{\alpha x}}{e^{\alpha y}}\right) \rightarrow i^{\alpha x} \quad \forall n. a \ge 0$$

$$\lim_{k \to 0} \left(\frac{e^{\alpha x}}{e^{\alpha x}}\right) \rightarrow i^{\alpha x} \quad \forall n. a \ge 0$$

$$EXAMPLE : Sketch \quad y = \frac{e^{2x} + e^{-x}}{e^{2x} - 1} \qquad x \rightarrow 0^{*}$$

$$() \quad no symmetry \\ 2) \quad pole is at $x=0$

$$y(x) \neq 0$$

$$s) \quad x \rightarrow +\infty \quad e^{2x} + e^{-x} \rightarrow e^{2x}$$

$$e^{2x} - 1 \rightarrow e^{2x} \qquad x \rightarrow \infty$$

$$(y \rightarrow \frac{e^{2x}}{e^{-x}} - e^{-x})$$

$$(y \rightarrow \frac{e^{2x}}{e^{-x}} - e^{-x})$$

$$(x \rightarrow -\infty) \quad e^{2x} + e^{-x} \rightarrow e^{2x}$$

$$(x \rightarrow -\infty) \quad e^{2x} + e^{-x} \rightarrow e^{-x}$$

$$(x \rightarrow -\infty) \quad e^{2x} + e^{-x} \rightarrow e^{-x}$$

$$(x \rightarrow -\infty) \quad e^{2x} - e^{-x}$$

$$(x \rightarrow -\infty) \quad e^$$$$

$$| = e^{y} \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{e^{y}} = \frac{1}{x}$$
This means $\int \frac{1}{x} dx - \log x + c$

$$\int_{1}^{\infty} \frac{1}{x} dx = \log x$$

$$\cdot \text{ Logarithmic growth / decay is slower than algebraic.}$$
EXAMPLE. Find derivative of $y = \arcsin x$.
$$\int \frac{1}{y} \frac{dx}{dx} = \log x$$

$$\cdot \log \frac{1}{y} = \frac{1}{x}$$

$$\int \frac{1}{y} \frac{dx}{dx} = \log x$$

$$\cdot \log \frac{1}{y} = \frac{1}{x}$$

$$\cdot \log \frac{1}{x} = \frac{1}{y} = \frac{1}{x}$$

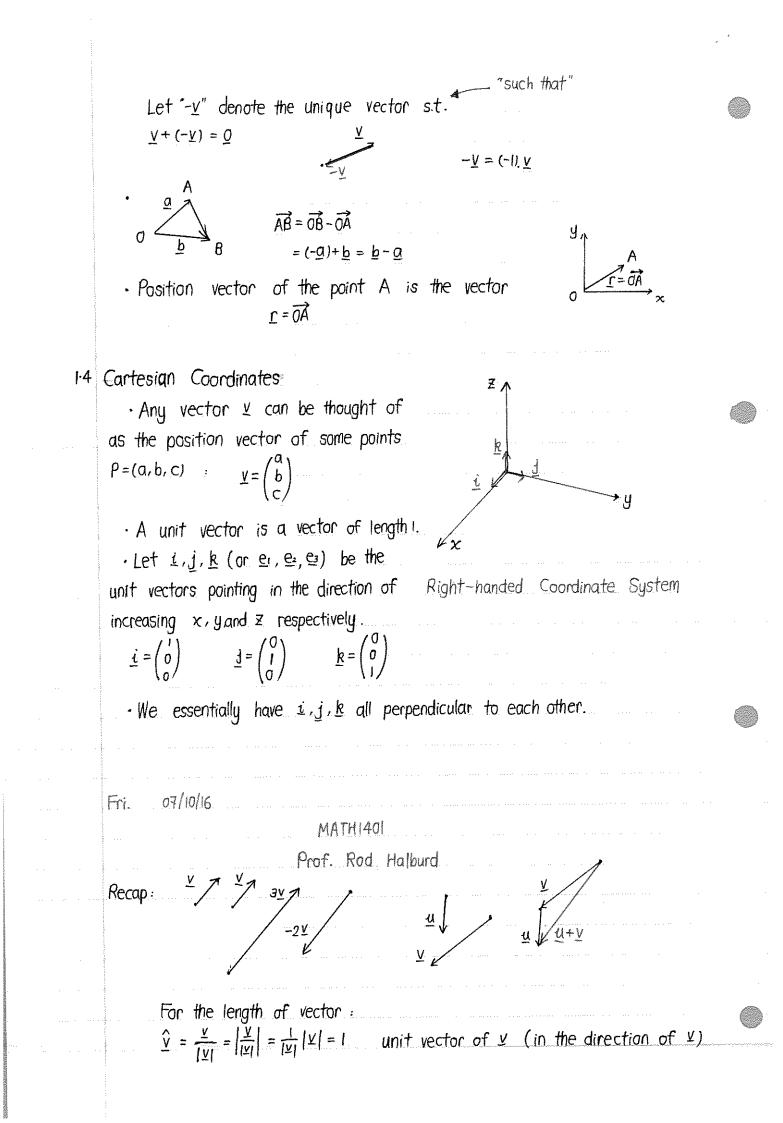
$$\cdot \ln \frac{1}{x} + \cos^{2} x = 0$$

$$\cdot \ln \frac{1}{x} + \frac{1$$

中 0.6 Hyperbolic Functions 双曲函数 • These are the odd and even part of ex sinh $x = \frac{e^{x} - e^{-x}}{2}$ cosh $x = \frac{e^{x} + e^{-x}}{2}$ • We can also define other hyperbolic trig functions. $tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$ coth $x = \frac{1}{tanh x}$ $coth x = \frac{1}{tanhx}$ $\operatorname{sech} x = \frac{1}{\cosh x}$ $cosech \chi = \frac{1}{sinh\chi}$ ____y=I u=-1 Sinhx coshx tanhx -Behaviour near 0, $\sinh x = \frac{e^{x} - e^{-x}}{2} = \frac{1 + x + \frac{x^{2}}{2!} - (1 - x + \frac{x^{2}}{2!} - \cdots)}{2}$ series form: $\sinh x = x + \frac{x^3}{3!} + - \rightarrow x$ when $x \rightarrow 0$ Similarly, $\cosh x \rightarrow 1$ $\tanh x \rightarrow x$ for small x - Behaviour as $x \to \pm \infty$: $\sinh x \to \pm e^{|x|} sgn(x)$ $\cosh x \rightarrow \frac{1}{2} e^{|x|}$ $tanhx \rightarrow sgn(x)$ · Hyperbolic identities are easier to prove than trig identities but similar. EXAMPLE: $\cosh^{2} x - \sinh^{3} x = 1$ $LHS = \left(\frac{e^{n} + e^{-x}}{2}\right)^{2} - \left(\frac{e^{x} - e^{-x}}{2}\right)^{2}$ $= \frac{1}{4} \left[e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x} \right]$ = 4 4 = 1 = RHS∴ true V x ∈ R Other trig. identities exist of course. $\cosh^2 x + \sinh^2 x = \cosh 2x$. We can differentiate them.

 $\frac{d}{dx}[\sinh x] = \cosh x$ $\frac{d}{dx}[\cosh x] = \sinh x$ $\frac{d}{dx}[\tanh x] = \operatorname{sech}^{2} x$ Note: Stanhxdx=ln/coshx · Inverse hyperbolic functions A artanhx arcoshx arsinhx tanh'x cosh"× $\sinh^{-1}x$ (NOT one-to-one function) $y = \cosh^{-1}x$ $\cosh y = x$ $\frac{e^{y}+e^{-y}}{2}=x$ Quadratic Equations $e^{2y} - 2xe^{y} + 1 = 0$ $e^{y} = \frac{2x \pm \sqrt{4x^{2} - 4}}{2} = x \pm \sqrt{x^{2} - 1}$ $y = \ln(x \pm \sqrt{x^2 - 1})$ So we say $\cosh^{-1}x = \ln(x + \sqrt{x^2 - 1})$ Similarly, arsinh $x = \ln(x + \sqrt{x^2 - 1})$ $\operatorname{artanh} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$ Mon. 03/10/16 Mathematical Methods 1: MATH1401 Prof. Rod Halburd r. halburd @ ucl. ac. uk Room 703 Office Hour: Monday at 11am Assessment Calculus Test 5% Problem Sheets [best (n-1) out of $n \ge n \ge 8$ or 9] (in Jan) 10% from problem sheets + vector test (week 5/6) + mid-sessional exan Final Exam 85%

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· .	Syllabus · Vectors
V	· Complex Numbers
	· Taylor Series
	Integration
	- Differential Equations
د و رو در	· Probability
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1.1 Introductio	- A scalar is a quantity, i.e. represented by a single (real) number
	eg. temperature, speed, distance
ין זער איז	- A vector has magnitude (i.e. length/size) and direction
	eg. force, velocity, displacement
	• We will represent any vector in R ³ (Euclidean 3-space, i.e. 3-dimensional space)
n an	as an arrow, with length represents its magnitude.
1.2 Notation	- Notations: B
	<u>V</u> (boldface letter) A AB
1.3	· Consider a displacement from A to B , and from B to C
Addition	$\beta \frac{v_2 = \vec{BC}}{c}$
Multiplicati	on VI = AB VI Parallelogram Rule
by a Scale	$\frac{1}{4} + \frac{1}{4} + \frac{1}$
111 (11 (11 (11 (11 (11 (11 (11 (11 (11	-Vector addition is commutative <u>v+u=u+v</u>
	associative $(\underline{u} + \underline{v}) + \underline{w} = \underline{u} + (\underline{v} + \underline{w})$
Члана, на полото и _{себен} устуст, столо стол, стор	• We denote zero vector by Q
	(i.e. the vector with 0 length) $\underline{V} + \underline{0} = \underline{0} + \underline{V} = \underline{V}$
1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 -	· Multiplication by a scalar:
	- Let v be a vector and let n be a non-negative scalar.
11111111111111111111111111111111111111	We define the vector $\lambda \mathbf{x}$ to be the vector pointing in the same direction as \underline{v}
······	with length $ x $, where $ y $ is the length of \underline{v} .
7 	<u>AV</u>
	-7f are loss than 0 than $3V$ where V is the second
	- If Λ is less than 0, then ΛY points in the opposite direction to Y and has length $ \Lambda Y $.



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EXAMPLE. Find a parametric equation for the line through (1.2.1) & (0, 1, -1)

$$y = p\bar{q} = {0 \choose 1} - {1 \choose 2} = {1 \choose 2}$$

$$g: (0+j-k) - (4+2j+k) = -1-j-2k$$

$$f(t) = \bar{q}\bar{p} + tx$$

$$= {1 \choose 2} + t {-1 \choose 2}$$

$$= {1 \choose 2+t}$$

$$\left[r(t) = {y \choose 1} + t {-1 \choose 2}\right]$$

$$= {1 \choose 2-t}$$

$$\left[r(t) = {y \choose 1} + t {-1 \choose 2}\right]$$

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$$\begin{array}{c|c} & \underbrace{\mathbf{x} = \mathbf{a} (\pm \mathbf{b}) \\ |\mathbf{x}| = \int \mathbf{a}^{\mathbf{a} \pm \mathbf{b}^{-1}} \\ |\mathbf{x}| = \int \mathbf{a}^{\mathbf{a} \pm \mathbf{b}^{-1}} \\ \\ \hline \\ & \text{Similarly}, \quad \underline{\mathbf{x}} = \mathbf{a} (\pm \mathbf{b}) (\pm \mathbf{c}) \\ |\mathbf{x}| = \int \mathbf{a}^{\mathbf{a} \pm \mathbf{b}^{-1}} \mathbf{c}^{\mathbf{c}} \\ \\ \hline \\ & \text{Iso} \\ \\ \hline \\ & \text{Iso} \\ \\ \hline \\ & \text{In, terms of vectors}, \\ \\ & \text{In, terms of vectors}, \\ \\ & \text{In, terms of vectors}, \\ \\ & \text{Iso} \\ \\ & \text{In, terms of vectors}, \\ \\ & \text{Iso} \\ \\ & \text{Iso}$$

$$\begin{split} \underline{b} = b \underline{i} + b \underline{i} + b \underline{k} \\ & \text{According to low of cosine :} \\ & [a - b]^{-} = [a]^{+} + [b]^{+} - 2[a][b] cos\theta \\ & \underline{[[a][b][cos\theta = a, b, + a, b, + c, b, b] = 0 + b]} \\ & \text{Scalar } \lambda \\ & & \lambda \underline{a} = (\lambda a) \underline{i} + (\lambda a) \underline{j} + (\lambda a) \underline{k} \\ \Rightarrow (\lambda a) \underline{i} + (\lambda a) \underline{j} + (\lambda a) \underline{k} \\ \Rightarrow (\lambda a) \underline{i} + (\lambda a) \underline{j} + (\lambda a) \underline{k} \\ \Rightarrow (\lambda a) \underline{i} + (\lambda a) \underline{j} + (\lambda a) \underline{k} \\ \Rightarrow (\lambda a) \underline{i} + (\lambda a) \underline{j} + (\lambda a) \underline{k} \\ \Rightarrow (\lambda a) \underline{i} + (\lambda a) \underline{j} + (\lambda a) \underline{k} \\ \Rightarrow (\lambda a) \underline{i} + (\lambda a) \underline{j} + (\lambda a) \underline{k} \\ \Rightarrow (\lambda a) \underline{i} + (\lambda a) \underline{i} + (\lambda a) \underline{k} \\ \Rightarrow (\lambda a) \underline{i} + (\lambda a) \underline{i} + (\lambda a) \underline{k} \\ \Rightarrow (a, b) \\ & = \underline{a}, (h, b) \\ & \text{Let } \underline{s} = c, \underline{i} + c \underline{i} + c \underline{k} \\ & = \underline{a}, (h, b) + (h, b) + (h, c) \underline{k} \\ & = \underline{a}, (h, b) + (h, b) + (h, c) \underline{k} \\ & = (a, b) + (a, b) + (a, b) + (c) \underline{k} \\ & = (a, b) + (a, b) + (a, b) + (c) \underline{k} \\ & = (a, b) + (a, b) + (a, b) + (c) \underline{k} \\ & = (a, b) + (a, b) + (a, b) + (c) \underline{k} \\ & = (a, b) + (a, b) + (a, b) + (c) \underline{k} \\ & = (a, b) + (a, b) + (a, b) + (c) \underline{k} \\ & = (a, b) + (a, b) + (a, b) + (c) + (a, b) + (c) \underline{k} \\ & = (a, b) + (a, b) + (a, b) + (c) + (b) + (c) \underline{k} \\ & = (a, b) + (a, b) + (a, b) + (a, b) + (c) + (b) + (c) \underline{k} \\ & = (a, b) + (a, b) + (a, b) + (c) + (a, b) + (c) +$$

c.d=0where d = q - c<u>⊆(a-c)</u>=0) expand $c.a - c^2 = 0$ $\lambda \widehat{b} \cdot a - \lambda^2 = 0$ substitute : $\Lambda(\underline{\hat{\mathbf{b}}},\underline{a}-\lambda)=0$ $\eta = \underline{a} \cdot \hat{\underline{b}}$ $\Rightarrow \underline{c} = (\underline{a}, \underline{\hat{b}}) \underline{\hat{b}}$ direction length • Given a unit vector û, we can uniquely write N V - Proja⊻ down y as a sum of a vector parallel to û and a vector orthogonal to a. Projay $\underline{v} = (\Pr_{\sigma_{j_{\widehat{\alpha}}}} \underline{v}) + (\underline{v} - \Pr_{\sigma_{j_{\widehat{\alpha}}}} \underline{v})$ 6 $= (\hat{\underline{u}}, \underline{\underline{v}})\hat{\underline{u}} + (\underline{\underline{v}} - (\hat{\underline{u}}, \underline{\underline{v}})\hat{\underline{u}})$ parallel to 2 orthogonal to 2 · Check that these vectors are orthogonal $[(\underline{\hat{u}},\underline{v}),\underline{\hat{v}}][\underline{v} - (\underline{\hat{u}},\underline{v})\underline{\hat{v}}] = (\underline{\hat{u}},\underline{v})(\underline{\hat{u}},\underline{v}) - (\underline{\hat{u}},\underline{v})^{2}\underline{\hat{u}},\underline{\hat{u}}$ <u>"û</u>.û = |û| = | $= (\widehat{\underline{u}},\underline{v})^2 - (\widehat{\underline{u}},\underline{v})^2$ = | ⇒ perpendicular to each other 14/10/16 Fri. MATH1401: Mathematical Methods 1 Prof. Rod Halburd Recap: $\underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos \theta$ $|\underline{a}| = \sqrt{\underline{a} \cdot \underline{a}}$ $\operatorname{proj}_{\mathfrak{A}} \underline{v} = (\widehat{\underline{u}} \cdot \underline{v}) \widehat{\underline{u}}$ direction length * • EXAMPLE: Write $\underline{v} = \underline{i} + 2\underline{j} + 3\underline{k}$ as the sum of a vector parallel to $\underline{u} = 3\underline{i} + 4\underline{j}$ and a vector perpendicular to u. Soln: $|u| = \sqrt{3^2 + 4^2} = 5$ W2 "soln" ≡ "solution" $\underline{\hat{\mathbf{u}}} = \frac{\underline{\mathbf{u}}}{|\underline{\mathbf{u}}|} = \frac{3\underline{\mathbf{i}} + 4\underline{\mathbf{j}}}{5} = \frac{3}{5}\underline{\mathbf{i}} + \frac{4}{5}\underline{\mathbf{j}}$ Let $\underline{W}_{i} = proj_{\underline{a}} \underline{V} = (\underline{\hat{u}}, \underline{V}) \underline{\hat{u}}$ U proj_{in}y $= \left(\frac{3}{5} + \frac{8}{5}\right) \left(\frac{3}{5!} + \frac{4}{5!}\right) = \frac{11}{5!} (3! + 4!)$ unit vector!

projection $v = W_1 + W_2$ - perpendicular vector where <u>W₂ = ⊻ - W</u> $= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - \frac{11}{25} \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ = . 1.8 Equation of a plane $Q(\chi, y, Z)$ · Let n be a normal vector to a plane. (perpendicular) and let P be a point on the plane. "orthogonal" Let Q = (x, y, z) be a general point on the plane. Now \vec{PQ} is orthogonal to <u>n</u>. either $50 \underline{n} \cdot \overrightarrow{PQ} = 0$ where $\underline{n} = n_1 \underline{i} + n_2 \underline{j} + n_3 \underline{k}$ $P = (\chi_0, y_0, Z_0)$ $\chi - \chi_0$ <u>y -y, - -0</u> n3/. Z-Z. $n_1 \chi + n_2 y + n_3 z = n_1 \chi_0 + n_2 y_0 + n_3 z_0$ EXAMPLE. Find a normal vector to the plane $3\chi + 2y - 2Z = 0$ $(\underline{n},\underline{r}=d)$ Note: Soln: r=a+nd1+µd2 $\underline{n} = (3\underline{i} + 2\underline{j} - 2\underline{k})$ Direction Cosines: $\underline{V} = |\underline{V}| \hat{\underline{V}}$ ⇒ $\hat{v} = u_1 \underline{i} + u_2 \underline{j} + u_3 \underline{k}$ $\hat{\mathcal{L}} = \underline{i} \cdot \hat{\mathcal{Y}} = \mathcal{U}_1 = \cos \alpha$ $m = j \cdot \hat{V} = U_0 = \cos\beta$ $m = \underline{k} \cdot \underline{\hat{v}} = \mathcal{U}_3 = \cos \gamma$ where α,β, X are the angles between ⊻ & i,j,k direction cosines EXAMPLE: Let A = (1,2,1) & B = (3,4,2) be two points. Find the direction cosines of the line AB. $\overrightarrow{AB} = \begin{pmatrix} 3\\4 \end{pmatrix}$ $-\binom{1}{2}$ $|\overline{AB}| = \sqrt{2^2 + 2^2 + 1^2} = 3$ $\hat{AB} =$

 $cos\alpha = \frac{2}{3}$ $\cos\beta = \frac{2}{3}$ $l^{2} + m^{2} + n^{2} = l$) COSY = = 1.12.2 Distance of a point from a plane The distance between a point Q & a plane P is distance between Q and pt the closest pt to Q on P. 'point" Note: <u>/(a-g), n</u> ∩ = $d = proj_{\hat{R}} \vec{QR}$ where A is a known Let R= (%, y., Z.) = 1 î. QR point on the plane be a pt on the plane Find the distance from (1,1,0) to the plane x+2y-2z=1• EXAMPLE, Soln: $\underline{n} = \underline{i} + 2\underline{j} - 2k$ normal : $|\underline{n}| = \sqrt{1^2 + 2^2 + (-2)^2} = 3$ $\hat{\Pi} = \frac{\Pi}{|\Pi|} = \frac{1}{2} (1 + 2j - 2k)$ Any point on the plane $R \equiv (1, 0, 0)$ is a point on the plane (that satisfies x+2y-2z=1) $Q \equiv (1,1,0)$ $\Rightarrow \vec{QR} = -j$ $d = |\hat{\underline{n}} \cdot \overline{QR}| = \frac{2}{3}$ distance 1.11 Vector Product / Cross Product / Wedge Product · Let's try to construct a product of vectors $\underline{u} \times \underline{v}$ giving a vector We want $(\Lambda \underline{U}) \times \underline{V} = \Lambda (\underline{U} \times \underline{V}) = \mathcal{M} \times (\Lambda \underline{V})$ associativitu $\underline{u} \times (\underline{v} + \underline{w}) = \underline{u} \times \underline{v} + \underline{u} \times \underline{w}$ distributivity $(\underline{v} + \underline{w}) \times \underline{u} = \underline{v} \times \underline{u} + \underline{w} \times \underline{u}$ • For two perpendicular unit vectors $\widehat{\mathfrak{U}}$, $\widehat{\mathfrak{V}}$, define $\hat{\underline{u}} \times \hat{\underline{v}}$ to be the unit vector perpendicular to $\hat{\underline{u}}$ and $\hat{\underline{v}}$, with direction given by the right hand rule. Λû×Ŷ not commutative đ Ŷ×û = -û×Ŷ <u>v × v</u> = <u>0</u> finally $\underline{i} \times \underline{i} = \underline{j} \times \underline{j} = \underline{k} \times \underline{k} = 0$

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$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\underline{i} \times \underline{j} = \underline{k}$ $\underline{j} \times \underline{i} = -\underline{k}$	<u>k</u>
$ \begin{array}{c} k \times \underline{i} = \underline{i} & \underline{i} \times \underline{k} + \underline{i} + \underline{i} \\ \underline{a} = \underline{b} \cdot \underline{i} + \underline{b} + \underline{b} \underline{k} \\ \underline{b} = \underline{b} \cdot \underline{i} + \underline{b} + \underline{b} \underline{k} \\ \underline{a} \times \underline{b} = (\underline{a} \cdot \underline{i} + \underline{a} \cdot \underline{i} + \underline{a} \cdot \underline{b} \times \underline{i} + \underline{a} + \underline{b} + $	<u>j×k=i k×j=-i</u>	
$\frac{b}{a} = b \cdot \frac{1}{2} + b \cdot \frac{1}{2} + b \cdot \frac{1}{2}$ $\frac{a \times b}{a} = (a \cdot \frac{1}{2} + a \cdot \frac{1}{2} + \frac{1}{2}$	$\underline{k} \times \underline{i} = \underline{j} \qquad \underline{i} \times \underline{k} = -\underline{j}$	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	• $a = a_1 i + a_2 j + a_3 k$	
$= a_{1}b_{1}\underline{i} \underline{i} + a_{1}b_{2}\underline{i} \underline{i} + a_{2}b_{3}\underline{i} \underline{i} + a_{2}b_{3}\underline{i} \underline{i} + a_{2}b_{3}\underline{i} \underline{i} + a_{2}b_{3}\underline{k} \underline{i} + a_{2}b_{3}\underline{i} \underline{i} \underline{i} + a_{2}b_{3}\underline{i} \underline{i} \underline{i} + a_{2}b_{3}\underline{i} \underline{i} \underline{i} + a_{2}b_{3}\underline{i} \underline{i} \underline{i} \underline{i} \underline{i} \underline{i} \underline{i} \underline{i}$	$\underline{b} = b_1 \underline{i} + b_2 \underline{j} + b_3 \underline{k}$	É U J
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\underline{a} \times \underline{b} = (a_1 \underline{i} + a_2 \underline{j} + a_3 \underline{k}) \times (b_1 \underline{i} + b_2 \underline{j} + b_3 \underline{k})$	The order cannot be reversed.
$= (a_{2}b_{1} - a_{2}b_{2})i + (a_{2}b_{1} - a_{2}b_{3})j + (a_{2}b_{2} - a_{2}b_{3})k $ $= (a_{2}b_{1} - a_{2}b_{2})i + (a_{2}b_{1} - a_{2}b_{3})k $ $= (a_{2}b_{1} - a_{3}b_{3})i - (a_{2}b_{2} - a_{2}b_{3})k $ $= (a_{2}b_{1} - a_{3}b_{3})i - (a_{2}b_{2} - a_{3}b_{3})i - (a_{2}b_{3} - a_{3}b_{3})i - (a_$	$= a_1 b_1 \underline{i} \times \underline{i} + a_1 b_2 \underline{i} \times \underline{j} + a_1 b_3 \underline{i} \times \underline{k} + a_2 b_1 \underline{j} \times \underline{i} + a_2 b_3 \underline{i} \times \underline{k} + a_3 b_3 \underline{i} \times \underline{k} + a_4 b_3 \underline{i} \times \underline{k} + a_4 b_3 \underline{i} \times \underline{k} + a_4 b_4 \underline{k} + a_5 b_3 \underline{i} \times \underline{k} + a_5 b_4 \underline{k} + a_5 b_5 \underline{k} + a_5$	$a_j \times j + a_2 b_3 j \times k + a_3 b_1 k \times i + a_3 b_3 k \times j + a_3 b_3 k \times k$
This defines the vector product $ b_1, b_2, b_3 $ = cross product $g \times b$ = $(a_1b_3 - a_3b_2)(f - (a_1b_3 - a_3b_2))(f - (a_1b_3 - a_3$	<u> </u>	<u>o i j -i o</u>
This defines the vector product $ b_1, b_2, b_3 $ = cross product $g \times b$ = $(a_1b_3 - a_3b_2)(f - (a_1b_3 - a_3b_2))(f - (a_1b_3 - a_3$	$= (a_2b_3 - a_3b_2)\underline{i} + (a_3b_1 - a_1b_3)\underline{j} + (a_1b_2 - a_2b_1)\underline{k}$	Note: $n = \begin{bmatrix} i & j & k \end{bmatrix}$
$= \operatorname{cross product} \underline{Q \times \underline{b}} \qquad = (\underline{a_1b_1 - a_1b_1}) \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{i} \underbrace{i - (a_1b_1 - a_1b_1)} \underbrace{i - (a_1b_1 - a_1b_1)}{$	This defines the vector product	
- Wedge product $a = a$ • Check: $g_{a}(\underline{a} \times \underline{b}) = \underline{b} \cdot (\underline{a} \times \underline{b}) = 0$ $\Rightarrow \underline{a} \times \underline{b}$ is orthogonal to $\underline{a} \times \underline{b}$ $\underline{a} \times \underline{b}$ orthogonal to $\underline{a} \times \underline{b}$ $\underline{b} \times \underline{b} \times \underline{c}$ $\underline{c} \times \underline$		$= (a_2b_3 - a_3b_2) \underline{i} - (a_1b_3 - a_3b_1) \underline{j}$
• Check: $ \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c}$		$+(a_1b_2-a_2b_1)\underline{k}$
$\begin{array}{c c} g.(\underline{a} \times \underline{b}) = \underline{b}.(\underline{a} \times \underline{b}) = 0 & g & g & g & g & g & g & g & g & g &$		<u>∧¤×b</u>
$\Rightarrow \underline{a \times b} \text{ is orthogonal to } \underline{a \times b} \qquad \underline{b}$ $\underline{a \times b} \text{ orthogonal to } \underline{a \times b} \text{ and } pts \text{ in the direction given by the}$ right hand rule. $\frac{a \times b}{b} \text{ orthogonal to } \underline{a \times b} \text{ and } pts \text{ in the direction given by the}$ right hand rule. $\frac{a \times b}{b} \text{ and } 2 \text{ vectors } \underline{a \times b} \text{ b}$ Let $\underline{\hat{u}} = \underline{\hat{a}} = \text{ unit vector in the direction of } \underline{a}$ Let $\underline{\hat{v}} = b = \text{ unit vector perpendicular to } \underline{\hat{u}} \text{ and in the plane spanned } by \underline{a \times b} \text{ a} \text{ and } \underline{\hat{u}} \text{ the plane spanned } by \underline{a \times b} \text{ a} \text{ and } \underline{\hat{u}} \text{ the plane spanned } by \underline{a \times b} \text{ a} $	· · · · · · · · · · · · · · · · · · ·	a contraction and a second
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		<u>P</u>
right hand rule. . We have 2 vectors $a \ b$ Let $\hat{u} = \hat{a} = unit$ vector in the direction of a Let \hat{v} be a unit vector perpendicular to \hat{u} , and in the plane spanned by $a\ b$ $\underline{a} = a_1 \hat{a}$ $\underline{b} = b_1 \hat{a} + b_2 \hat{2}$ $\underline{a} + b_2 \hat{a} + a_b \hat{2} \hat{v} \hat{v}$ $= a_1 b \hat{a} \hat{u} \hat{u} + a_b \hat{2} \hat{v} \hat{v}$ $a + b_2 \hat{a} \hat{u} \hat{v} \hat{v}$ $a + b_2 \hat{a} \hat{v} \hat{v} \hat{v}$ $a + b_2 \hat{a} \hat{v} \hat{v} \hat{v}$ $a + b_2 \hat{a} \hat{v} \hat{v} \hat{v} \hat{v}$ $a + b_2 \hat{u} \hat{v} \hat{v} \hat{v}$ $a + b_2 \hat{v} \hat{u} \hat{v} \hat{v} \hat{v}$ $a + b_2 \hat{v} \hat{v} \hat{v} \hat{v} \hat{v} \hat{v} \hat{v} \hat{v}$	3	direction given by the
. We have 2 vectors $\underline{a} \underline{b} \underline{b}$ Let $\underline{\hat{u}} = \underline{\hat{a}} = unit$ vector in the direction of \underline{a} Let $\underline{\hat{v}}$ be a unit vector perpendicular to $\underline{\hat{u}}$, and in the plane spanned by $\underline{a} \underline{b}$ $\underline{a} = a_1 \underline{\hat{u}}$ $\underline{b} = b_1 \underline{\hat{u}} + b_2 \underline{\hat{v}}$ $\underline{a} = a_1 \underline{\hat{u}} \times (b_1 \underline{\hat{u}} + b_2 \underline{\hat{v}})$ $= a_1 b_1 \underline{\hat{u}} \times \underline{\hat{u}} + a_1 b_2 \underline{\hat{u}} \times \underline{\hat{v}}$ unit vector $\underline{a} = [\underline{a}] \underline{b} = [\underline{a}] \underline{\hat{u}} \times \underline{\hat{v}} $ $= [\underline{a}] \underline{b} = [\underline{a}] \underline{b} \times \underline{\hat{v}} $ $= [\underline{a}] \underline{b} = [\underline{a}] \underline{b} \times \underline{\hat{v}} $ $= [\underline{a}] \underline{b} = [\underline{a}] \underline{b} = [\underline{b}] \underline{s} = \underline{b} $ $\underline{a} \times \underline{b} = -\underline{b} \times \underline{a}$ $\underline{a} \times (\underline{b} \times \underline{c}) \neq (\underline{a} \times \underline{b}) \times \underline{c}$ in general $eg. \underline{a} \times (\underline{a} \times \underline{b}) \neq (\underline{a} \times \underline{a}) \times \underline{b}$ $[\underline{a} \times \underline{b}] = [\underline{a}] \underline{b} \sin \theta = area of parallelogram]$ $ \underline{b} \sin \theta$ $\underline{b} = [\underline{b}] \underline{b} \sin \theta$ \underline{a}	5	0 0
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$\begin{array}{c} \underline{a} = a_{1} \underline{\hat{u}} \\ \underline{b} = b_{1} \underline{\hat{u}} + b_{2} \underline{\hat{v}} \\ \underline{a} \times \underline{b} = a_{1} \underline{\hat{u}} \times (b_{1} \underline{\hat{u}} + b_{2} \underline{\hat{v}}) \\ = a_{1} b_{1} \underline{\hat{u}} \times \underline{\hat{u}} + a_{1} b_{2} \underline{\hat{u}} \times \underline{\hat{v}} \\ \overline{u} & unit vector \\ \underline{a} = a_{1} \underline{b} \underline{\hat{u}} \times \underline{\hat{u}} + a_{1} b_{2} \underline{\hat{u}} \times \underline{\hat{v}} \\ \underline{a} & unit vector \\ \underline{a} = \underline{a} \\ \underline{a} = \underline{a} \\ \underline{b} = \underline{a} \underline{b} = \underline{a} \underline{b} \underline{\hat{u}} \times \underline{\hat{v}} \\ = a_{1} b_{2} \\ \underline{a} = a_{1} b_{2} \\ \underline{a} = a_{1} b_{2} \\ \underline{a} = a_{1} b_{1} \\ \underline{a} \times \underline{b} = -\underline{b} \times \underline{a} \\ \underline{a} \times (\underline{b} \times \underline{c}) \neq (\underline{a} \times \underline{b}) \times \underline{c} \text{in general} \\ \underline{a} \times (\underline{b} \times \underline{c}) \neq (\underline{a} \times \underline{b}) \times \underline{c} \text{in general} \\ \underline{a} \times (\underline{b} \times \underline{c}) \neq (\underline{a} \times \underline{a}) \times \underline{b} \\ \underline{a} = \underline{a} \underline{b} \underline{s} i n \theta = area \text{ of parallelogram} \\ \underline{b} \underline{s} i n \theta \\ \underline{a} = \underline{a} \underline{b} \underline{s} i n \theta = area \text{ of parallelogram} \\ \underline{b} \underline{s} i n \theta \\ \underline{a} \\ \underline{a}$	· ·	<u>q</u>
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Find the eqn of the plane containing the "eqn" = "equation" (non-collinear) pts P1, P2, P3. Soln: $\overrightarrow{P_1P_3}$ & $\overrightarrow{P_1P_2}$ lie in the plane. so a normal vector is given by $n = P_1 P_3 \times P_1 P_3$ so Pi is a pt on the plane & <u>n</u> is a normal vector [see previous example] EXAMPLE 2: Find the distance between the two skew lines $\underline{\Gamma} = \underline{O} + \underline{J} \underline{u}$ <u>r</u>=b+µ⊻ Mon. 17/10/16 MATH401: Mathematical Methods 1 Prof. Rod Halburd • Recap: $\underline{a} \times \underline{b} = (a_2b_3 - a_3b_2)\underline{i} + (a_3b_1 - a_1b_3)\underline{j} + (a_1b_2 - a_2b_1)\underline{k}$ $|\underline{a} \times \underline{b}| = |\underline{a}||\underline{b}| \sin \theta$ where $\theta \in [0, T]$ <u>∧a×p</u> -= area of parallelogram → direction is perpendicular to 9 & b and given by the 9K ЧP right hand rule. • Two alternative way of calculating axb: $g \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ Method 1: determinant $= \frac{i}{b_2} \begin{vmatrix} a_2 & a_3 \\ -j \end{vmatrix} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \frac{b_1}{b_2} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_3 \end{vmatrix}$ $= \alpha S - \beta Y$ where Index Notations 1.9 Method 2: Eik where i, j, k \in {1,2,3} (i) E123 =1 (ii) Eigk changes sign when we swap two indices For example, - Eijk = Ejik if any 2 indices are the same for even permutations of (1,2,3) for odd permutations of (1,2,3) Eijk = $0 \quad (2,1,3) \rightarrow (1,2,3) \qquad 1 \quad \text{permutation (odd)} \Rightarrow \mathcal{E}_{213} = -1$ $(3, 1, 2) \rightarrow (1, 3, 2) \rightarrow (1, 2, 3) 2 \text{ permutations (even)} \Rightarrow \mathcal{E}_{312} = 1$

 $e_1 = i$, $e_2 = j$, $e_3 = k$ ei × ej = ZEijkek under Einstein symmation convention · Einstein Summation Convention: repeated indices are summed over (from 1 to 3). EXAMPLE: $a = q_1 e_1 (= a_1 e_1 + a_2 e_2 + a_3 e_3)$ $a \times b = (a_i \underline{e}_i) \times (b_j \underline{e}_j)$ index notation = $a_i b_j (e_i \times e_j)$ = Eijkaibjek \cdot Q. Find the distance from the pt P(-1,1,-1) to the plane contains the pts A (1,0,0), B (-1,2,1) & C (0,0,1) |AP. Î]= d unit normal
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 2 vectors in the plane are $\vec{AB} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \& \vec{AC} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ $\underline{n} = \overrightarrow{AB} \times \overrightarrow{AC}$ A normal to the plane is $= \begin{bmatrix} 1 & 1 & 1 \\ -2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ 12 $\left[\underline{n} \right] = \sqrt{2^2 + 1^2 + 2^2} = 3$ $\therefore \quad \hat{\Pi} = \frac{\Pi}{|\Pi|} = \frac{1}{3} \begin{pmatrix} 2\\ 1\\ 2 \end{pmatrix} = \frac{2}{3} \underline{i} + \frac{1}{3} \underline{j} + \frac{2}{3} \underline{k}$ $\overrightarrow{AP} = (-2)\underline{t} + \underline{j} - \underline{k}$ $\Rightarrow \text{ distance } d = |\overrightarrow{AP}. \widehat{D}| = |-\frac{4}{3} + \frac{1}{3} - \frac{2}{3}| = \frac{5}{3}$ Distance between two (skew) lines 1.12.3 $\cdot r_i = a + \lambda \underline{u}$ $\mathbf{r}_2 = \mathbf{b} + \mu \mathbf{V}$ $\therefore \ \underline{\forall} \times \underline{\forall}$ is perpendicular to both $\underline{\forall}$ and $\underline{\forall}$ (i.e. to the lines) unit vector $\hat{\underline{n}} = \frac{\underline{u} \times \underline{v}}{|\underline{u} \times \underline{v}|}$ · Let w be a vector connecting any pt on the 1st line with any pt on the 2nd, then we want the length of the projection of \underline{w} in the direction $\hat{\underline{n}}$. $d = \left| \hat{n} \cdot (b - q) \right|$ 1.13.1 Volume of a parallelopiped $V = \left((\underline{a} \times \underline{b}) \cdot \underline{c} \right)$

(scalar triple product) C $\lceil a, b, c \rceil = (\underline{a} \times \underline{b}) \in$ = [b, c, a] = [c, a, b] = -[b, a, c]· Three_Identifies $\delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$ · EykEimn = SimBkn - SinSkm (summing over i) · EimnEimn = 284 (summing over m and n) (summing over i, j and k) • EijkEijk =6 (Extremely Helpful Notes see 19/10/16 Problem Class, 19/10/16 Applied Tutorial & printed notes 1401 - Vector products and indices) 1 21/10/16 Fri. MATH1401: Mathematical Methods 1 Prof. Halburd Recap: $(\underline{a} \times \underline{b}) \times \underline{c}$ · e1=i, e2=j, e=k $\underline{e}_{m} \times \underline{e}_{n} = \underline{\mathcal{E}}_{mn1} \underline{e}_{1} + \underline{\mathcal{E}}_{mn2} \underline{e}_{2} + \underline{\mathcal{E}}_{mn3} \underline{e}_{3} = \sum_{p=1}^{3} \underline{\mathcal{E}}_{mnp} \underline{e}_{p} \quad \text{where } m, n, p \in \{1, 2, 3\}$ Emmp is the coefficient of ep in the expansion of emxen e₂ $e_1 \times e_2 = e_3 = 0e_1 + 0e_2 + e_3$ $E_{121} = E_{122} = 0$ $= \hat{c}_{121} \underline{e}_1 + \hat{c}_{122} \underline{e}_2 + \hat{c}_{123} \underline{e}_3$ $\mathcal{E}_{123} = 1$ · All Eijk can be deduced from (ii) swapping 2 indices changes the sign eq. $\mathcal{E}_{123}^{(2)}$ (swapping | & 2) $\rightarrow \mathcal{E}_{213} = -\mathcal{E}_{123}$ (iii) If 2 indices are the same -+ 0 e.q. E131 (has 2 1s) → E131=0 Einstein Summation Convention repeated "j" means we're summing over "j" aj is a scalar (if j=1, then $a_j=a_i$) \underline{c}_j is a vector (if j=2, then $\underline{c}_j=\underline{c}_j$)

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,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	Similarly, <u>b=bkek</u> Therefore, <u>a×b</u> =(ajej)×(bkek)	
	$= \alpha_j b_k (c_j \times c_k)$	
	$= Q_j b_k \mathcal{E}_{jkl} \mathcal{E}_i \qquad (\underbrace{\mathcal{D}}_{ijkl} \overset{3}{\mathcal{D}}_{ijkl} \overset{3}{\mathcal{D}}_{ijkl})$	
•	Kronecker Delta	
*****	$S_{ij} = \begin{cases} i = j \\ i \neq j \end{cases}$	
	$\underline{a} \cdot \underline{b} = q_j b_j$	******
	δijaj=ai since we're summing over "j"	
	$= S_{i1}Q_1 + S_{i2}Q_2 + S_{i3}Q_3$,
	$\delta_{ij}a_ib_j = a_jb_j = g.b$	1
	$= a_1b_1 + a_2b_2 + a_bb_3$	
	3 Identities:	
	$\mathcal{E}_{ilk}\mathcal{E}_{imn} = \mathcal{E}_{im}\mathcal{E}_{km} - \mathcal{E}_{in}\mathcal{E}_{km} \qquad \left(\overset{3}{\mathfrak{D}} \right)$	
	$\mathcal{E}_{imn}\mathcal{E}_{imn} = 2\delta \mathcal{U}$ $(\tilde{\Sigma}\tilde{\Sigma})$	
	$\frac{1}{2} \frac{1}{2} \frac{1}$	
	<u>axb</u> = Eijkarbjek = dkek	
	where de = Eigealbj	
	What is (a < b) < c ? Anything that has been summed	
	$(\underline{a} \times \underline{b}) \times \underline{c} = \mathcal{E}_{ijk} d_i \mathcal{C}_{j} \underline{e}_{k}$ over is a dummy index.	
	= Eijk (Emni anbn) Cjek	
	= Eik Eimn ambn cj ek The 1st index must be the same.	
	= (SjmBkn-SjnBkm)ambnCj <u>Ck</u>	·····
	= Sjmamsknbn Cjek - SjnbnskmamCjek	
	$= (a_j C_j) b_k e_k - (b_j C_j) a_k e_k$	
	= (g.c)b - (b.c)a	- 10 11
	Triple Products	•
<u>()</u>	Scalar triple product	
	$(\underline{g} \times \underline{b}) \cdot \underline{c} = (\underline{b} \times \underline{c}) \cdot \underline{a} = (\underline{c} \times \underline{a}) \cdot \underline{b} \qquad c \xrightarrow{+} b (clockwise)$	
	$= -(\underline{C} \times \underline{b}).\underline{a} = -(\underline{b} \times \underline{a}).\underline{c} = -(\underline{a} \times \underline{c}).\underline{b}$	
3	Vector triple product	
	$\mathbf{g} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{g} \cdot \mathbf{c})\mathbf{b} - (\mathbf{g} \cdot \mathbf{b})\mathbf{c} \qquad \text{Note: } \mathbf{g} \times (\mathbf{b} \times \mathbf{c}) \neq (\mathbf{g} \times \mathbf{b}) \times \mathbf{c}$	·····
	• Jacobi Identity: $g \times (b \times c) + c \times (g \times b) + b \times (c \times g) = 0$	

Wednesday 12/10/16 MATHI401 Problem Class Prof. Wilson (Helen) helen wilson Quel-acuk GUESTIONS on Moodle ① | □ | = √ □. □ to find the length of to □ $\frac{because}{2} = \frac{2}{2} \frac{|2||2|}{2} \cos\theta \quad \text{where } \theta = 0$ $\frac{\partial}{\partial \theta} = \frac{\partial}{\partial \theta} \frac{1}{2} \frac{\partial}{\partial \theta} \frac{\partial}$ to the third side and half its length. Let $\overrightarrow{AB} = \underbrace{b}, \overrightarrow{AC} = \underbrace{c}, \underbrace{dhen}$ BC = C-E $\overrightarrow{AD} = \underline{\pm} \underline{b}$, $\overrightarrow{EC} = \underline{\pm} \overrightarrow{BC} = \underline{\pm} (\underline{c} - \underline{b})$ · AB+DE+EC -AC DE = 날으 .'. parallel and - lengths CF What do we need to show? 7 DE parallel to BC and $\left| \overrightarrow{DE} \right| = \frac{1}{2} \left| \overrightarrow{BC} \right|$ · We know : AD = = AB because D is The midpoint of AB and AE = = AR $S_0 \vec{DE} = \vec{DA} + \vec{AE}$ $= -\overrightarrow{AD} + \overrightarrow{AE}$ =-之和+之和 $= \frac{1}{2} (\overrightarrow{AC} - \overrightarrow{AB})$ = ±BC which shows both results we needed. 326 5 = -1+2j+2k 121 $\frac{\text{Pind the magnitude of } 9+2b-3c}{2+2b-3c} = \left(\frac{3}{-2}\right) + 2\left(\frac{2}{-4}\right) = 3\left(\frac{2}{-2}\right)$ $\frac{1}{z}\left(\frac{-1}{z}\right) = \left(\frac{1}{-16}\right)$ $9 + 2b - 3c = \sqrt{(00 + 256 + 12)} = \sqrt{(07)}$

(4) Find the angle between the vectors (2, -3, 6) and (1, 2, 2) $\left(\frac{-3}{2}\right) \left(\frac{1}{2}\right)$ $\frac{\binom{2}{-3}\binom{2}{2}}{\sqrt{2^2+3^2+6^2}} = \frac{2-6+12}{7\cdot 2} = \frac{8}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{\theta}{21}\right)$ a.b = |a||b| caseThe angle is $\cos^{-1}\left(\frac{2}{2i}\right)$ (i) if |a+b| = |a-b|, then a and b = to show:(i) if |a+b| = |a-b|, then a and b = then |a|=|b|(ii) if |a+b| = |a-b|, then a and b = the perpendicular. $M_{\pm}: (i) \quad (a+b) \cdot (a-b) = 0$ $\frac{a}{a} \cdot (\underline{a} - \underline{b}) + \underline{b} \cdot (\underline{a} - \underline{b}) = 0$ $\underline{a} \cdot \underline{a}^* - \underline{a} \cdot \underline{b} + \underline{b} \cdot \underline{a} - \underline{b}^* = 0$ 6.a=a.b g==6 $\underline{a} \cdot \underline{g} = \underline{b} \cdot \underline{b}$ $= \left| \underline{\alpha} \right|^{2} = \left| \underline{b} \right|^{2}$ And since both length are non-negative. <u>|9|=|b</u> (ii) $| \underline{a} + \underline{b} | = | \underline{a} - \underline{b} |$ Absalute Va $50 |a+b|^{2} = |a-b|^{2}$ (a+b), (a+b) = (a-b)(a-b)Sphare both sides a.a + 2a.b + b.b = a.a - 2a.b + b.b4q.b = 0 $\underline{a} \cdot \underline{b} = 0$ so \underline{a} and \underline{b} must be perpendicular

	Wednesday 19/10/16 MATH1401 Problem Class — Index Notation
	Dr. Wilson
,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	$ \mathbf{e}_1 = \underline{i}, \mathbf{e}_2 = \underline{j}, \mathbf{e}_3 = \underline{k}$
	$\underline{a} = (a_1, a_2, a_3)$
م در شری می از این از این از این	$= a_1 \underline{i} + a_3 \underline{k}$
	$= Q_1 \underbrace{e_1} + Q_2 \underbrace{e_2} + Q_3 \underbrace{e_3}$
	= 2 akek ket - vector
	Ref vector
ļ	2. Einstein Summation Convention:
	The notation Write fusing the Einstein summation
\bigcirc	akbk means 2 akbk convention
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	<u>Always 3D space</u>
	• Dot product:
	$\underline{a} \cdot \underline{b} = (a_1, a_2, a_3) \cdot (b_1, b_2, b_3)$
****	$= a_1 b_1 + a_2 b_2 + a_3 b_3$
	$=\sum_{k=1}^{3} \alpha_{k} \delta_{k}$
****	OR with summation convention,
	$\underline{a} \cdot \underline{b} = a_k b_k$
	• $S_{ij} = \begin{cases} 1 & i = j \end{cases}$
***	l ₀ i≠j
0	What is Sijaj?
	$\delta_{ij}a_{j} = \sum_{i=1}^{n} \delta_{ij}a_{j}$ means "sum over the repeated index"
	$= S_{i1}a_1 + S_{i2}a_2 + S_{i3}a_3$
	= ai It effectively replaces j' with i'
*****	So, δij is the (3×3) identity matrix.
5	• Rules :
#	1 A repeated index (in one expression) is called a dummy index. Its name
	does not matter.
	e.g. aikCk = ayCj
	And it can only repeat twice. aibici
<b>()</b> #	
	of the 3 possible values.
	$a_i = b_{ij} C_j$ (as an example)

	What does it mean ?	
**************************************	It means 3 things (at once) .	0
	$a_1 = b_1 C_1 + b_{12} C_2 + b_{13} C_3$ (i=1)	
	AND $a_1 = b_{21}C_1 + b_{22}C_2 + b_{23}C_3$ ( <i>i</i> =2)	anan an
	AND $a_3 = b_{31}C_1 + b_{32}C_2 + b_{33}C_3$ (i=3)	
	i (free index) labels which equation	******
	j (dummy index) is summed within each equation	
ar anna an ann an an an an an an an an an	$ (1 {ijk} = {123} \text{ or } {231} \text{ or } {312} $	
б.	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	e)
	( o otherwise	;r={n=+++++++++++++++++++++++++++++++++++
	Note:	
2007-2011-01-01-01-01-01-01-01-01-01-01-01-01-	(I) Eijk = Ejki take 1 st to the last → same group	
a	(I) Eijk = -Eiki swap any 2 of them -* different groups	
xm x x xm mmar w mmar wa 1 = 55 (m a 4 ] a 5 / a 1 4 a dama 1 a 1 x − 1 a 1 1 a 1 1	(Ⅲ) Any two the same → given zero eg. Eikk = 0	
7.	Cross Product	
	<u>a×b</u> = <u>e</u> iEijkajbk	
	Remember $a_i = \delta_{ij}a_j$	
	* We can always use sij to 'convert' j to i.	
	i.e. Sij Xjmnpg = X1mnpg (very handy)	
	Lit replaces "j with "?"	
<b>B</b>	The $\varepsilon$ fact	~~~~
	1) EighEim = Sip Skg-Sig Skp . Make sure to rotate until we get the same	
	ndex to the front	an 1990 an tha an th
	· Combine the other two	
n an an an ann ann an an an an an an an	<u>Swop are of them</u>	******
	so $\underline{a} \times (\underline{b} \times \underline{c}) = \underline{e} i \underline{\mathcal{E}}_{ijk} a_j (\underline{b} \times \underline{c})_k$	*****
	= <u>Ci</u> EijkQjEkpgbpCg	
	What I mean by $(b \times c)_k = \mathcal{E}_{kpg} b_p c_q$ is $b \times c = \mathcal{E}_k \mathcal{E}_{kpg} b_p c_q$	
	= Eiaj bp Cg Eijk Ekpg Rotate!	
	$= \underline{e}iQ_{j}b_{p}C_{g}\mathcal{E}_{kj}\mathcal{E}_{kpq}$	2000.000mm
2011-11.11.11.11.11.11.11.11.11.11.11.11.1	= $ei Q_j b_p C_q (\delta_{ip} \delta_{jq} - \delta_{iq} \delta_{jp})$	
m==10104144€=============================	= EiQjbpCq SipSjg - EiQjbpCg SigSjp	
	$\delta_{ip}b_p = b_i  \delta_{jq}C_q = C_j  \delta_{iq}C_q = C_i  \delta_{jp}b_p = b_j$	
	$\underline{a} \times (\underline{b} \times \underline{c}) = \underline{e} i \underline{a} j \underline{b} i \underline{c} j - \underline{e} i \underline{a} j \underline{b} j \underline{c} i$	-]w]m1[m16]m2m1
	= $b_i \underline{e}_i a_j C_j - C_i \underline{e}_i a_j b_j = \underline{b} (\underline{a}, \underline{c}) - \underline{c} (\underline{a}, \underline{b})$	

····	Wed. 19/10/16	
	Applied Tutorial	
aganan ang ang ang ang ang ang ang ang a	[a×b]; = Cykqjbk [it entry]	
	• EXAMPLE O:	
	$Prove  (\underline{a} \times \underline{b}). (\underline{c} \times \underline{d}) = (\underline{a} \cdot \underline{c})(\underline{b} \cdot \underline{d}) - (\underline{a} \cdot \underline{c})(\underline{b} \cdot \underline{c}) - (\underline{a} \cdot \underline{c}) - (\underline{a} \cdot \underline{c})(\underline{b} \cdot \underline{c}) - (\underline{a} \cdot \underline{c})(\underline{c} \cdot \underline{c}) - (\underline{c} \cdot \underline{c})(\underline{c} \cdot \underline{c}) - (\underline{c} \cdot \underline{c})(c$	. <u>d)(b.c)</u>
	Proof: LHS ≈ ( <u>a × b</u> ) _i ( <u>c × d</u> ) _i	
	= EijkqjbkEimnCmdn	
	= QjbrCmdn (EyjrEimn)	
	= QjbrCmdn (Sjm8rn - Sjn8rm)	
	= Qj Cm Sjmbkdn Skn - Qjdn Sjn bkl	Cm Skn
0	= Qj Cj bkdk - Qjdj bkCk	an 1999 de la 1999 de la margen d La margen de la marge
annan, ar 1909, 92 (a nan sa annan ar fa f	$= (\underline{a}.\underline{c})(\underline{b}.\underline{d}) - (\underline{a}.\underline{d})(\underline{b}.\underline{c}) = RHS$	
	• EXAMPLE $@$ : $Prove \underline{a} \cdot (\underline{b} \times \underline{c}) = \underline{b} \cdot (\underline{c} \times \underline{a}) = \underline{c} \cdot (\underline{g} \times \underline{a})$	<u>ط</u>
	$P_{roof}$ , $\underline{a} \cdot (\underline{b} \times \underline{c}) = a_i (\underline{b} \times \underline{c})_i$	Note:
	- A. Errhy Ch	
	= Qi Eijkbj Ck	1) The 1 st index must be
		The 1 st index must be corresponding to the entry
	= EijkQibjCk = bj EjkiQiCk	corresponding to the entry
	= EijkaibjCk	corresponding to the entry © Ejki aick = (S×a)j
	= EijkQibjCk = bj EjkiQiCk	convesponding to the entry © Eikiaich = (cxa)j = (axc)j
	$= \mathcal{E}_{ijk} \mathcal{Q}_{i} \mathcal{D}_{j} \mathcal{C}_{k}$ $= \mathcal{D}_{j} \mathcal{E}_{jkl} \mathcal{Q}_{i} \mathcal{C}_{k}$ $= \mathcal{D}_{j} (\subseteq \times \underline{\mathcal{Q}})_{j}$ $= \underline{\mathcal{D}}_{i} (\subseteq \times \underline{\mathcal{Q}})^{\dagger}$ $= \mathcal{C}_{k} \mathcal{E}_{kij} \mathcal{Q}_{i} \mathcal{D}_{j}$	corresponding to the entry © Ejki aick = (S×a)j
	$= \mathcal{E}_{ijk} \mathcal{Q}_{i} \mathcal{D}_{j} \mathcal{C}_{k}$ $= \mathcal{D}_{j} \mathcal{E}_{jkl} \mathcal{Q}_{i} \mathcal{C}_{k}$ $= \mathcal{D}_{j} (\subseteq \times \underline{\mathcal{Q}})_{j}$ $= \underline{\mathcal{D}}_{i} (\subseteq \times \underline{\mathcal{Q}})^{\dagger}$ $= \mathcal{C}_{k} \mathcal{E}_{kij} \mathcal{Q}_{i} \mathcal{D}_{j}$	convesponding to the entry © Eikiaich = (cxa)j = (axc)j
	$= \mathcal{E}_{ijk} \mathcal{Q}_{i} \mathcal{D}_{j} \mathcal{C}_{k}$ $= \mathcal{D}_{j} \mathcal{E}_{jkl} \mathcal{Q}_{i} \mathcal{C}_{k}$ $= \mathcal{D}_{j} (\mathcal{L} \times \mathcal{Q})_{j}$ $= \mathcal{D}_{i} (\mathcal{L} \times \mathcal{Q})^{\dagger}$	corresponding to the entry $\bigcirc E_{jki} \underline{a_ic_k} = (\underline{c} \times \underline{a})_j$ $\neq (\underline{a} \times \underline{c})_j$ The order $\underline{N}^1$
	$= \mathcal{E}_{ijk} \mathcal{Q}_{i} \mathbf{b}_{j} \mathbf{C}_{k}$ $= \mathbf{b}_{j} \mathcal{E}_{jkt} \mathcal{Q}_{i} \mathbf{C}_{k}$ $= \mathbf{b}_{j} (\mathbf{\underline{C}} \times \underline{g})_{j}$ $= \underline{\mathbf{b}}_{\cdot} (\mathbf{\underline{C}} \times \underline{g})^{\dagger}$ $= \mathbf{C}_{k} \mathcal{E}_{kij} \mathcal{Q}_{i} \mathbf{b}_{j}$ $= \mathbf{C}_{k} (\underline{\mathbf{G}} \times \underline{\mathbf{b}})_{k}$	convesponding to the entry © Eikiaich = (cxa)j = = (axc)j
	$= \mathcal{E}_{ijk} \mathcal{Q}_{i} \mathbf{b}_{j} \mathbf{C}_{k}$ $= \mathbf{b}_{j} \mathcal{E}_{jkt} \mathcal{Q}_{i} \mathbf{C}_{k}$ $= \mathbf{b}_{j} (\mathbf{\underline{C}} \times \underline{g})_{j}$ $= \underline{\mathbf{b}}_{\cdot} (\mathbf{\underline{C}} \times \underline{g})^{\dagger}$ $= \mathbf{C}_{k} \mathcal{E}_{kij} \mathcal{Q}_{i} \mathbf{b}_{j}$ $= \mathbf{C}_{k} (\underline{\mathbf{G}} \times \underline{\mathbf{b}})_{k}$	corresponding to the entry $\bigcirc E_{jki} \underline{a_ic_k} = (\underline{c} \times \underline{a})_j$ $\neq (\underline{a} \times \underline{c})_j$ The order $\underline{N}^1$
	$= \mathcal{E}_{ijk} \mathcal{Q}_{i} \mathbf{b}_{j} \mathbf{C}_{k}$ $= \mathbf{b}_{j} \mathcal{E}_{jkt} \mathcal{Q}_{i} \mathbf{C}_{k}$ $= \mathbf{b}_{j} (\mathbf{\underline{C}} \times \underline{g})_{j}$ $= \underline{\mathbf{b}}_{\cdot} (\mathbf{\underline{C}} \times \underline{g})^{\dagger}$ $= \mathbf{C}_{k} \mathcal{E}_{kij} \mathcal{Q}_{i} \mathbf{b}_{j}$ $= \mathbf{C}_{k} (\underline{\mathbf{G}} \times \underline{\mathbf{b}})_{k}$	corresponding to the entry © Eiki aich = (S × a)j = (a × c)j The order 11!
	$= \mathcal{E}_{ijk} \mathcal{Q}_{i} \mathbf{b}_{j} \mathbf{C}_{k}$ $= \mathbf{b}_{j} \mathcal{E}_{jkt} \mathcal{Q}_{i} \mathbf{C}_{k}$ $= \mathbf{b}_{j} (\mathbf{\underline{C}} \times \underline{g})_{j}$ $= \underline{\mathbf{b}}_{\cdot} (\mathbf{\underline{C}} \times \underline{g})^{\dagger}$ $= \mathbf{C}_{k} \mathcal{E}_{kij} \mathcal{Q}_{i} \mathbf{b}_{j}$ $= \mathbf{C}_{k} (\underline{\mathbf{G}} \times \underline{\mathbf{b}})_{k}$	corresponding to the entry © Eiki aich = (S × a)j = (a × c)j The order 11!

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	MATH1401: Mathematical Methods 1	1999	
nne 400 - 1 mil 10 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1	Chapter 2. § Complex Numbers §		
	Introduction		
	$x+iy$ , where $x, y \in \mathbb{R}$ , is a complex number.	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	
mmh(mhm()h) par a se an	Addition: $(\chi_1 + i y_1) + (\chi_2 + i y_2) = (\chi_1 + \chi_2) + i(y_1 + y_2)$		
-	Multiplication: $(x_1 + iy_1)(x_2 + iy_2) = x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2$		
	$= (\chi_1 \chi_2 - y_1 y_2) + i(\chi_1 y_2 + y_1 \chi_2)$	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	
••••	Division: $\frac{1}{x+iy}$ where $(x,y) \neq (0,0)$		
n fa far frihas fri fait star a sa ser ar sa ser a	= 1 x-iy multiply numerator &	denominator	
- Q	bu complex capiugate	2 	
	$= \frac{\chi - iy}{\chi^{i} + y^{i}}$		
1999 C 1, P. 199 M 10000 - C C C C C C C.	$= \frac{\chi}{\chi^{2} + y^{2}} + i\left(\frac{-y}{\chi^{2} + y^{2}}\right)$		
	Def. A complex number is a pair of real numbers	<u>9</u>	
	(x,y) with the operations:	$(\mathbf{x},\mathbf{y}) \equiv \mathbf{x} + \mathbf{i}\mathbf{y}$	
	$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$	10	
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	$(x_1, y_1)(x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + y_1 x_2)$	ź	
	$(\chi, y)^{-1} = \left(\frac{\chi}{\chi^2 + y^2}, -\frac{y}{\chi^2 + y^2}\right)$		
mana (anang 2003) pang ang ang ang ang ang ang ang ang ang	(e. ~ (e. ~)		
	Note: Write $(x, y) = x + iy$ (1,0))'='i'	
0	$= \chi (1,0) + y(0,1)$ (0,1)	'≡'j'	
	(x, 0) is a 'copy' of the real numbers.		
1	z = x + iy (assume x, y are real)	E1+E2 AE2	
	where $\mathbf{x} = \operatorname{Re}(\mathbf{Z})$ real part of \mathbf{Z}		
nume) (s) and s a start of a start	y = Im (Z) imaginary part of Z	17.	
•	$\overline{z} = x - iy = \overline{z}^*$ (complex) conjugate of \overline{z}		
•	$ z = \sqrt{x^2 + y^2} \text{modulus of } z$	x+iy - 1:	
•	$\theta = \arg(z)$ argument of z	z y	
	$(defined up to integer multiples of 2\pi)$	χ.	
	$-\pi < \arg(z) \leq \pi$	*****	
	$x = r\cos\theta$, $y = r\sin\theta$		
		1144100001882211111111111111111111111111	
	$Z = \Gamma(\cos\theta + i\sin\theta)$ polar form		

		r B
	Geometry in the complex plane	
3.5.1	<u>Circle</u> : <u>complex numbers</u>	
100-100 marca and a second	(The set of all $z \in C$ s.t.) $ z-z_0 = \rho$, for some $\rho > 0$	
**************************************	$\sqrt{\text{set of pts distant } e \text{ from } z_0}$	
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	$\checkmark$ circle of radius $\ell$ , centred at $z_0$ $Z_0$	**************************************
	· Algebraic	
	$Z = \chi + i y \qquad Z_0 = \chi_0 + i y_0$	
	$e^{2} =  z - z_{0}  =  (x - x_{0}) + i(y - y_{0}) $	**************************************
	$e^2 = (\chi - \chi_0)^2 + (y - y_0)^2$ which is eqn of a circle	
<u>3·2·2</u>	<u>Line:</u> Note: $ Z - Z_1  =  Z - Z_2 $  Z + 3i  =  Z + (5-2i)	
, 		
amma yangi bala a wara a an eena manana ya gabababa a an eena an aya bababa	$( a-b  \equiv \text{ distance from } a \text{ to } b')$	anan sana sana sana sana sana sana sana
	✓ pts equal-distant from -3i and -5+2i	
ana ana ang ang ang ang ang ang ang ang	line, perpendicular to line joining these 4-31 pts & through the mid-point perpendicular bisector	v, m, v, m, m, m, m, v, m, v, m,
	$\sqrt{z} = x + iy$	
santa an sum (sel ) o berna e estre us se e se anter e norme (seta de la deste de server estre en server)	• Algebraic :	
	$ \chi + i(y+3)  =  (\chi+5) + i(y-2) $	
	$\chi^{2} + (y+3)^{2} = (\chi+5)^{2} + (y-2)^{2}$	ment for the form and a start of the start of the start many manual formation at a st
s een sen mennementen konstruction en en oos se sen mennementen konstruction en een oor	$\chi^2 + y^2 + 6y + q = \chi^2 + 10\chi + 25 + y^2 - 4y + 4$	
	y = x+2	n a a a a a a a a a a a a a a a a a a a
3.3	<u>Euler's</u> Formula	V. Second a Second Second V. Se
www.wallania.com.com.wallania.com.com.com.com.com.com.com.com.com.com	$e^{i\theta} = \cos\theta + i\sin\theta$ Note. $(e^{i\theta})^n = e^{in\theta}$	n server að server sem lýar 2018 við skall efni af minna fri meðir á se sem sem sem sem sem sem sem sem sem
	EXAMPLE: (find nth roots)	
	Given Zo, find all Z s.t. $Z^n = Z_0$ .	nn waar wa watan watan ta
	Since $z = r(\cos\theta + i\sin\theta) = re^{i\theta}$ ,	
	$Z_0 = \Gamma_0 e^{i\theta_0},$	
agunada ar ar 1919 ar an an aguna dha dunat a ta ar ar an an an annan an an an an an an an an	$\therefore \Gamma^{n} e^{in\theta} = \Gamma_{0} e^{i\theta_{0}}$	
	Take modulus of both sides : $r^{n} = r_{0}$	******
sur a secure of the first start of the second secure of the second secure of the second s	$\Rightarrow \Gamma = \Gamma_0^{1/n}$	
	The usual n th root of a non-negative real number $(re^{i\theta})^n = r_0 e^{i(\theta_r + 2k\pi)}$ $k = \cdots, -2, -1, 0, 1, 2, \cdots$	
\$#\$4664444114411111111111111111111111111		₩₽₩₩ <u>₽₩</u> \$₩₩ <u>₽₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩</u>
	Equate arguments: $N\theta = \theta_0 + 2k\pi$	*********************

, z

-		
4	$\theta = \frac{\theta_0}{n} + \frac{2k\Pi}{n}$	
$\bigcirc$	e.g. if $n = 3$ , $\theta = \frac{1}{2}\theta_0 + \frac{2}{3}k\pi$	Re
		± 00
3.4	De Moivre's Theorem	
•	$e^{i\theta} = \cos\theta + i\sin\theta$	
	$(e^{i\theta})^n = (\cos\theta + i\sin\theta)^n$	
	$= \cos(n\theta) + i\sin(n\theta)$	MAANJA CANA MANA MANDO DO D
	$\Rightarrow \left[ (\cos \theta + i \sin \theta)^n = \cos(n\theta) + i \sin(n\theta) \right]$	
وی در این می از می این می این در این می این می این می این می این می در این ( سی از این	EXAMPLE: Find cos30 in terms of sino & cos0.	xXx8x8x10x111111111111111111111111111111
	$\cos 3\theta + i\sin 3\theta = (\cos \theta + i\sin \theta)^3$	
	$= \cos^3\theta + 3\cos^2\theta(i\sin\theta) + 3\cos^2\theta(i\sin^2)^2 + (i\sin\theta)^3$	ig to Pascal's 4
0	$= (\cos^3\theta - 3\cos\theta\sin^2\theta) + i(3\cos^2\theta\sin\theta - \sin^3\theta)$	V - ···
~	Real part: cos30 = cos30 - 3cos0 sin30	
Name of the Subsection and the second se	•	
(*)=>+(1(==)+(*)==>>+(*)>>+(*)>>+(*)>+(*)>+(*)>+(*)>+(*)>+		aaamamamaanaa aa ahaa ahaa ahaa ahaa aha
	Mon. 24/10/16	
	MATH1401: Mathematical Methods 1	*****
<u>3</u> ·4·1	Trignometric Identities Prof. Halburd	1) m 10 m
•	$e^{z} = e^{x+iy} = e^{x}e^{iy} = e^{x}(\cos y + i\sin y)$	******
******	$e^{i\theta} = \cos\theta + i\sin\theta$	
	$\left(e^{-i\theta} = \cos\theta - i\sin\theta\right)  \textcircled{0}$	
	$\frac{0+0}{2}: \cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$ $\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$	
¥	$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$	10 / 100 / 100 / 100 / 100 / 100 / 100 / 100 / 100 / 100 / 100 / 100 / 100 / 100 / 100 / 100 / 100 / 100 / 100
مرسور و المراجع ( المراجع ) و المراجع ( المراجع ) (	EXAMPLE:	
	Find cos ⁴ 0. [binomial expansion]	
	$\cos^4\theta = \left[\frac{1}{2}(e^{i\theta} + e^{-i\theta})\right]^4  \text{pascal's triangle}  14641$	
	$= \frac{1}{16} \left[ (e^{i\theta})^4 + 4 (e^{i\theta})^3 (e^{-i\theta}) + 6 (e^{i\theta})^2 (e^{-i\theta})^2 + 4 (e^{i\theta}) (e^{-i\theta})^3 + (e^{-i\theta})^4 \right]$	
	$=\frac{1}{16}(e^{4i\theta}+4e^{2i\theta}+6+4e^{-2i\theta}+e^{-4i\theta})$	۵۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰۰
	$= \frac{1}{16} \left( e^{4i\theta} + e^{-4i\theta} \right) + \frac{4}{16} \left( e^{2i\theta} + e^{-2i\theta} \right) + \frac{6}{16}$	
	$= \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$	
		**************************************
<b>3</b> .5	Roots of Unity	94994444494955555555555555555555555555
	example: Find $z_{st}$ . $z^6 = 64i$	handaddillillillilliddaddd gygdin yfyryr fyryr yfyryr yfyryr yfyryr yfyryr yfyryr yfyryr yfyryr yfyryr yfyryr y
······	$Z^{6} = \overline{64} e^{i \left( \frac{\pi}{2} + 2k \pi \right)}$	*******
¹¹¹¹ ///mease.en.org.un.org.un.org.un.org.un.org.un.org.un.org.dowed.dow	[643] ⁴ (154)	ett för hänna händ til det det känna stat känna sänna sän

 $\Rightarrow z = \sqrt[n]{64}e^{i\left(\frac{\pi}{12} + \frac{k\pi}{3}\right)} = 2\left[\cos\left(\frac{\pi}{12} + \frac{k\pi}{3}\right) + i\sin\left(\frac{\pi}{12} + \frac{k\pi}{3}\right)\right]$ when k = -3,  $Z_1 = 2 \left[ \cos(-\frac{11\pi}{12}) + i \sin(-\frac{11\pi}{12}) \right]$ when k = -2,  $Z_2 = 2 \left[ \cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right) \right]$ when k = -1,  $Z_3 = 2 \left[ \cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4}) \right]$ when k = 0,  $Z_4 = 2 \left[ \cos(\frac{\pi}{12}) + i \sin(\frac{\pi}{12}) \right]$ When k = 1,  $z_5 = 2 \left[ \cos(\frac{5\pi}{12}) + i \sin(\frac{5\pi}{12}) \right]$ when k=2,  $Z_{6} = 2\left[\cos(\frac{3\pi}{4}) + i\sin(\frac{3\pi}{4})\right]$ · Show that the sum of first nth roots of unity is 0. (see HW3)  $Z^n = I = e^{i(2k\pi)}$  $\Rightarrow Z = e^{\frac{1(2kT)}{n}}$ Denote  $w = e^{i\left(\frac{2\pi}{n}\right)}$ , then  $w^{\mathfrak{o}} + w^{\mathfrak{i}} + w^{\mathfrak{o}} + \dots + w^{\mathfrak{n}-\mathfrak{i}} = 0$ where n=0, 1, 2, ..., n-1

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namante e du-merca l'evidendi e di l'esclutte di escere vi eschare e cen	$Z^n = 1$
adami (1999) dan se terreta de ter	Let $z = re^{i\theta} = r(\cos\theta + i\sin\theta)$ , then
analan (11) (11) (11) (11) (11) (11) (11) (12) (12	$(z^n =) \Gamma^n e^{in\theta} = 1$
******	["[cos(nθ) + isin(nθ)] = [
13mm) (1.mm) (1.	$Re: r^{n}cos(n\theta) = r \Rightarrow r \neq 0$
anna ann ann ann ann ann ann ann ann an	$Im: \Gamma^{n}sin(n\theta) = 0 \implies sin(n\theta) = 0$
<u>(</u>	$\frac{\xi \sin \mathbf{r} = 0  \text{iff}  \chi = MT  M \in \mathbb{Z}_3}{\xi \sin \mathbf{r} = 0  \text{iff}  \chi = MT  M \in \mathbb{Z}_3}$
	$\Rightarrow n\theta = MT M \in \mathbb{Z} (*)$
allanda di dama dan sedan se se se se se se se de la se	Since $-\pi < \theta < \pi$ ,
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ntant te tomone a secone ce can non-conservation and a second	$-n < M \leq n$ (#)
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major fajon (stor) and an even error and an even	$\Gamma^{(-1)}M = 1$
ethildret familiada pağınde yhtiğinin yanğı yayılış yayamı va a	Since $r > 0$ , $(-1)^{m} = 1 \Rightarrow M$ even
	So we get
	$M = 2m, \mathbf{r} = 1$
	$(\#) \Rightarrow -n < 2m \le n$ $-\left \frac{n}{2}\right  \le m \le L\frac{n}{2} \right  = means  \text{``the greatest integer that is smaller}$ $+ \ln n = \frac{n}{2}$
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	So roots are $z = \cos\left(\frac{2m\pi}{\Omega}\right) + i\sin\left(\frac{2m\pi}{\Omega}\right) - \lfloor\frac{n}{2}\rfloor \le m \le \lfloor\frac{n}{2}\rfloor$
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Mon. 24/10/16 (continued) MATH1401: Mathematical Methods 1 Prof. Halburd Chapter 3. § Differential Calculus § 3.1 Differentiation and Continuity y A ÿ↑ ુ_ુ ક્(≭) ¥=ft⊀ı f(x+h) tangent approximation र्रा र। ٥ アン  $\hat{\chi}$ 0 X+h x  $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = slope of the tangent to <math>y = f(x)$  at point x (gradient) The equation of the tangent to y = f(x) at  $x = x_0$  is  $\frac{y-f(x_0)}{x-x_0} = f'(x_0)$ f(x) $f(x_0)$  $\Rightarrow \mathcal{Y} = f(\mathbf{x}_0) + f'(\mathbf{x}_0) (\mathbf{x} - \mathbf{x}_0)$ ✓ Def.  $Lf = f(x_0) + f'(x_0)(x - x_0)$  is the linearisation of f at  $x = x_0$ Lf is an approximation to f for x near x. ✓ EXAMPLE. Use a linearisation to estimate Jioz. Soln: Let  $f(x) = \sqrt{x}$ We want f(1:02) and we know f(1)=1  $f'(x) = \frac{1}{2\sqrt{x}}$  $f'(1) = \frac{1}{2}$ Lf(x) = f(1) + f'(1)(x-1)In other words, I've chosen  $x_0 = 1$  $\therefore$  Lf(1.02) = 1+ $\frac{1}{2}(1.02-1)$ = 1-01  $\approx f(1.02) = \sqrt{1.02}$ 3.2 Taylor Series

 $\cdot$   $\checkmark$  Using the definition of the derivative , we were able to find an approximation of a function near a point. We suspect that we can get better approximations by taking into account higher derivatives.  $f(x) = a_0 + a_1 (x - x_0) + a_2 (x - x_0)^2 + \dots + a_n (x - x_0)^n + \dots$  $f(x) = \sum_{n=0}^{\infty} Q_n (x - x_0)^n$ For x near  $x_0$ ,  $f(x_0) = q_0$ differentiate  $\frac{f'(x) = q_1 + 2q_2(x - x_0) + 3q_3(x - x_0)^2 + ... + hq_n (x - x_0)^{n-1} + ...}{2mm}$  $f'(x_0) = a_1$ wrt x $f''(x) = 2a_2 + 2.3a_3 (x - x_0) + \dots + n(n - 1)a_n (x - x_0)^{n-2} + \dots$  $f''(x_0) = 2Q_2$  $f''(x_0) = 6a_3$  $f''(x) = 2.3 q_3 + ...$  $f^{(n)}(x_{\circ}) = n!a_n$ √ Def. If f and all its derivatives at x. exist, then the Taylor Series  $\frac{df \ f \ at \ x = x_o \ is \ p' \ f^{(n)}(x_o)}{p_{n}} \frac{f^{(n)}(x_o)}{p_{n}'} \frac{(x - x_o)^n}{p_{n}'}$  $\sqrt{f(x)} = \sum_{n=0}^{\infty} \frac{f^{(m)}(x_0)}{n!} (x - x_0)^n \text{ tangent}$ =  $f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!} (x - x_0)^2 + \dots$ = parabola [approximate a fn locally (close to the pt  $\infty =$  )] 9 1 3rd order approx. y=fix 0 order approx.  $f(x)=f(x_0) \rightarrow \text{const. fn}$ 1st order approx.  $f(x) = f(x_0) + f'(x_0)(x - x_0) \rightarrow \text{tangent at } x = x_0$  $x = \frac{2^{nd} \text{ order approx.}}{f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f'(x_0)}{2!}(x - x_0)^2 + \dots }$  $\chi_{a}$  $\sqrt{f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \frac{f'''(x_0)}{4!}(x - x_0)^4 + \dots}$ remainder is bounded by c. | x-x. | 3 where C is a const 4.12796438 ... error e.q. 4-1 ≈0.1 ≈ 0.01 4.13 4.128 ≈ 0:001

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۹ ب	/ EXAMPLE O	
	$\sqrt{\text{EXAMPLE}} \odot :$ $f(x) = e^x$	
	$f(x) = e^{x}$	$e^{x} = \sum_{n=0}^{\infty} \frac{x^{n}}{n!}$ $= 1 + x^{2} + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \dots$
19.000000000000000000000000000000000000	at $x=0$ , $f^{(m)}(0)=1$	
	Therefore, Taylor Series $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = \sum_{n=0}^{\infty} \frac{x^n}{n!}$	or e ^x at x=0 is Note: 0!=1
	• Def.	
		7 (i.e. $x_0 = 0$ ) is also called a Maclaurin
	V EXAMPLE @;	mm.///////////////////////////////////
0	Find the Maclaurin Series	$af f(x) = \sin x$
		f(0) = 0
ан талаан байтай дайна дайн тэрээлээ. Эм		: '(0) = 1
nn e e ann ann an de 22 is an fadanda a a ann ar		$\begin{array}{c c} (0) = 0 & repeats + all the even & power \\ \hline \\ $
		$f^{(n)}(0) = 0$ $f^{(n)}(0) = 0$ , n even
	$\Rightarrow f^{(n)}(0) = 0  \text{if } n  \text{is ev}$	
	$\Leftrightarrow f^{(2k)}(0) = 0$	
	♥ when n is odd, n	= 2k+1
and the second	$\sin \chi = \sum_{k=1}^{\infty} \frac{f^{(2k+1)}(0)}{(2k+1)!} \chi^{2k+1}$	=2R+1] / check where it starts
0		k-even
1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 -	$\Rightarrow f^{(2k+1)}(0) = \begin{cases} -1 \\ -1 \end{cases}$	k add $\sin x = \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k+1)!} \chi^{2k+1}$
	$\Leftrightarrow f^{(2k+1)}(0) = (-1)^{k}$	k add $\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \chi^{2k+1}$ $= \chi - \frac{\chi^3}{3!} + \frac{\chi^5}{5!} + \dots$
1. Denset a 111 - 11 - 11 - 11 - 11 - 11 - 11 -	Therefore, Maclaurin Seri	
	$\sum_{k=0}^{\infty} \frac{f^{(2k+1)}(0)}{(2k+1)!} (x-0)^{2k+1} =$	$\tilde{r}_{-(-1)k}^{k}$ -2k+1
n Jachard anna a' sa an an taona an a	k=0 (2kti)!	k=0 (2k+1)!
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Ø	Prof. Halb	

V EXAMPLE @:  $f(x) = \cos x$ Soln: f(x) = cosx + f(0)=1 f'(0) = 0 $f'(x) = -\sin x$ f''(0) = -1 $f''(x) = -\cos x$ f"(0)=0 >  $f''(x) = \sin x$  $\Rightarrow f^{(m)}(0) = \begin{cases} 0 & n \quad \text{odd} \\ (-1)^k & n = 2k \end{cases}$ n=2k (even)  $\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$  $\Rightarrow \cos x = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$  $= 1 - \frac{x^2}{2!} + \frac{\chi^4}{4!} - \cdots$  $=\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \chi^{2k}$  $\bigvee \text{ EXAMPLE } \oplus : \qquad \text{complex numbers} \\ \textbf{z} \in \textbf{C}, \quad \exp(\textbf{z}) = \sum_{n=0}^{\infty} \frac{\textbf{z}^n}{n!}$ Since  $e^{i\theta} = \exp(i\theta)$ ,  $\exp(\mathbf{Z}) = e^{i\theta} = \sum_{n=n}^{\infty} \frac{(i\theta)^n}{n!}$  $= \sum_{n=1}^{\infty} \frac{i^n \theta^n}{n!}$  $= \sum_{\substack{n \text{ even} \\ n \text{ even} \\ n \text{ even} \\ n \text{ odd} \\ = \sum_{\substack{k=0} \\ k=0}^{2k} \frac{i^{2k} \theta^{2k}}{(2k)!} + \sum_{\substack{k=0} \\ k=0}^{\infty} \frac{i^{2k+1} \theta^{2k+1}}{(2k+1)!}$   $= \sum_{\substack{k=0} \\ R=0}^{\infty} \frac{(-1)^{k} \theta^{2k}}{(2k)!} + i \sum_{\substack{k=0} \\ k=0}^{\infty} \frac{(-1)^{k} \theta^{2k+1}}{(2k+1)!}$  $\nabla$  $= \cos\theta + i\sin\theta$ 3.3 Binomial Expansion  $f(x) = (1+x)^n$ f(0) = 1 $f'(x) = \prod (1+x)^{n-1}$ f'(0) = n $f''(x) = n(n-1)(1+x)^{n-2}$ f''(0) = n(n-1) $f^{(n)}(x) = n(n-1)...(n-r+1)(1+x)^{n-r} f^{(n)}(0) = n(n-1)(n-2)...(n-r+1)$ Therefore, the maclaurin series of  $(1+x)^n$  is  $(|+\chi)^n = \sum_{h=0}^{\infty} \frac{f^{(r)}(0)}{r!} \chi^n$  $= \sum_{r=0}^{\infty} \binom{n}{r} \times^{r} \quad \text{where} \quad \binom{n}{r} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}$ î n choose ri

This is called binomial expansion.  $\checkmark$  This expansion converges to  $(1+\chi)^n$  for  $|\chi| < 1$  even when n is not a positive integer. n∈R. If n is a positive integer, the series is finite  $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ JEXAMPLE O: Find the Maclaurin series of  $\frac{1}{\sqrt{1+x}}$  to order 3. (x³) Soln:  $\frac{1}{\sqrt{1+\chi}} = (1+\chi)^{-\frac{1}{2}}$ =  $1 + {\binom{-1/2}{1}\chi} + {\binom{-1/2}{2}\chi^2} + {\binom{-1/2}{3}\chi^3} + \dots$  $= 1 + \frac{f_{1/2}}{1!} \chi + \frac{(-1/2)(-3/2)}{2!} \chi^{2} + \frac{(-1/2)(-3/2)(-5/2)}{3!} \chi^{3} + \dots$  $= 1 - \frac{1}{2} \chi + \frac{3}{8} \chi^{2} - \frac{5}{16} \chi^{3} + \dots$ (<del>X</del>) VEXAMPLE @: Find the Taylor Series of  $\frac{x}{\sqrt{1-x^2}}$  about x=0 up to the  $x^3$  term. In this case, the fn is relatively complicated. Thus, use previous result from EXAMPLE O Soln:  $\frac{x}{\sqrt{1-x^2}} = x \cdot \frac{1}{\sqrt{1-x^2}}$ Change x to  $(-x^2)$  in (x).  $\frac{1}{\sqrt{1-x^2}} = (-\frac{1}{2}, (-x^2) + \frac{3}{8}, (-x^2)^2 - \frac{5}{16}, (-x^2)^3 + \dots$  $= 1 + \frac{1}{2} \chi^2 + \frac{3}{2} \chi^4 + \frac{5}{16} \chi^6 + \dots$ So,  $\frac{x}{1-x^2} = x + \frac{1}{2}x^3 + \frac{3}{8}x^5 + \frac{5}{16}x^3 + \dots$ V EXAMPLE 3 Find the series expansion of arctanx. Soln. We know that  $\int \frac{1}{1+x^2} dx = \arctan x + c$ And  $\frac{1}{1+\chi^2} = (1+\chi^2)^{-1}$  $= 1 + \frac{(-1)}{1!} \chi^{2} + \frac{(-1)(-2)}{2!} (\chi^{2})^{2} + \frac{(-1)(-2)(-3)}{3!} (\chi^{2})^{3} + \dots$  $= 1 - \chi^{2} + \chi^{4} - \chi^{6} + \dots$ So,  $\arctan x = \int (1 - x^2 + x^4 - x^6 + ...) dx$  $= \chi - \frac{1}{3}\chi^{3} + \frac{1}{5}\chi^{5} - \frac{1}{7}\chi^{7} + \dots$ 3.4 Implicit Differentiation

VEXAMPLE O. Suppose that y is defined implicitly by  $x^3 + 3xy + 2x^2y^5 = 6$ Find  $\frac{dy}{dx}$  at x=1, y=1. Note: this pt is on the curve. Soln: Take  $\frac{d}{dx}$  of both sides:  $\frac{1}{7x^6} + 3y + 3x\frac{dy}{dx} + 4xy^5 + 10x^2y^4\frac{dy}{dx} = 0$ sub  $\chi = 1, y = 1$ :  $\frac{y = 1}{3}, \frac{y = 1}{3}$   $\frac{y = 1}{3}, \frac{dy}{dx} + 4 + 10 \frac{dy}{dx} = 0$   $13 \frac{dy}{dx} = -14$   $\frac{dy}{dx} = -\frac{14}{13}$ V EXAMPLE @. Find  $\frac{d}{dx}(x^{*})$ Soln: Let y=x* Natural logarithm: 'ln' = 'log' = 'loge' Take ln() of both sides Iny = lnx* hy = xlnx Differentiate wrt  $\chi$  ,  $\frac{dy}{dx} \frac{d(\ln y)}{dx} \frac{d}{dx} \frac{(\ln y)}{dx} = \ln x + 1$  $\Rightarrow \frac{d}{dx}(x^{*}) = \frac{dy}{dx} = (\ln x + 1) \chi^{*}$ Alternative Method:  $x^{x} = (e^{\ln x})^{x} = e^{x \ln x}$  $\Rightarrow \frac{d}{dx}(x^{x}) = \frac{d}{dx}(e^{x \ln x})$  $= (-ln x + 1) e^{xln x}$  $=(\ln x+1)x^{x}$ 3.5 Inverse Functions · How to find the inverse? 'y≥χ y=,f(x)  $y = f(x) \Leftrightarrow x = f^{-1}(y)$ y = f(x)only works if the fn is bijective (one-to-one fn)

 $y = f^{-1}(x)$  is the reflection of y = f(x) in the line y = x. · y = sinx The area that can produce an inverse CANNOT find the inverse without restrictions  $\overline{x}$ Note:  $(\sin x)^{-1} = \frac{1}{\sin x}$ sin'x = arcsinx . EXAMPLE: Find d arcsin x Soln: Let y = arcsinx i.e. siny = xarcsin> Differentiate wrt x  $\cos y \frac{dy}{dx} = 1$   $\Rightarrow \frac{dy}{dx} = \frac{1}{\cos y}$   $\Leftrightarrow \frac{d}{dx} (\operatorname{arcein} x) = \frac{\pm 1}{\sqrt{1 - x^2}}$ since  $\cos y = \pm \sqrt{1 - \sin^2 y}$  $= \frac{1}{\int -\infty^2}$ since arcsinx is increasing 3.6 Hyperbolic Functions  $\Rightarrow \frac{\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}}{\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}}$ •  $e^{i\theta} = \cos\theta + i\sin\theta$  $e^{-i\theta} = \cos\theta - i\sin\theta$ • Def.  $\cosh x = \frac{e^x + e^{-x}}{2}$  $\sinh x = \frac{e^{x} - e^{-x}}{2}$  $\sqrt{If}$  (x, y) = (cost, sint), we have  $x^{2} + y^{2} = \cos^{2}t + \sin^{2}t = 1$ If  $(x,y) = (\cosh t, \sinh t)$ ; we have 94  $\chi^2 - y^2 = \cosh^2 t - \sinh^2 t = 1$ χ

proof: $\cosh^2 x = \frac{1}{4}(e^{2x} + 2 + 6)$ $\sinh^2 x = \frac{1}{4}(e^{2x} - 2 + 6)$		
$\Rightarrow \cosh^2 x - \sinh^2 x = 1$		
✓ We also have $\cosh^{2} x = \frac{1}{4}(e^{2x}+2+e^{-2x})$ $= \frac{1}{4}(e^{2x}+e^{-2x})+\frac{1}{2}$ $= \frac{1}{2}\cosh 2x + \frac{1}{2}$		
$\Rightarrow \cosh 2x = 2\cosh^2 x - 1$	and an	
$\sqrt{\sinh^2 x + \cosh^2 x} = \cosh 2x$ 2sinh ² x +1 = cosh2x		
$sech^2 x = 1 - tanh^2 x$ - cosech [*] x = 1 - coth ² x		
• Differentiation $\frac{d}{dx} \sinh x = \cosh x$ $\frac{d}{dx} \cosh x = \sinh x$ $\frac{d}{dx} \cosh x = \sinh x$ $\frac{d}{dx} \tanh x = \operatorname{sech}^{3} x$ • Taylor Series $\cosh x = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$ $\sinh x = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$	$\frac{d}{dx} \operatorname{coth} x = -\operatorname{cosech}^{2} x$ $\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \operatorname{tanh} x$ $\frac{d}{dx} \operatorname{cosech} x = -\operatorname{cosech} x \operatorname{coth} x$	····
• Differences between hyperbolic $   \sqrt{\cos x} = \frac{1}{2}(e^{ix} + e^{-ix}) $ $   \Rightarrow \cos(ix) = \frac{1}{2}(e^{-x} + e^{x}) = \cosh x $ $   \sqrt{\sin x} = \frac{1}{2i}(e^{ix} - e^{-ix}) $ $   \Rightarrow \sin(ix) = \frac{1}{2i}(e^{-x} - e^{-x}) = -\frac{1}{2i} $ • Even / Odd Fn ? $   \sqrt{\cosh x} = \frac{1}{2}(e^{x} + e^{-x}) $ When $x >> 1$ , $\cosh x \sim \frac{1}{2}e^{2x}$ $   x \ll -1,  \cosh x \sim \frac{1}{2}e^{2x} $ i.e. $[x  >> 1, \cosh x \sim \frac{1}{2}e^{2x}$	$-\frac{1}{i} = -\frac{i}{i^{2}} = -\frac{1}{-1} = i$ $\frac{1}{2}(e^{x}-e^{-x}) = i\sinh x$ $-\frac{1}{2}e^{-x} \int \sqrt{2}e^{x}$	····· ···· ··· ··· ···
⇒ coshx is an even functi f(-x)=f(x)		

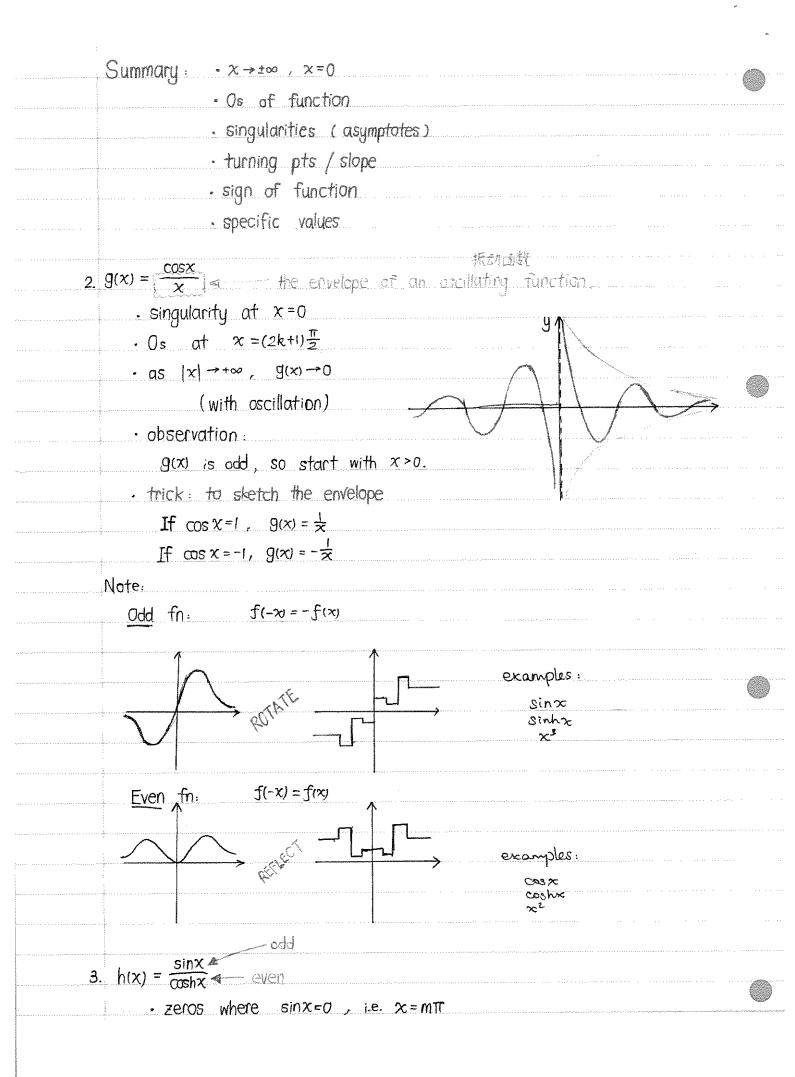
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••	and it has a minimum of 1 at $x=0$ . $\sqrt{\sinh x} = \frac{1}{2}(e^{x}-e^{-x})$	
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	$= \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$	milaimonaileanaame
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	$(a_0 + a_1x + a_2x^2 + \dots)(b_0 + b_1x + b_2x^2 + \dots) \Rightarrow DO \text{ NOT expand all the}$	terms
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	constant linear quadratic you go. term term term	

② Dividing Series EXAMPLE :  $2+3\chi+4\chi^{2}+...$ Find the series up to the x² term. Soln: We know that  $\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \dots$ So  $\frac{1}{2+3\chi+4\chi^{2}+...} = \frac{1}{2} \cdot \frac{1}{1+(\frac{3}{2}\chi+2\chi^{2}+...)} \cdot \frac{\chi}{\chi}$  cubic terms  $= \frac{1}{2} \left\{ 1 - \left(\frac{3}{2}\chi + 2\chi^{2} + \left[ \right] \right) + \left(\frac{3}{2}\chi + 2\chi^{2} + \left[ \right] \right)^{2} - \dots \right\}$ sub:  $= \frac{1}{2} \left\{ \left| -\frac{3}{2}\chi + \left(\frac{9}{4} - 2\right)\chi^{2} + ... \right\} \right\}$  $= \frac{1}{2} - \frac{3}{4}\chi + \frac{1}{8}\chi^{2} + \dots$ 3.8 Curve Sketching · Basic Steps, O Basic symmetries . even/odd/neither  $\checkmark$  even function: f(-x) = f(x) [reflection in the y-axis]  $\sqrt{\text{odd}}$  function: f(-x) = -f(x) [reflection in the origin] even odd . 2 Behaviour of  $x \rightarrow +\infty$  &  $x \rightarrow -\infty$ O Vertical asymptotes / pts where the fin is not defined Extreme pts (minima / maxima) & stationary pts f'(x) = 0EXAMPLE: Sketch  $y(x) = \frac{\sqrt{x^2+1}}{(x+1)^2}$ Soln: No obvious symmetries.  $\chi \rightarrow +\infty$  :  $\chi^2 + 1 \rightarrow \chi$  $(\chi + 1)^2 \rightarrow \chi^2$ 

so  $y(x) \rightarrow \frac{x}{x^1} = \frac{1}{x}$ ,  $x \gg 1$  $\chi \to -\infty$ ;  $\chi^2 + I \to -\chi$  $\sqrt{\chi^2} = |\chi|$  $(\chi + 1)^2 \rightarrow \chi^2$ so  $y(x) \rightarrow \frac{-x}{x^2} = -\frac{1}{x}$ , x <<-1For x near -1,  $y(x) = \frac{\sqrt{(-1)^2 + 1}}{(x+1)^2} = \frac{\sqrt{2}}{(x+1)^2} > 0$ since y(x) is not defined at x = -1So,  $y(x) \rightarrow +\infty$  as  $x \rightarrow 1^{-}$  &  $x \rightarrow 1^{+}$  x approaches 1 from below  $\cdot$  - from above  $U(x) = \frac{\sqrt{x^{2+1}}}{(x^{+1})^2}$  $y'(x) = \frac{\frac{1}{2} \cdot 2x(x^2+1)^{-\frac{1}{2}}(x+1)^2 - \sqrt{x^2+1} \cdot 2(x+1)}{(x+1)^4}$  $= (\chi+1)^{-3} (\chi^{2}+1)^{-\frac{1}{2}} [\chi(\chi+1) - 2(\chi^{2}+1)]$  $= -(x+1)^{-3}(x^{2}+1)^{-\frac{1}{2}}(x^{2}-x+2)$ no real roots | △ = -7 < 0 => no real roots  $\Rightarrow$  No stationary pts. (i.e.  $f'(x) \neq 0 \forall x$ ) -1 x

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Wed. 02/11/16 MATH1401: Help Class Prof. Wilson § Hyperbolic Functions §  $f(x) = \frac{\sinh x}{\cosh x - 2}$ 1. Sketch y=fix where · What happens to y=fix as  $\chi \to +\infty$ ?  $e^{-x} \rightarrow 0$  looks similar to  $f(x) = \frac{\frac{1}{2}(e^{x} - e^{-x})}{\frac{1}{2}(e^{x} + e^{-x}) - 2} \xrightarrow{\ddagger} \frac{\frac{1}{2}}{\frac{1}{2}e^{x} - 2} = \frac{\frac{1}{2}}{\frac{1}{2} - 2e^{-x}} \xrightarrow{\frac{1}{2}} = 1$ Similarly, as  $x \rightarrow -\infty$ ,  $e^{x} \rightarrow 0$  $f(x) \xrightarrow{-\frac{1}{2}e^{-x}}_{\frac{1}{2}e^{-x}-2} \xrightarrow{-\frac{1}{2}}_{\frac{1}{2}-2e^{x}} \xrightarrow{-\frac{1}{2}}_{\frac{1}{2}} = -1$  Singularities ? [ | f(∞) → +∞ ] (asymptotes) This happens where  $\cosh x - 2 = 0$  $x = \pm \cosh(2)$ • Zeros of f(x) [pts y=0]  $\checkmark$  For us,  $\sinh x = 0$ ⇒ x = 0 Only one zero. (*)  $\sqrt{\ln x} < -\cosh^{-1}(2)$ , we must get from -1 to  $\pm \infty$ . However, from (X), we know that it cannot pass through O  $\Rightarrow f(x) \rightarrow -\infty$  as  $x^{-} \rightarrow -\cosh^{-1}(2)$ Similarly, f(x) > 1 in  $\cosh^{-1}(2) < x < \infty$  $\frac{df}{dx} = \frac{\cosh x (\cosh x - 2) - \sinh x . \sinh x}{(\cosh x - 2)^2}$  $= \frac{COSh^2x - sinh^2x - 2cOShx}{(COShx - 2)^2}$  always -ve since  $cosnx \ge 1$ <u>y</u> 1  $= \frac{1-2\cosh x}{(\cosh x-2)^2} = always +ve$ ⇒ gradient always -ve. -cosh"(2) | cosh-1(2) 0



• no singularities since  $\cosh x \neq 0$  $\cdot ds x \rightarrow +\infty$ ,  $\cosh x \approx \frac{1}{2}e^{x}$ so  $\frac{\sin x}{4e^x} \rightarrow 0$ sign(h(x)) = sign(sinx) since coshx > 0For functions, <u>f think of "add" as -1"</u> even  $\times$  odd = odd "even" ds +1  $odd \times odd = even$ even  $\times$  even = even 4.  $tanh x = \frac{sinhx}{coshx}$  $\cdot$  Os at sinhx = 0 ⇒ <u>x</u>=0 singularity at 0 •  $x \rightarrow +\infty$ : tanh  $x \rightarrow 1$  [divide top 8 bottom by  $e^{x}$ ]  $x \rightarrow -\infty$ : tanh  $x \rightarrow -1$  [divide top & bottom by  $e^{-x}$ .] • turning pts .  $\frac{d}{dx}(\tanh x) = \frac{1}{\cosh^2 x} > 0$ ⇒no turning pts Show that  $\binom{n}{r-1} + \binom{n}{r} = \binom{n+1}{r}$ LHS =  $\frac{n!}{(r-1)!(n-r+1)!} + \frac{n!}{r!(n-r)!}$  $= \frac{n!}{(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n-1)!(n$  $= \frac{n!}{(r-1)!(n-r)!} \cdot \frac{n+1}{(n-r+1)r}$  $\frac{(n+1)!}{r!(n-r+1)!} = \binom{n+1}{r} = RHS$  $\overline{\mathcal{D}}$ 

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4-2	Level Set (c+d,f) and (-c+d,t)	<b>C</b> 1
	Level Set $(c+d, f)$ and $(-c+d, f)$ Def. where $c^2 = a^2 - b^2$	5
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And a second	A set of the form $\{(x,y) \in \mathbb{R}^2 : f(x,y) = c\}$ for some given constant c is	***************************************
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19145/11141/011111111111111111111111111111	EXAMPLE Q: A contour plot	1997 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 -
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underschell (anner under sicht)) ( justen der sichten er geber der sichtet)	This is r a circle, if c > 0 (centred at (0,0))	•
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	empty set $\rightarrow 0$ , if C<0	• • • •
•	Example O:	united on [1/1/1/1/1/1/1/1/1/1/1/1/1/1/1/1/1/1/1/

(c=0) ₽=≫ y=-X  $f(x,y) = x^2 - y^2$ The level sets are f(x,y)=c  $\Leftrightarrow \chi^2 - y^2 = C$ ₹_ % This is { a hyperbola, if c>0  $y=\pm x$  if C=0  $y^{2}+C=x^{2}$  if C<0 C=3 C=1 4.3 Graph in 3 Dimensions  $\checkmark$  The graph of a function f(x, y) in  $3d(\mathbb{R}^3)$  is the set of all (x, y, z) s.t. z = f(x, y) $\checkmark$  Also, for functions f(x, y, z), we can plot level sets, level curves, etc. · Recall : ax+by+cz=1 is a plane in  $\mathbb{R}^3$ . In particular, y=2x+1 is a plane in  $\mathbb{R}^3$ . the line (in  $\mathbb{R}^2$ ) y=2x+1 being streched up in Z direction (since z can be anything you like) ✓ The 3 special planes x=0, y=0, and z=0 are called the coordinite planes. y=z plane x-z plane x-y plane y=0 plane  $\checkmark$  In plots of f(x, y, z) = 0, we intersect the graph •и with planes parallel to the coordinate planes. (i.e. x = k, or y = k, etc.) • EXAMPLE O:  $Z = \chi^2 + U^2$ Take slices Intersect this with a plane of the form z = k.  $\begin{cases} \overline{Z} = \chi^2 + y^2 \\ z = k \end{cases} \Rightarrow \chi^2 + y^2 = k$ k>0 circles circles k=0 (0,0) k<0 Ø  $\boldsymbol{\chi}$ 

ξ. ŝ		
****	Now intersect surface with the coordinate plane $x$	=0. Z
	$\begin{cases} \chi = 0 \\ z = \chi^2 + y^2 \end{cases} \Rightarrow z = y^2  \text{parabola} \end{cases}$	
旋转抛物	az = $x^2 + y^2$ So we have a paraboloid (parabola rotated arc	und an axis)
·····	• EXAMPLE @.	
844x84yaammii yaammii yaammii yaamii yaa	$\frac{x^{2}}{4} + y^{2} + z^{2} = 1$	na - Andrea Marine, 2003, an andrea de marine para constante de la dela de la dela de la dela de la dela de
,	Intersect with plane $x=k$ . take slices in x direct	tion 🚽
eereening (states)	$\Rightarrow y^2 + Z^2 = 1 - \frac{k^2}{4} \qquad (set \ x \ as \ a \ constant$	· · · · · · · · · · · · · · · · · · ·
3m, 4 a mar 2 a c a c a c a c a c a c a c a c a c a	∴ a circle -2 <k<2< td=""><td>NO-2</td></k<2<>	NO-2
mandyn fel fan e regen an	a point $k = \pm 2$	y
and and are a few states of the states of th	Ø [k]>2	xx ===================================
	Now intersect with $z = 0$	
****	$\Rightarrow \frac{x^2}{4} + y^2 = 1  an  ellipse  (focus on x axis sin$	
<b>柄</b> 球	So we have an ellipsoid $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 1$	
	EXAMPLE 3	namenan kanan k
	$x^2 - y^2 + z^2 = 1$	адалат олтин Титерин титин тайдар унун ологит ил 1999 50 da Baran унун байлаг тайдагаан ул оронуу унун оронуу у
	Intersect with plane y=k.	an in a second and a second a
	$\Rightarrow \chi^2 + \chi^3 =  +k^2 $	
an a	: a circle ¥k	
	⇒ always circles with the smallest circle at k=0	x Ny
	Now intersect with $z=0$ .	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
O	⇒ x²~y²=	y y y y y y y y y y y y y y y y y y y
	in x-y plane	aan oo aa ahaa ahaa ahaa ahaa ahaa ahaa
单叶双曲面	So we have an [1-sheeted hyperboloid], $\frac{x^2}{a} + \frac{y^2}{b} - \frac{z^2}{c} =$	
	Advice: Go with circles first!	
y ann feldiginn a' a chuirte Eigeagan georrann aisem a' rusa	Let's revisit this example using an alternative method.	
	Suppose we take slices z=k first	
dhafan en gan en gannann generald og generald og generalden.	$\Rightarrow x^2 - y^2 = 1 - k^2$	
**************************************		
martslendurrennarnstansjundurstennsj	$\frac{1}{ k  < 1}$ $\lim_{k \to \infty}  y  = \pm x  \text{if }  k  = \pm 1$	$\rightarrow \times$ ( $\rightarrow$
	hyperbola if $ \mathbf{k}  > 1$	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
	We cannot take $x=0/y=0$ since we do not have a successe.	Immetrical shape in this

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		2
	Take y=c.	100-104 102 (40-10-10) (40-10-10) (40-10-10)
1971 - 1972 - 1974 - 1974 - 1974 - 1974 - 1974 - 1974 - 1974 - 1974 - 1974 - 1974 - 1974 - 1974 - 1974 - 1974 - F		80214] wysymago-egolamos (w 21212) (Minespergenerginger, 1111-05412-06
anlar Mekselar Denominanlar (desete 1900ard menmunal)	x ² - y ² - z ² = 1	
nangan () ang mang mang mang mang mang mang mang	Intersect with $x=k_{-}$	Al Gold G. Stada I. Samar Magazing a Galian ( 175 Samatan Galiana).
*9=4,4=1 (%=0,000,000,000,000,000,000,000,000,000,	This is $\Rightarrow y^2 + z^2 = k^2 - 1$	ande das sets and a set of the set of the set of the set of the sets of the sets of the sets of the sets of the
	rotational symmetric. circle, if 1k1>1	and a second
Recommendation () () () () () () () () () () () () ()	about $x - axis$ . point [(0,0)], if $k=\pm 1$	511114999999999999999999999999999999999
	je iki < 1 y	94999999999999999999999999999999999999
	Now intersect with z=0.	ad and a calend a specific processing and a specific data with the first statement.
	$\Rightarrow \chi^2 - y^2 = 1$ hyperbola	
双叶双曲面	So we have a 2-sheeted hyperboloid $\frac{x^2}{a} - \frac{y^2}{b} + \frac{z^2}{c} = 1$	********
4:4	Partial Derivative	
	. Def.	#255529-27299-2749-2749-2549-2549-2549-2549-2549-2549-2549-25
	The partial derivative of a function f(x,y) is	gerreren ander son ander son ander an ander an ander an ander an ander a son and a son and a son and a son and
pronounces	-f(x+h, u) - f(x, u)	a San San San San San San San San San Sa
; ^ş	$\frac{\partial x}{\partial h} h = h$	angangsisalasaninga nganana jujunga anjasina si mara
	$\frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}  (if they exist)$	15911111111111111111111111111111111111
anna an ann ann ann ann ann ann ann ann	ay have h	anaka (mini faloning manaka ka faloning manaka
	✓ Treat y as a constant if partial derivative wrt ».	
	Treat x as a constant if partial derivative wrt y.	with the set of the se
	V EXAMPLE O:	nn blan san san san san san san san san san s
	$f(x, y) = x^2 + y^2$	enderste staten ander en
	$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$	00040-01-01-01-01-01-01-01-01-01-01-01-01-01
	$\frac{\partial X}{\partial x} = \frac{1}{h^2 \partial x^2 + h^2 + (1^2) - [x^2 + 4^2]}$	aanggaaanguna Dawadaan Coroo Coroonaa aa
	$= \lim_{\substack{h \neq 0}} \frac{\left[ (x+h)^2 + y^2 \right] - \left[ x^2 + y^2 \right]}{h}$	saaaaaa aha waxaa ka
99999999999999999999999999999999999999	$= \lim_{h \to \infty} (2x+h)$	
	= 2%	ngga pagangan ang kang pananang sa sa sama sa kamang mumitin
99999999999999999999999999999999999999	V EXAMPLE 2:	
	$g(x, y) = xe^{y} + x + 2y$	
	$\frac{\partial q}{\partial x} = e^{y} + i \qquad (\text{treat } y \text{ as if } i \text{ is a constant})$	

*

4  $\frac{\partial q}{\partial y} = xe^{y} + 2$  (treat x as if it is a constant) · Other notation:  $\frac{\partial f(x,y)}{\partial x} = f_x(x,y)$  $\frac{\partial f(x, y)}{\partial y} = f_y(x, y)$ 子公安-四 means nothing ! Note: · Also, can be differentiated again  $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = (f_x)_{\otimes} = f_x$  subscript means wrt x  $\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} = (f_x)_y = f_{xy}$ Similarly,  $\frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) = \frac{\partial^2 f}{\partial x \partial y} = (f_y)_x = f_{yx}$  $\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = (f_y)_y = f_{yy}$ VEXAMPLE O.  $f(x, y) = y^2 e^{x} + x^2 + 3y$  $\frac{\partial f}{\partial x} = y^2 e^x + 2x$  $\frac{\partial f}{\partial y} = 2ye^{x} + 3$  $\frac{\partial^2 f}{\partial x^2} = y^2 e^x + 2$  $\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial y} \right) =$ 2e*  $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) = 2y e^x$  $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) = 2y e^x$  $\sqrt{In general}$ , if  $\frac{\partial^2 f}{\partial x \partial y}$  and  $\frac{\partial^2 f}{\partial y \partial x}$  are continuous, then they're equal. ✓ EXAMPLE Q: Find hx, hy & hz where  $h(x,y,z) = x^{y} + y^{z} + z^{x}$ [same def. for fns of 3 variables] (fns' = 'functions'  $h_x = \frac{\partial h}{\partial x} = y x^{y-1} + (\ln z) z^*$  $h_y = \frac{\partial h}{\partial y} = (\ln x) x^y + z y^{z-1}$  $h_z = \frac{\partial h}{\partial z} = (\ln y) y^z + \chi z^{x-1}$ Z* = e^{xlog}z differentiate wrt  $x: \frac{\partial z^{x}}{\partial x} = (\ln z) z^{x}$ 4.4.1 Equation of Tangent Planes • Graph  $z = f(x, y) \leftarrow z$  as a function of x and y. 77-tangent plane Ques. Find the tangent plane at (xo, yo) (xo, yo, f(xo, yo)) - We want to find 2 tangent vectors in the plane.

- Intersect 
$$z = f(x, y)$$
 with the plane  $y = y$ .  
 $z = f(x, y_0)$   
Slope of  $z = f(x, y_0)$  at  $x = x_0$  (in  $x = z$  plane) (is  $f_x(x_0, y_0)$   
So a tangent vector is  $z = f_x(x_0, y_0) \frac{1}{2} + \frac{1}{2}$ 

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	• Def.	
	Let û be a unit vector, the directional derivative	há há
	of $f(x,y)$ in the direction $\hat{u} = u_1 i + u_2 j$ is	۲ <u>r+hû</u>
	$D_{\underline{a}}f(x,y) = \lim_{h \neq 0} \frac{f(\underline{r}+h\underline{\hat{u}}) - f(\underline{r})}{h}$	+9j
ndandan dijamaji yang yang maja ara na masa ana dikan kadan dija a		than an a
2228542885110000000000000000000000000000000000	$= \lim_{h \to 0} \frac{[f(x+hu_1, y+hu_2)] - f(x, y)}{h - partial derivative wrt x (i.e. y held constant)}$	19
and a stand and a stand and a stand a s	$= \lim_{h \to 0} \left[ \frac{f(x+hu_1, y+hu_2)-f(x, y+hu_2)\overline{u_1}}{h\overline{u_1}} + \frac{f(x, y+hu_2)-f(x, y)\overline{u_2}}{h\overline{u_2}} \right]$	หมายการการการการการการการการการการการการการก
Managana a germoorren recenen ourrenne ad dae de bestoen ar oorondaa	$= \frac{\partial f}{\partial x} u_1 + \frac{\partial f}{\partial y} u_2$ in order to make hu, the denominator (1)	ante-anno marante agrega a calanta marante a de de set a provinsi servico de seta da da da
Mahadh y digantag bayayan mgar ammor anakan ngantag kangi asara	$= (\nabla f) \cdot \hat{\mathcal{U}} \qquad \text{in order to make here have the denominator (1)}$ $= (\nabla f) \cdot \hat{\mathcal{U}} \qquad \text{where } \nabla f = \frac{\partial f}{\partial x} \cdot \frac{\partial f}{\partial y} \cdot \frac{\partial f}{\partial y} = \text{gradient of } f$	becquse $\Delta x = hu_1$ )
*#####################################	$\frac{-(\sqrt{3}) \cdot \underline{\omega}}{\sqrt{\ln R^3} (3d)},$ where $\sqrt{f} = \frac{1}{2x^2} + \frac{1}{2y^2} = \frac{1}{2y^2} = \frac{1}{2y^2} = \frac{1}{2y^2} + \frac{1}{2y^2} = \frac{1}{2y^2} + \frac{1}{2y^2} = \frac{1}{2y^2} + \frac{1}{2y^2} = \frac{1}{2y^2} + \frac{1}{2y^2} + \frac{1}{2y^2} = \frac{1}{2y^2} + \frac{1}{2y^2} + \frac{1}{2y^2} = \frac{1}{2y^2} + \frac{1}{2$	
	$D_{\underline{\alpha}}f = \frac{\partial f}{\partial x}u_1 + \frac{\partial f}{\partial y}u_2 + \frac{\partial f}{\partial z}u_3 = (\nabla f) \cdot \underline{\hat{u}}$	,,
**************************************	where $\nabla f = \frac{\partial f}{\partial x} \underline{i} + \frac{\partial f}{\partial y} \underline{j} + \frac{\partial f}{\partial z} \underline{k}$	na y na gun an ann an ann an Calineann ann an an ann an ann an ann an ann a
	V EXAMPLE :	ородовани на полото на 1999 родини и на основните на проблат, на одного и подала в е е ото
eelenburnan maanaa eenaan ya ahaa ahaa ahaa ahaa ahaa ahaa ah	Find the directional derivative of $f(x, y, z) = (x+2y)^3 + e^{-z}$ at (1)	(-1,0) in the
Naamada aa ahaa ahaa ahaa ahaa ahaa ahaa	direction $\hat{\mathcal{U}} = \frac{1}{3} (-2i + j + 2k)$ .	and an and a second
192001100001111000000000000000000000000	$\nabla f = \frac{\partial f}{\partial x} \frac{1}{2} + \frac{\partial f}{\partial y} \frac{1}{2} + \frac{\partial f}{\partial z} \frac{k}{z}$	
[∇f] (xa, ye, Za)		
- √√f-at-(x₀,y	$ _{3, \overline{z_{0}}}' = \nabla f _{(1, -1, 0)} = 3\underline{i} + 6\underline{j} - \underline{k}$	
	$\therefore D_{a}f = (\nabla f) \cdot \hat{u}$	a ay mino (amono ya ani una a ani mino ya ingono ya ani ana ana ana ani bio ya ina ani ani ani ani ani ani ani
	$= \begin{pmatrix} 3\\ 6\\ -1 \end{pmatrix} \cdot \begin{pmatrix} -\overline{3}\\ -\overline{3}\\ -\overline{3}\\ 2 \end{pmatrix}$	****
	$(-1)'(\frac{1}{3})'$ = $-2+2-\frac{2}{3}$	19 m
enterin altra di anti manimeterrati angli (se se sa angli angli a		
	$=-\frac{2}{3}<0$ This implies that f decreases in this direct Recall:	<u>ion,</u>
44mm/mg/mg/a/1/1/1/2/mm/ma/mg/a/a/a/a/mm/pa/1/20000000		ret Antoninka (Sperfrahrum attention om koppoming for er om med stationer attentioner som attentioner attention
	$D_{\underline{a}}f = \widehat{\underline{u}}(\nabla f) \qquad \text{since }  \widehat{\underline{u}}  = i$ $=  \nabla f  \cos \theta \qquad \text{where } \theta  \text{is the angle between } \widehat{\underline{u}} \& \nabla f$	nnnntynnsfylms fylger e ombornne minut egnoged grinn in ser andalde ardel fylger for mynus ogdar
**************************************	$\checkmark$ Fix f. (since Daf depends on both function f and direction $\hat{u}$ )	
*****	$\sqrt{Daf}$ has a max, $ \nabla f $ , when $\theta = 0$	е алт файраран се на саласан калан каландар улук калан се на калан калан калан калан калан калан калан калан к
	$\Rightarrow \mathfrak{V}$ points in the same direction as $\nabla f$ .	
Taalingun yhnet a	$\sqrt{Daf}$ has a minimum, $- \nabla f $ , when $\theta = \pi$	
	$\Rightarrow \widehat{u}$ points in the opposite direction as $\nabla f$	

 $\sqrt{Daf} = 0 \Leftrightarrow \hat{\mathcal{U}}$  is orthogonal to  $\nabla f$ . • Consider a level surface f(x, y, z) = c, for some constant c.  $\sqrt{D_{\hat{u}}f} = 0$  for  $\hat{\underline{u}}$  tangent to the surface Z  $\Rightarrow$  ( $\nabla f$ ) is normal to the surface. Fri. 18/11/16 MATH1401, Mathematical Methods 1 Prof. Halburd  $\operatorname{Recap}: \sqrt{D_g f(x,y)} = \lim_{h \to +\infty} \frac{(x+hu_1, y+hu_2) - f(x, y)}{h}$  $= (\nabla f)$ .  $\hat{u}$ where  $\hat{u} = u_1 \hat{i} + u_2 \hat{j}$  (unit vector)  $\nabla f = \frac{2f}{2x} \cdot \frac{1}{2} + \frac{2f}{3y} \cdot \frac{1}{2} + (\frac{2f}{3z} \cdot \frac{k}{2}) \rightarrow if f(x, y, z)$  is a function of (gradient of f) 3 variables  $\sqrt{\text{Given } f(x,y,z)}$  $\forall f$  is some vector at  $(x_0, y_0, z_0) \rightarrow \text{consider different directions } \mathcal{U}$ ~ Vf(x0, y0, Zo) fastest  $\frac{u}{\theta = \pi}$  to vector  $\sqrt{1}$  If we have a surface given as a level set f(x,y,z) = cdirection of tangent does not change Z tangent plane normal vector surface tangent to surface are in direction  $\hat{u}$  s.t. Daf=0 $\Leftrightarrow (\nabla f) : \widehat{\mathcal{U}} = 0$ If  $\nabla f(x_0, y_0, z_0) \neq 0$ , then it is a normal vector to the surface at  $(x_0, y_0, z_0)$ . ✓ EXAMPLE: Find the tangent plane to  $\chi^2 + 2y^2 + 3Z^2 = 6$  at (1, 1, 1). Soln: The surface is f(x,y,z)=6 where  $f(x,y,z)=x^2+2y^2+3z^2$  $\nabla f = 2x i + 4y j + 6z k$ 

 $\nabla f(1,1,1) = 2\underline{i} + 4\underline{j} + 6\underline{k} = 2(\underline{i} + 2\underline{j} + 3\underline{k}) \leftarrow \text{coefficient are all +ve}$ ⇒ continue to move in the same direction ----So a normal vector at (1,1,1) is  $\underline{\Pi} = \underline{\hat{U}} + 2\hat{j} + 3\underline{k}$ The tangent plane is  $\mathbf{n} \cdot \mathbf{P} \cdot \mathbf{P} = 0$ where  $P_0 = (1, 1, 1)$ , P = (x, y, z).  $\binom{1}{2}, \binom{\chi-1}{y-1} = 0$ x - 1 + 2(y - 1) + 3(z - 1) = 0

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Wed. 16/11/16 MATH1401: Mathematical Methods 1 Applied Tutorial 1 level set: $0 \{(x,y) : f(x,y) = c\}$ where $f(x,y) = x^2 + 2y^2$ when c=1, $\{(x,y): x^2+2y^2=1\} \longrightarrow ellipse$ $\{(x,y): x^2+2y^*=0\} \longrightarrow \text{point } (0,0)$ C=0. $g: \mathbb{R}^2 \to \mathbb{R}$ 2 graph (g) = $\{(x, y, z) : z = f(x, y)\} \rightarrow$ surface $f \cdot R \rightarrow R$ 3 graph $(f) = \{(x, f(x)) : x \in \mathbb{R}\}$ f(x)0 x 2. Functions of Multiple Variables g = y=a^ 39(Xo, Yo) Эx · gradient of a function means rate of charge of f in the x-dire $\frac{\partial g}{\partial x}(x,y)$ (<u>\7q)(%,y)</u> = €R²◀ - means 2D Z_Y رالا، کر) <u>الور</u> (۲،۲) <u>الور</u> rate of change of f in the Fix y=y. -ve <u>29</u> g <u><u><u>y</u>-direction</u></u> $x + ve \frac{\partial 9}{\partial x}$ <u>99</u> & H+ve an $-ve \times diffection$ moves in one x direction \Rightarrow 90 ⊕ve -ve x +ve x direction $\frac{\partial 9}{\partial x}$ & moves in +ve x direction $\Rightarrow 94$ direction, -ve VC2 90 <u>ज्य</u> ४ +ve x direction +ve moves in -ve x direction \Rightarrow 94 10 3월 & x moves in ove x direction $\Rightarrow 9^{\textcircled{}}$ ⊜ve ⇒ gradient points into direction of greater increase of g $(\text{same sign} \Rightarrow g\uparrow)$ · gradient : perpendicular to the level set Any point on the same level set has the same g(x,y)Wed. 23/11/16 MATH1401: Mathematical Methods 1

Help Class	
Prof. Wilson	
§ gradient §	
$1. \nabla f'' = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$. <u></u>
grad f"	
•Q1. Suppose $\underline{V} = 3x^2\underline{i} + \underline{Z}\underline{j} + (\underline{y} + \underline{Z}^3)\underline{k}$	
Find f s.t. $v = \nabla f$. $*$ "constant" (depends on $y \otimes Z$.	, but not x)
Solo: $\frac{\partial f}{\partial x} = 3x^2 \Rightarrow f = x^3 + \overline{A(y, z)}$ O	
$\frac{\partial f}{\partial y} = z \qquad \Rightarrow f = yz + \beta(x, z) \qquad @$	
$\frac{\partial f}{\partial z} = y + z^3 \qquad \Rightarrow f = yz + \frac{1}{2}z^4 + C(X, y) \textcircled{3}$	
$Q = 0$; $0 = B(x, z) = \frac{1}{4}z^4 - C(x, y)$	
$C(x, y) = -\frac{1}{4}z^4 + B(x, z)$	
no y dependence	
$\Rightarrow C(x, y) = C(x) \text{ and } $	
$B(x,z) = \frac{1}{4}z^4 + C(x)$	
$f = x^3 + A(x, z)$	
$f = yz + z^{4} + C(x)$	
$\Rightarrow f = x^3 + yz + \frac{1}{4}z^4 + D$ general solution	
Q_2 . $Y = YZ\hat{j} + Yk$	
Soln: $\frac{\partial f}{\partial y} = y z$ $\frac{\partial f}{\partial z} = y$ $f = \frac{1}{2}y^2 z + A(z)$ 0 $\frac{\partial f}{\partial z} = y$ $f = zy + B(y)$ 0	
$\frac{\partial f}{\partial r} = 0$	
a function o	£
$O - Q: O = yz (\frac{1}{2}y - 1) + A(z) - B(y)$	ji
A(Z) =- YZ (= yZ (= y - 1) + B(y) Cannot get Zy out of A	v \ <u>(</u> <i>Z</i>)
Not possible.	····
Not every vector held $y(x, y, z)$ is a gradient ∇f .	
Summary: Mixed Derivative Theorem	
TOP Well-benaved tunctions 1(x, y, z),	
For well-behaved functions $f(x, y, z)$, $\frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right)$	

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	$\frac{\partial}{\partial y}(V_1) = \frac{\partial}{\partial x}(V_2)$
(P)MPPPA humana jajanjanan umP)n) humana kamana kana	$\frac{\partial}{\partial x}(v_1) = \frac{\partial}{\partial x}(v_2)$ These are sufficient conditions
	$\frac{\partial}{\partial z}(V_2) = \frac{\partial}{\partial y}(V_3)$ (if all is smeath, differentiable, etc.)
	$\frac{z}{z} = f(x,y) [2D]$
adaado 10 aaadad aadaaadaaaaadaaaaa) yoo oo yoo aayoo yoo ahayoo	$\sqrt{\hat{u}}$. ∇f is the slope I feel if I
иневании склопае у де у поссе у рами (5 с работает с рашие с де волого)	walk in the direction of 2.
	✓ The direction of
••\$9947.6.57.0000.4.570000.0000000000000000000000	steepest increase of f
hmadimmaaaan 	$\sqrt{ \nabla f }$ is the max slope.
record in hime between a constant and a known (1972)	$\sqrt{\text{perpendicular to } \nabla f}$ is the line/contour $f = \text{constant}$.
	. g(∞,y,ℤ) [3D]
and a second	∇g points in the direction of increasing g.
eereeer to dooreee fundade fabrikking (almos fabrikking) aan aan aan aan aan aan aan aan aan aa	129 tells us how fast g increases in that direction.
	• A surface can be defined as g = constant.
	∇g is perpendicular to the surface.
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	Prof. Halburd		
Anaratin parkating and a second s	Chapter 5. § Integration §		##311A16rv1vvAnner(ammerga(1);122
41	Introduction : Integral Calculus	<u>f(x)</u>	
t Terrer far fan maan de hyskelyjn dyn de fyfyl on yn yn yn ar an fan fan fan f		<u> </u>	ang the second
	Area $\int_{a}^{b} f(x) dx$ defines integral.	The	
stanilastanakti (estjani); entjennan sustanana (den			nd (ed 1-fest) de la ferre a construction de construction de construction de construction de construction de co
Deven mener pour Distance and the second	<u>Riemann Integrals</u>		~~~
and the second	fix mid-point y=f(v)		χ
Miching Juliera		nammaa amaanga madaanaa madaa ay ay ay ay ahaa ahaa ahaa ahaa aha	-594
			y=f(v)
Anathard ann ann an Anghailte ann an Anatharachaigea	0 a x x+h V		9-10
**************************************	$\sqrt{\text{Let } F(x)} = \int_{a}^{x} f(v) dv$		atananat fi fo (keel marga ana ana ana ana
and a second	$F(x) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h}$		******************************
	$= \lim_{x \to 0} \frac{\int_{a}^{x+h} f(x) dx - \int_{0}^{x} f(x) dx}{\int_{0}^{x+h} f(x) dx - \int_{0}^{x} f(x) dx}$	0	V
nga a su a	h→o h	- The number on top is approx. t	he area
, and an is a more marked by the day by the second and is a marked by	$= \lim_{h \to 0} \frac{\int x^{+h} f(y) dy}{h}$	of triangle. (: mid-point)	and a second
11000000000000000000000000000000000000	$= \lim_{h \to 0} \frac{h}{h} \text{ area of } \square$ $= \lim_{h \to 0} \frac{ hf(x) }{h} = \lim_{approx.} h.f(mid-point)$	- The partition does not need to	be
energy the second se	h≁o h approx.	regular. The approximation works	; as
**************************************	= f(x)	long as the width of each subd	ivision
Normatuseeeeeeeeeeeeeeeeeeeeeeeeeeeeee		tends to 0.	9991934995454-1-1-2-2-2-2-2-2-2-2-2-2-2-2-2-2-2-2-2-
eenmataankikk 5-foodhirood aanaoonaans faarakki boossa	Fundamental Theorem of Calculus	untun kan mengenakan kan kan kan kan kan kan kan kan ka	the second s
attalatt för föranda mannatt till föra för att för sön som sön som som	$\frac{d}{dx}\int f(x)dx = f(x)$		annan an ann an tha fairt fa a suir an
·····	Improper Integrals		1941-14593-universite universite de la constante de la constante de la constante de la constante de la constant
1965-0197-01-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0	Suppose that f is defined in [a,b] ex	cept at CE(a,b) then	ini datatan oran siya ya ya ya
est ff Sfare ff Mark Skaardag fan serie aan ar gegen en gege	$\int_{a}^{b} f(x) dx = \lim_{\substack{i \in \neq 0 \\ i \in \neq 0}} \int_{a}^{c-\varepsilon} f(x) dx + \lim_{\varepsilon \to 0^{+}} \int_{c+\varepsilon}^{b} f(x) dx$	x) dx	40414505544440
. В став с с с с с с с с с с с с с с с с с с с	e approaches 0 from the +ve side		There a Deressian and the region of a state of the second
	Both limits must exist.	чилэлтандарадарадарадарадагаагаагаагаарадардуулдуулуунын нааффарадирадагаагаарадардуулуу	tale de la la la compaça e compação e com
			territeritettettetterriterierettetterri

EXAMPLE D. How to integrate $f(x) = \frac{1}{x^2}$? [-±] y, (for theorem) $\int_{-1}^{1} \frac{1}{x^2} dx = \left| -\frac{1}{x} \right|_{-1}^{1} = (-1) + (-1) = -2$ And this is WRONG. c-£ C ٩o Right way. Consider $\lim_{\varepsilon \to a} \int_{-1}^{a-\varepsilon} \frac{1}{x^2} dx$ $= \lim_{\varepsilon \to 0^+} (-x^{-1}) \Big|_{-1}^{-\varepsilon}$ $= \lim_{\varepsilon \to 0^+} (\varepsilon^{-1} - 1)$ Also, $\lim_{\epsilon \to 0^+} \int_{\epsilon}^{t} \frac{1}{x^2} dx = \infty$ Hence, integral $\int_{1}^{1} \frac{1}{x^2} dx$ doesn't converge. ✓ EXAMPLE @. $\int_{-1}^{1} \frac{1}{x} dx = \lim_{c \to 0^{-}} \int_{-1}^{c} \frac{1}{x} dx + \lim_{c \to 0^{+}} \int_{c}^{1} \frac{1}{x} dx$ not continuous $= \lim_{c \to 0^{-}} \ln|x| \Big|_{-1}^{c} + \lim_{c \to 0^{+}} \ln|x| \Big|_{-1}^{c}$ must integrate separately $= \lim_{c \to 0^{+}} (\ln|c| - \ln|-1|) + \lim_{c \to 0^{+}} (\ln|-\ln c)$ indeterminate (doesn't converge) χ Ø 1 V EXAMPLE ③. $\int_{0}^{\infty} e^{-x} dx = \lim_{L \to \infty} \int_{0}^{L} e^{-x} dx$ $= \lim_{L \to +\infty} (-e^{-x}) \Big|_{0}^{L}$ $= 1 - \lim_{t \to t_{m}} e^{-L} = 1$

Warman and James and Carlow and Car	Recop: y = f(x)
	$9(x, y) = f(x) - y = 0$ $\frac{\partial y}{\partial x}$
grad grad	alent of the first of the second seco
derivat	tive wrt $x \left(\nabla g = \frac{\partial g}{\partial x} i - \frac{\partial g}{\partial y} j \text{ in general} \right)$
Amm ^{ann 1} statumt d'amatésatut (1,112) a 11,11,111 a 11,111	<u>T</u> . (79) = 0
Maglufe(Anc)Aurumounounoneurumeeynobenetingingelyg(ghenegig	$\Rightarrow \underline{T} = \underline{i} + f'(\underline{x}) \underline{j} \text{tangent vector}$
	V
4-3	Integration by Parts
	$\frac{d(uv)}{dx} = \frac{du}{dx} v + u \frac{dv}{dx}$
**************************************	$\Rightarrow \int u \frac{dy}{dx} dx = -uv - \int v \frac{dy}{dy} dx$
	$\int \int dx dx = \int \int dx dx$
	e.g. differentiate the polynomial & integrate trig
1918	Especially useful for 'inverse fins'
uuuut fuuddaa ay a	e.g. o $\int unx dx = \int 1. \ln x dx$
ellekinelligumassikasissaan vasmusseussessissiga	u=lnx $v'=l$
ananan ana ang ang ang ang ang ang ang a	
nemate and the second	$u = \arcsin x$ $v' = 1$
•	Iterated Integration (Reduction Formula)
eessiin oo dharaan ahaa ahaa ahaa ahaa ahaa ahaa aha	EXAMPLE:
AAADD	$\sqrt{\ln f \sin^{n} x dx}$
en and a fair and a	$= \int \sin x dx$
100 met la commeto a calificação para do como se	$U = \sin^{n-1} x$ $V' = \sin x$
Хевистичный колоникалартери Герекого со со на со село с	$u' = (n-1)\sin^{n-2}x\cos x v = -\cos x$
*****	$I_n = -\sin^{n-1}x\cos x + \int \cos^2 x (n-1)\sin^{n-2}x dx$
uthan(aya yan ya kumaka yamayka ya	$= -\sin^{n+1}x\cos x + \int (1-\sin^2 x)(n-1)\sin^{n-2}x dx$
1427.0 ° 10,0 ° 10,0 ° 10,0 ° 10,0 ° 10,0 ° 10,0 ° 10,0 ° 10,0 ° 10,0 ° 10,0 ° 10,0 ° 10,0 ° 10,0 ° 10,0 ° 10,0	$= -\sin^{n-1}x\cos x + (n-1)\int (\sin^{n-2}x - \sin^n x) dx$
engleng gans son son son ny managatana da anatana da atanàn desara any	$= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x dx - (n-1)\int \sin^n x dx$
talan menangkan kana tana panatapan seja pana panta panta panta pana pana pana	$\therefore n I_n = (n-1) I_{n-2} - \sin^{n-1} x \cos x \qquad I_{n-2} \qquad I_n$
**************************************	\checkmark Use this iteratively to reduce I_n to evaluating
	$L_0 = \int dx = x + c$
Taddimitry (1997)	$I_1 = \int \sin x dx = -\cos x + c$

4.4	Substitution Method	la seren a construction a a construct
	$\int f(x) dx = \int f(y) \frac{dy}{dy}$	
	$\int f'(x) e^{f(x)} dx = e^{f(x)} + C$	
	$\int \frac{f'(x)}{f(x)} dx = \ln[f(x)] + C$	
	$\int f'(x) [f(x)]^{n} dx = \frac{[f(x)]^{n+1}}{n+1} + C$	49944479
	V EXAMPLE:	
	Find Se ^{sinx} cosxdx	
	Soln: $u = sin x$ $u' = cos x$.	
www.timitiinelikerneikeenserningenzernensernensernettillt	$\int \cos x e^{\sin x} dx = \int \cos x e^{u} \frac{dx}{du} du$	
Devision of the 2002 of the 2002 of the 2004 of the	= Je ^u du	
99000,0000,000,000,000,000,000,000,000,	= e _n +c	
anannan an an 2000 ta fa fa fan de Stan La denama an	= e ^{sin×} +c	Ø
	$\sqrt{\alpha^2 - \chi^2}$ $\chi = \alpha \sin \alpha$	
ngestengenningenengenensensensenselskolistel deleterere	$\sqrt{a^2+\chi^2}$ $\chi = asinh u$	4 42 200 2 11111 12 2 2 2 2 2 2 2 2 2 2 2 2
9499999	$\sqrt{x^2-a^2}$ $\chi = a \cosh u$	
Ğığı olaş Şaranın və ara başı kaşı kaşı kaşı kaşı kaşı kaşı kaşı k	$\frac{1}{\sigma^2 + \chi^2} \qquad \chi = a \tan \theta$	ees geen www.markees.com/www.com/
	Some FP3 Formulae:	
beleden and an and a second	$\int \int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin(\frac{x}{a}) + C$	ununtitä pääninget nämint tääniman
5411114405-611111-5-000-68848464111111564146	$\int \frac{1}{\sqrt{x^2 + \alpha^2}} dx = \operatorname{arsinh}(\frac{x}{\alpha}) + c$	
*&&&&	$\int \frac{1}{\sqrt{2^2 - a^2}} dx = \operatorname{arcosh}(\frac{\pi}{a}) + C$	
500110500110533511000000000000000000000	$\int \frac{1}{q^2 - x^2} dx = \frac{1}{q} \operatorname{artanh}(\frac{x}{q}) + C$	
wbw192000000000000000000000000000000000000	$\int \frac{d^2 - x^2}{d^2 + x^2} dx = \frac{1}{a} \arctan(\frac{x}{a}) + C$	Q
-64565054446955446654466449485855555555	$\int \frac{1}{2\pi^{2} - a^{2}} dx = \frac{1}{2a} \ln \left \frac{x + a}{x - a} \right + c$	mi)-64244-6644-6744-64444
#\$	J.X-U	999-001-001-001-00-00-00-00-00-00-00-00-00-
4:5	Integrating Rational Functions (Partial Fractions)	444.6969.446-29-27.7777777744.4762-444
Bettereit of an		
	$R(x) = \frac{P_m(x)}{Q_n(x)}$ where $P_m(x)$ is a polynomial of degree m ,	ter og geren en en der en d
	$Q_n(x)$ is a polynomial of degree n .	
Sennangenangenangnangnangenergenergenerge	How to find SR(x) dx ?	
	O If $m \ge n$, use polynomial division (long division) to obtain a rational	
en an		
	function with $m < n$. @ Factorise $Q_n(x)$ (over \mathbb{R})	
1	3	

4	$Q_n(\chi) = (\chi - a_1)^{C_1} (\chi - a_2)^{r_2} \dots (\chi - a_n)^{r_n} (\chi^2 + b_1 \chi + c_1)^{s_1} \dots (\chi^2 + b_n \chi + c_n)^{s_n}$
	The ration function R(x) can be expressed in terms of partial fractions as
1999-10-11-11-11-11-11-11-11-11-11-11-11-11-	$R(x) = \frac{P_m(x)}{r}$
1400-014747993347454044444444	Qn(X) r, terms r, terms
The degree	$= \left[\frac{A_{1}}{x-a_{1}} + \frac{A_{2}}{(x-a_{1})^{2}} + \dots + \frac{A_{r}}{(x-a_{l})^{0}}\right] + \dots + \left[\frac{B_{l}}{x-a_{l}} + \dots + \frac{B_{r_{M}}}{(x-a_{M})^{r_{M}}}\right]$
numerator	is one $\int \sqrt{C_1 X + D_1} degree I$ $C_1 X + D_1 = I = \int C_1 X + D_1 = I$
more than	[(X - D)X - C)] = [(X - D)X - C)]
-on-denomin	dtor degree 2
Фифентенного состание селетение на различие с состание с состание с состание с состание с состание с состание с	√ EXAMPLE.
telefond al al a fair a fair a fair an ann an fair	Find $\int_0^\infty \frac{dx}{(x+\eta)^2(x^2+\mu)}$
-bahambahannangerbektigindyadawiyanananan	
0	Soln: $(x+2)^{3}(x^{2}+1) = \frac{A}{x+2} + \frac{B}{(x+2)^{2}} + \frac{Cx+D}{x^{2}+1}$ the denominator
ternenderskellerskolmmelselserererer en mersemensererskynstelsky	repeated terms $I \equiv A(x+2)(x^{2}+1) + B(x^{2}+1) + (Cx+D)(x+2)^{2}$

n kanan dan makaman pelaran yan yan yan mana peperanan kanan penangan se	
n 1900 a fa an ann an an ann an an ann a	e.g. sub $x = -2$: $B = \frac{1}{5}$
	equate coefficients:
n ben like to om tet stormen ben gemen gebrevelt op men om kar besom ge	$I = (A+C) \chi^{3} + (2A + \frac{1}{5} + 4C + D) \chi^{2} + (A+4C+4D) \chi + (2A + \frac{1}{5} + 4D)$
**************************************	χ ³ : A+C=0
VRR88841000000000000000000000000000000000	$x^2: 2A + \frac{1}{5} + 4C + D = 0$
***************************************	x : A + 4C + 4D = 0
	$x^{\circ}: 2A + \frac{1}{5} + 4D = 1$
	$\therefore A = \frac{4}{25}$
148400 (mark) or a data mini (482) dag dami basa sa bira da mini da	B=±
ni olimini matta ang pagmana ing manana ang pagmana ing pagmana ang pagmana ang pagmana ang pagmana ang pagman	$c = -\frac{4}{25}$
talktalappan moorek ajapjan thebid paret t	$D = \frac{3}{25}$
Martin Community of Section 2010 and and a section of the Section	
ta ta da mang ta ya a a a a a a a a a a a a a a a a a	
50000000000000000000000000000000000000	Man 21/11/16
	MATH1401: Mathematical Methods 1
	Prof. Halburd
	Recap:
<u>(</u>	Improper Integral
<u>r</u>	- Incyini

Of continuous on [a,b] except at x=c $\int_{a}^{b} f(x) dx = \lim_{\varepsilon \to 0^{+}} \int_{a}^{c-\varepsilon} f(x) dx + \lim_{\varepsilon \to 0^{+}} \int_{c+\varepsilon}^{b} f(x) dx$ hoth limits must even hoth limits must exist ac @ f continuous on (a,b] but not at x=a $\int_{a}^{b} f(x) dx = \lim_{\epsilon \to 0^{+}} \int_{a+\epsilon}^{b} f(x) dx$ if this limit exists (1) f continuous on $[a, +\infty)$, then $\int_{a}^{\infty} f(x) dx = \lim_{b \to +\infty} \int_{a}^{b} f(x) dx$ Partial Fractions (Cont.) 4'5 ✓ EXAMPLE: (cont.) $\int_{0}^{\infty} \frac{dx}{(x+2)^{2}(x^{2}+1)} = \int_{0}^{\infty} \left[\frac{4}{25}(x+2)^{-1} + \frac{1}{5}(x+2)^{-2} + \left(-\frac{4}{25}x + \frac{3}{25}\right)(x^{2}+1)^{-1}\right] dx$ $= \lim_{h \to \infty} \int_{0}^{h} \left[\frac{4}{25} (x+2)^{-1} + \frac{1}{5} (x+2)^{-2} - \frac{4x}{25(x^{2}+1)} + \frac{3}{25(x^{2}+1)} \right] dx$ $= \lim_{b \to \infty} \left\{ \frac{4}{25} \ln |x+2| - \frac{1}{5} (x+2)^{-1} - \frac{2}{25} \ln (x^2+1) + \frac{3}{25} \arctan x \right\}_{0}^{b} \right\}$ $= \lim_{b \to \infty} \left\{ \frac{2}{25} \ln \frac{\binom{b+2}{b+2}}{\frac{b^2+1}{b^2+1}} - \frac{1}{5(b+2)} + \frac{3}{25} \arctan b \right\} \xrightarrow{\text{Always remember}} \frac{1}{15(b+2)} \xrightarrow{\text{Alway$ tan $=\frac{31}{50}-\frac{4}{25}\ln 2+\frac{1}{10}$ Double Angle Formulae 4.6 COS24+SIN24 =1 $1 + \tan^{2} u = \sec^{2} u = \frac{1}{\cos^{2} u}$ $\Rightarrow \cos u = \frac{1}{1 + \tan^2 u}$ -(cosu>0)

Similarly, since tanu So, $\cos 2u = \cos^2 u - \sin^2 u = \frac{1 - \tan^2 u}{1 + \tan^2 u}$ $\sin 2u = 2\sin u \cos u = \frac{2\tan u}{1 + \tan^2 u}$ Put $u = \frac{\theta}{2}$, $t = \tan u = \tan \left(\frac{\theta}{2}\right)$, then . $\cos\theta = \frac{1-t^2}{1+t^2}, \quad \sin\theta = \frac{2t}{1+t^2}$ $\cos\frac{\theta}{2} = \frac{1}{\sqrt{1+t^2}}, \quad \sin\frac{\theta}{2} = \frac{t}{\sqrt{1+t^2}}$ $t = \tan \frac{\theta}{2} \Rightarrow \theta = 2 \operatorname{arctant}$ $\Rightarrow d\theta = 2\cos^2\frac{\theta}{2}dt$ $\Rightarrow d\theta = \frac{2}{1+t^2} dt$ V EXAMPLE: Find $\int_{\sigma}^{\frac{\pi}{2}} \frac{d\theta}{2+\sin\theta}$ Soln: change the limits Let $t = tan^{\frac{\theta}{2}}$, then $\frac{dt}{d\theta} = sec^{2}(\frac{\theta}{2})$ $\int_{\sigma}^{1} \frac{\frac{2dt}{1+t^{2}}}{\left(2+\frac{2t}{1+t^{2}}\right)} = \int_{\sigma}^{1} \frac{2}{2+2t^{2}+2t} dt$ $= \int_{\sigma}^{1} \frac{1}{t^{2}+t+1} dt$ $= \int_{0}^{1} \frac{1}{(t+\frac{1}{2})^{2}+\frac{3}{4}} dt$ $Let \quad t+\frac{1}{2} = \frac{13}{2} \tan u, \text{ then } \frac{1}{3} \sec^{2} u$ $I = \int_{arctan \frac{13}{2}}^{arctan \sqrt{3}} \frac{1}{\frac{3}{4}(\tan^{2} u+1)} \cdot \frac{13}{2} \sec^{2} u du$ $=\int_{\frac{\pi}{3}}^{\frac{\pi}{3}}\frac{4}{3}\frac{\sqrt{3}}{2}du$ $=\frac{2\sqrt{3}}{3}[u]_{\frac{\pi}{2}}^{\frac{\pi}{2}}$ $=\frac{\sqrt{3}}{4}\pi$ Fri. 25/11/16

MATH1401: Mathematical Methods 1 Prof. Halburd Indeterminate forms of limits (facts): $\frac{0}{0}/\frac{\infty}{\infty}$ e.g. $\lim_{x \to 0^+} x^a (\log x)^b$ a,6>0 $\lim_{x\to 0^+} x^{\alpha} (\mathbf{e}^{x})^{b}$ a.b >0 ✓ exponential grows faster than power power grows faster than log. ✓ L'Hôpital's Rule $\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)} \quad \text{if } f(x_0) = g(x_0) = 0 \quad \text{or } \pm \infty.$ EXAMPLES: $\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\cos x}{1} = 1$ $\lim_{x \to \infty} \frac{\ln x}{x^n} = \lim_{x \to \infty} \frac{\frac{1}{x^{n-1}}}{nx^{n-1}} = \lim_{x \to \infty} \frac{1}{nx^n} = 0$

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۴ ۵	Fri. 25/11/16 (cont.)
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·····	Prof. Halburd
nin an an an dh'ann ann ann an an gann an an ann an an an an an an an an an	Chapter 6. § (Ordinary) Differential Equations (ODEs) §
	Def.
unaan oo moo oo mood aadh in taliiniin dhaadaadaadaada	The order of an ODE is the order of the highest derivative appearing
nganna, 1, mar 1, ma	in the equation.
پېږې د ۱۹۹۵ د د دې ورو د د د د د د د د د د د د د د د د د د	<u>Def</u> .
an yaa ayaa ayaa yaa yaa ya ya ya ya ya ya	The most general form of an n th order ODE is
eeeerstelette (televensingen felsen ander andere (televenset of andere	$F(x, y, \frac{dy}{dx}, \dots, \frac{d^n y}{dx^n}) = 0$
6.1	First Order Differential Equations
6.1.1	Separable Equations
**************************************	If an ODE is separable, then
₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩	$\frac{dy}{dx} = f(x)g(y) \qquad \text{Note:} \frac{dy}{dx} = x^2 + y^2 \text{is not separable}$
garimuszugigesztekinek jakonagonagonagonaranianianiaa	$\Leftrightarrow \frac{dy}{g(y)}, \frac{dy}{dx} = f(x)$
	integrate wrt x :
	$\int \frac{d}{dx} \frac{dx}{dx} = \int f(x) dx$
y z a y manum anna an a y a y a y a y a y a y a y a y	$\Leftrightarrow \int \frac{dy}{g(y)} = \int f(x) dx$
rear and the second	This gives a one-parameter family of solns.
	Note: The parameter is an integration constant.
aanteen en het de het de het en de de aante de de het de het de het het het het het het het het het he	EXAMPLE:
the the second secon	Solve the ODE $\propto \frac{dy}{dx} + 3y = 2$, given the initial condition $y(1) = 2$.
eeelikaan delemaa delema ka	Soln: (i.e. $y=2$ when $x=1$)
597999 5015-5015-0015-0015-001-001-001-001-001-	$\sqrt{\frac{x dy}{dx}} = 2 - 3y$
******{***}***************************	$\int \frac{dy}{2-3y} = \int \frac{dy}{x} don't \text{ forget the absolute value}$
tteren an	$-\frac{1}{2}\ln[2-3y] = \ln[x] + c$
unnen an the state of the	$\ln 2-3y = \ln x ^{-3}$ another constant e^{-3c}
n de Oli de la Colonna de Colonna de Santa de Santa de Santa de Santa de Colonna de Colonna de Colonna de Colon	$2^{-3} y = C_0 x^{-3}$ (where $C_0 = \pm e^{-3x}$)
	✓ sub $y(1) = 2$: 2-6 = Co = -4 ⇒ $y = \frac{1}{3}(2 + \frac{4}{3}) = \frac{2}{3}(1 + \frac{2}{3})$
latender en transfordet til Statender och en en ander ander en en ander en en ander en en en en en en en en en	:

	$\frac{dy}{dx} + \alpha(x)y = f(x) \leftarrow doesn't include any y terms (*)$	
4	Idea .	
	\checkmark We multiply both sides of (*) by a function $I(x)$ (called an	
	integrating factor) to make LHS an exact derivative (of a product).	
	$I(x)\frac{dy}{dx} + a(x)I(x)y = f(x)I(x) \qquad (#)$	
	V We want the LHS to be	
	$\frac{d}{dx}(Iy) = I\frac{dy}{dx} + \frac{dI}{dx}y$	
	\sqrt{Then} , we want: $\frac{dI}{dx} = aI$ (by comparing what we have and what	
	integrate wrt x: we want)	• • • • • • •
	$\int \frac{1}{I} \cdot \frac{dI}{dx} dx = \int a(x) dx$	
	$\Leftrightarrow \int \frac{dI}{I} = \int a dx a' \text{ represents } (a(x))'$	
	$\frac{\ln I }{\ln I } = \int a(x) dx + c$	
	$I = \widehat{c} \exp(\int a(x) dx) \qquad (\widehat{c} = e^{c})$	
	Choose \hat{C} . $I(x) = \exp(\int a(x) dx)$	
	\sqrt{Then} , (#) becomes	
	$\frac{d}{dx}(Iy) = I(x)f(x)$	
	(Integrate) $\Rightarrow I(x) y(x) = \int I(x) f(x) dx$	
	$\Rightarrow y(x) = \frac{1}{T(x)} \int I(x) f(x) dx$	
	$\int \frac{1}{x} dy = g(x)$ $\int \frac{dy}{dx} + c(x)y = g(x)$	
	$b(x)\frac{dy}{dx} + c(x)y = g(x)$	
	O divide by b(x) to get a standard eqn	
	O compute the integrating factor I(x).	
	Image: Image	
	\oplus integrate to find $\mathfrak{g}(x)$.	•••••
	S find particular soln.	
	✓ EXAMPLE Ø:	

 $y(\pi) = 1$. Soln: $\frac{dy}{dx} + \frac{2}{x}y = \frac{\cos x}{x^2}$... 2 Then $I(x) = \exp(\int \frac{2}{x} dx) = \exp(2\ln x) = x^2$ Multiply \mathcal{Q} by $I = \alpha^2$. $\chi^{1} \frac{dy}{dx} + 2\chi y = \cos \chi$ $\Rightarrow \frac{d}{dx} (x^2 y) = \cos x$ $\Rightarrow x^2 y = \sin x + c$ $\mathcal{Y}(\mathbf{T})=\mathbf{1} \quad : \quad \mathbf{T}^2=\mathbf{C}$ $\Rightarrow y = \frac{\sin x + \pi^2}{x^2}$ ✓ EXAMPLE @. Solve $x^2 \frac{dy}{dx} + (1+x)y = \frac{1}{x}$ Soln: $\frac{dy}{dx} + (\frac{1+x}{x^2})y = \frac{1}{x^3}$... 0 Then $I(x) = \exp\left(\int \frac{1+x}{x^2} dx\right) = \exp\left(\int \frac{1}{x^2} dx + \int \frac{1}{x^2} dx\right)$ $= \exp\left(-\frac{1}{x} + \ln|x|\right)$ $= \chi e^{-\frac{1}{x}}$ Multiply **(b)** by I(x): $xe^{-\frac{1}{x}}\frac{dy}{dx} + (\frac{1}{x}+1)e^{-\frac{1}{x}}y = \frac{1}{x^2}e^{-\frac{1}{x}}$ $\Rightarrow \frac{d}{dx} \left(x e^{-\frac{1}{x}} y \right) = \frac{1}{x^2} e^{-\frac{1}{x}} \int f(x) e^{f(x)}$ $xe^{-\frac{1}{2}}y = \int \frac{1}{2}e^{-\frac{1}{2}}dx = e^{-\frac{1}{2}} + c$ ⇒ $\Rightarrow y = \frac{1}{x} + \frac{c}{xe^{-1/x}}$ · Bernoulli's Equation $\frac{dy}{dx} + yP(x) = y^{n}Q(x) \qquad n \neq 1$ This can be reduced to a linear equation by introducing $Z = y^{1-n}$ $\Rightarrow \frac{dz}{dx} = (1-n)y^n \frac{dy}{dx}$ (substitute) $\Rightarrow \frac{dz}{dx} = (1 - n)y^{-n}(y^nQ(x) - yP(x))$

	$= (n-1)y^{-n} \cdot y P(x) - (n-1)Q(x)$	····
	= $(n-1) y^{1-n} P(x) - (n-1) Q(x)$	
	= (n-1)zP(x) - (n-1)Q(x) - general 1 st order linear ODE	
6.2	Second Order Differential Equations	
	$\frac{d^2y}{dx^2} + a(x)\frac{dy}{dx} + b(x)y = f(x) \qquad (1)$	
	\checkmark If $f(x)=0$, we have	
	y'' + a(x)y' + b(x)y = 0 (2)	
	which is said to be homogeneous.	
	$\sqrt{f(x)}$ is sometimes called a forcing function.	
	\checkmark Suppose that $y_1 \& y_2$ are solns of (2).	
	Let $y(x) = \alpha y_1(x) + \beta y_2(x)$ where α, β are constants.	
	Then,	
	$y'' + \alpha y' + by = (\alpha y_1 + \beta y_2)'' + \alpha (\alpha y_1 + \beta y_2)' + b(\alpha y_1 + \beta y_2)$	
	$= \alpha \{ y_{1}'' + \alpha y_{1}' + b y_{1} \} + \beta \{ y_{2}'' + \alpha y_{2}' + b y_{2} \} = 0$	
	\Rightarrow y solves (2).	
	√ If y₁ & y₂ are independent solns,	
	(i.e. one is not a multiple of the other for 2 nd order ODEs)	
	then $y = \alpha y_1 + \beta y_2$ is the general soln.	
6.2.1	Constant Coefficient Homogeneous Linear Second Order ODEs	
¥	y" + ay' + by = 0 (3) a, b constant	
	√ We want to look for solns of (3) of the form	
	$\frac{y(\chi)}{2} = e^{\lambda \chi} \qquad \cdots (4)$	
	$\Rightarrow y' = \lambda e^{\lambda x}$	
	$y'' = \Lambda^2 e^{\lambda x}$	
	substitute into (3): $(\Lambda^2 + a\Lambda + b) e^{\Lambda \pi} = 0$	
	$\Leftrightarrow \Lambda^{3} + a\lambda + b = 0 \qquad \dots (5) \qquad \text{Characteristic Eqn.}$	
	Auxiliary Egn.	
	$\sqrt{\text{Case 1}}$ (5) has 2 different real roots $\Lambda_1 \& \Lambda_2$.	
	Then , egn (3) has 2 roots	
	$e^{\lambda_1 \times} \& e^{\lambda_2 \times}$	V

÷,

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	(not multiple of each other)
	So the general soln is
hantatelen telefolk for for en	$\frac{y(x) = \alpha e^{\lambda_1 x} + \beta e^{\lambda_2 x}}{\alpha_1 \beta_1} \qquad \alpha_1 \beta_2$
₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩₩	$\sqrt{Case 2:}$ (5) has 1 real repeated root
999 - 1999 - 1999 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1	$n = \frac{-\alpha \pm \sqrt{\alpha^2 - 4b}}{2} = -\frac{\alpha}{2} \qquad \text{since} \alpha^2 - 4b = 0$
and a second	Only 1 solo of son 141 due to 1 secontrol and
historediliseeselemmentem seerestaan possed eelssoode	Only 1 soln of eqn (4). due to 1 repeated root Let's look for another soln $(5) \Leftrightarrow (7 + \sqrt{5})^2 = 0$
	$\frac{1}{10000000000000000000000000000000000$
999 fearmath adhanak kananata fea Qipping Jangar e dearana	$y = g(x)e^{\lambda x} \qquad (\lambda = -\frac{a}{2}) \qquad \Leftrightarrow a^2 = 4b$
9	$\Rightarrow \mathbf{y}' = (\mathbf{g}' + \lambda \mathbf{g}) e^{\lambda \mathbf{x}}$
iningeneration of the second	$y'' = (g'' + 2\lambda g' + \lambda^2 g) e^{\lambda x}$
	So eqn (3) becomes
an a' chuin an an an ann an Anna an Ann	$(g'' + 2\lambda g' + \lambda^2 g) + \alpha (g' + \lambda g) e^{\lambda x} + bg e^{\lambda x} = 0$
neer Mardin de Leerneer d'America e de la construction de la construction de la construction de la construction	$\{g(\lambda^{2}+a\lambda+b)+g'(2\lambda+a)+g''\}e^{\lambda x}=0$
an a	disappears because disappears because $n = -\frac{a}{2}$ this is a characteristic
aan na aana ahaa ahaa ahaa ahaa ahaa ah	egn. $q^{\prime\prime}e^{\lambda x} = 0$
**************************************	⇔ g"=0
ann an tha an tha ann an tha an t	$\Rightarrow g(x) = \alpha x + \beta$
	Conclusion: repeated root
	$\frac{y(x) = (\alpha x + \beta)e^{\lambda x}}{\beta + \beta}$
	$\sqrt{\text{Case 3. (5)}}$ has 2 (distinct) complex roots π_i where
	$\begin{array}{l} n_{1} = p + ig \\ n_{1} = p - ig \end{array} \right\} \text{ they must be complex conjugate} \\ \end{array}$
	General soln is
	$C_{1}e^{(p+ig)x} + C_{2}e^{(p-ig)x} $ by Euler's Formula $e^{iy} = \cos \psi + i \sin \psi$
	where $e^{(p+ig)x} = e^{px}e^{igx} = e^{px}(\cos gx + i\sin gx)$
	where $e^{(p+ig)x} = e^{px}e^{igx} = e^{px}(\cos gx + i\sin gx)$ $e^{(p-ig)x} = e^{px}e^{-igx} = e^{px}(\cos gx - i\sin gx)$
	So.
	$\frac{1}{2} \left[e^{(p+ig)x} + e^{(p-ig)x} \right] = e^{px} \cos gx \left[= \operatorname{Re}\left(e^{(p+ig)x} \right) \right]$
	$\frac{1}{2i} \left[e^{(p+ig)x} + e^{(p-ig)x} \right] = e^{px} singx \left[= Im \left(e^{(p+ig)x} \right) \right]$
	So, the general soln is
	·····································

* *****

• EXAMPLE: Solve $y' + y' + y = 0$ where $y(0) = 0$ y'(0) = 1 Soln: Characteristic Eqn. $n^{++} n^{+} 1 = 0$ $(n + \frac{1}{2})^{+} = -\frac{\pi}{4}$ $n = -\frac{1}{2} \pm \frac{1\pi}{2}$ Then. $e^{nx} - e^{-\frac{1}{2}x} e^{\frac{1}{2}x} = e^{\frac{1}{2}x} (\cos \frac{\pi}{2} x + i \sin \frac{\pi}{2}x)$ $\Rightarrow Re(e^{nx}) = e^{-\frac{1}{2}x} \cos \frac{\pi}{2}x$ Im $(e^{nx}) = e^{-\frac{1}{2}x} \cos \frac{\pi}{2}x$ General soln: $y(x) = e^{-\frac{\pi}{2}} (a \cos \frac{\pi}{2} x + \beta \sin \frac{\pi}{2} x)$ Since $y(0) = 0$, $0 = \alpha$ $\Rightarrow y(x) = \beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x + \frac{\pi}{2} \beta e^{-\frac{\pi}{2}} \cos \frac{\pi}{2}x$ Since $y(0) = 1$, $\beta = \frac{2\pi}{3}$ $\Rightarrow PS = y(x) = \frac{2\pi}{3} e^{-\frac{\pi}{3}} \sin (\frac{\pi}{2}x)$ $e^{\frac{\pi}{2}} (\frac{\pi}{2}) = \frac{2\pi}{3} e^{-\frac{\pi}{3}} \sin (\frac{\pi}{2}x)$ $f(x) = -\frac{2\pi}{3} e^{-\frac{\pi}{3}} \sin (\frac{\pi}{2}x)$ $f(x) = -\frac{\pi}{3} e^{-\frac{\pi}{3}} \sin (\frac{\pi}{3}x)$ $f(x) = -\frac{\pi}{3} e^{-\frac{\pi}{3}} \sin (\frac{\pi}{3}x)$ $f(x) = -\frac{\pi}{3} e^{-\frac{\pi}{3}} \sin (\frac{\pi}{3}x)$ $f(x) = -\frac{\pi}{3} e^{-\frac{\pi}{3}} e^{-\frac{\pi}{3}} \sin (\frac{\pi}{3}x)$ $f(x) = -\frac{\pi}{3} e^{-\frac{\pi}{3}} e^{-\frac{\pi}{3}} \sin (\frac{\pi}{3}x)$ $f(x) = -\frac{\pi}{3} e^{-\frac{\pi}{3}} e^{-\frac{\pi}{3}} e^{-\frac{\pi}{3}} \cos (\frac{\pi}{3}x)$ $f(x) = -\frac{\pi}{3} e^{-\frac{\pi}{3}} e^{-\frac{\pi}{3}} e^{-\frac{\pi}{3}} \cos (\frac{\pi}{3}x)$ $f(x) = -\frac{\pi}{3} e^{-\frac{\pi}{3}} e^{-\frac{\pi}{3}} e^{-\frac{\pi}{3}} e^{-\frac{\pi}{3}} e^{-\frac{\pi}{3}} e^{-\frac{\pi}{3}} e^{-\frac{\pi}{3}} e^{-\frac{\pi}$		$y(x) = e^{p^{x}} (\alpha \cos g x + \beta \sin g x)$	
y'(0) = 1 Soln: Characteristic Eqn. $\Lambda^{++} \Lambda^{+} (= 0$ $(\Lambda + \frac{1}{2})^{+} = -\frac{3}{4}$ $\Lambda = -\frac{1}{2} \pm \frac{13}{2}$ Then. $e^{\Lambda x} = e^{-\frac{1}{2}x} e^{i\frac{\pi}{2}x} = e^{\frac{\pi}{4}x} (\cos \frac{\pi}{2} x + i\sin \frac{\pi}{2}x)$ $\Rightarrow Re (e^{\Lambda x}) = e^{-\frac{\pi}{4}x} \cos \frac{\pi}{2}x$ $Im (e^{\Lambda x}) = e^{-\frac{\pi}{4}x} \cos \frac{\pi}{2}x$ $Im (e^{\Lambda x}) = e^{-\frac{\pi}{4}x} \cos \frac{\pi}{2}x$ $General soln:$ $y(x) = e^{-\frac{\pi}{4}x} (a \cos \frac{\pi}{2}x + \beta \sin \frac{\pi}{2}x)$ Since $y(0) = 0$, $0 = \alpha$ $\Rightarrow y(x) = \beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x$ $y(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x$ $y(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x$ Since $y(0) = 1$, $\beta = \frac{2\pi}{2}$ $\Rightarrow PS y(x) = \frac{2\pi}{2}e^{-\frac{\pi}{2}} \sin (\frac{\pi}{2}x)$ $6^{2\cdot 2} Inhomogeneous Equations$ $y'' + ay' + by = f(x)$ $y'' + ay' + by = f(x)$ $y''' + ay'' + by = f(x)$ $Yate. (1)$ $y''' + ay'' + by = f(x)$ $y''''''''''''''''''''''''''''''''''''$			ç
Soln: Characteristic Eqn. $\lambda^{++} \lambda + 1 = 0$ $(\lambda + \frac{1}{2}t)^{+} = -\frac{\pi}{4}$ $\lambda = -\frac{1}{2} \pm \frac{1}{2}$ Then, $e^{\lambda x} = e^{-\frac{1}{2}x} e^{\frac{1}{2}x} = e^{-\frac{1}{2}x} (\cos \frac{\pi}{2}x + i \sin \frac{\pi}{2}x)$ $\Rightarrow Re(e^{\lambda x}) = e^{-\frac{1}{2}x} \cos \frac{\pi}{2}x$ $Im(e^{\lambda x}) = e^{-\frac{1}{2}x} \sin \frac{\pi}{2}x$ General soln. $y(x) = e^{-\frac{\pi}{4}} [a \cos \frac{\pi}{2}x + \beta \sin \frac{\pi}{2}x]$ Since $y(0) = 0$, $0 = \alpha$ $\Rightarrow y(x) = \beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x$ $y'(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x + \frac{15}{2}\beta e^{-\frac{\pi}{2}} \cos \frac{\pi}{2}x$ Since $y(0) = 1$, $\beta = \frac{4\pi}{3}$ $\cdot \Rightarrow PS$ $y'(x) = \frac{2\pi}{3} e^{-\frac{\pi}{2}} \sin (\frac{\pi}{2}x)$ (*) $f^{2} - \frac{1}{2}\beta e^{-\frac{\pi}{2}} \sin (\frac{\pi}{2}x)$ (1) y'' + ay' + by = f(x) (1) y'' + ay' + by = f(x) (1) y'' + ay' + by = f(x) Take the difference. Note, $(y_{1} \pm y_{2})'' = y'' \pm y'_{n}$			ladadaad di Soot e Ashadaad ah Abbilana
$h^{+} h + 1 = 0$ $(h + \frac{1}{2})^{+} = -\frac{3}{4}$ $h = -\frac{1}{2} \pm \frac{13}{2}$ Then, $e^{hx} = e^{-\frac{1}{2}x} e^{i\frac{13}{2}x} = e^{-\frac{1}{2}x} (\cos \frac{13}{2}x + i\sin \frac{13}{2}x)$ $\Rightarrow Re(e^{hx}) = e^{-\frac{1}{2}x} \cos \frac{13}{2}x$ $Im(e^{hx}) = e^{-\frac{1}{2}x} \cos \frac{13}{2}x$ General soln. $g(x) = e^{-\frac{3}{2}} (a\cos \frac{13}{2}x + \beta \sin \frac{13}{2}x)$ Since $y(0) = 0$, $0 = \alpha$ $\Rightarrow y(x) = \beta e^{-\frac{3}{2}} \sin \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $y(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{3}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{3}{2}} \cos \frac{13}{2}x + 13$	1999-1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1997 - 1	<u><u>y'(0)</u> = 1</u>	
$(\lambda + \frac{1}{2})^{*} = -\frac{3}{4}$ $\lambda = -\frac{1}{2} \pm \frac{13}{2}$ Then, $e^{\lambda x} = e^{-\frac{1}{2}x} e^{i\frac{\pi}{2}x} = e^{\frac{1}{2}x} (\cos \frac{\pi}{2} x + i\sin \frac{\pi}{2}x)$ $\Rightarrow Re(e^{\lambda x}) = e^{-\frac{1}{2}x} \cos \frac{\pi}{2}x$ Im $(e^{\lambda x}) = e^{-\frac{1}{2}x} \cos \frac{\pi}{2}x$ General soln. $g(x) = e^{-\frac{\pi}{4}} (\alpha \cos \frac{\pi}{2}x + \beta \sin \frac{\pi}{2}x)$ Since $y(0) = 0$, $0 = \alpha$ $\Rightarrow y(x) = \beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x$ $g(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x + \frac{\pi}{2}\beta e^{-\frac{\pi}{2}} \cos \frac{\pi}{2}x$ Since $y(0) = 1$, $\beta = \frac{2\pi}{3}$ $\therefore \Rightarrow PS$ $g(x) = -\frac{2\pi}{3} e^{-\frac{\pi}{2}} \sin(\frac{\pi}{2}x)$ $f(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} \sin(\frac{\pi}{2}x)$ $f(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} \sin(\frac{\pi}{2}x)$ $\beta = 2\frac{\pi}{3}$ $\therefore \Rightarrow PS$ $g(x) = -\frac{2\pi}{3} e^{-\frac{\pi}{2}} \sin(\frac{\pi}{2}x)$ $f(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} \sin(\frac{\pi}{2}x$		Soln: Characteristic Egn:	handfelja (Januariana (Januaria (Januaria (Januaria) 2014 martine (Januariana (Januaria) 2014 martine (Januaria)
$\lambda = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$ Then, $e^{\lambda x} = e^{\frac{1}{2}x} e^{\frac{\sqrt{3}}{2}x} = e^{\frac{1}{2}x} (\cos \frac{\pi}{2}x + i\sin \frac{\pi}{2}x)$ $\Rightarrow Re(e^{\lambda x}) = e^{-\frac{1}{2}x} \cos \frac{\pi}{2}x$ $Im(e^{\lambda x}) = e^{-\frac{1}{2}x} \sin \frac{\pi}{2}x$ General soln: $y(x) = e^{-\frac{1}{2}x} (\alpha \cos \frac{\pi}{2}x + \beta \sin \frac{\pi}{2}x)$ Since $y(0) = 0$, $0 = \alpha$ $\Rightarrow y(x) = \beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x$ $y'(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x + \frac{13}{2}\beta e^{-\frac{\pi}{2}} \cos \frac{\pi}{2}x$ Since $y'(0) = 1$, $\beta = \frac{2\pi}{3}$ $\Rightarrow PS. y(x) = \frac{2\pi}{3} e^{-\frac{\pi}{2}} \sin (\frac{\pi}{2}x)$ (*) $6^{2\cdot2\cdot 2} Inhomogeneous Equations. y'' + ay' + by = f(x) (*)\int Suppose that y_1 & y_2 are solns. y'' + ay'_1 + by_1 = f(x) (1)y''_1 + ay'_1 + by_1 = f(x) (1)y''_1 + ay'_1 + by_1 = f(x) Take the difference : Note, (y_1 \pm y_2)'' = y''_1 \pm y'_2$			10000° 101100° 00000 1 1000000° 1° 101° 000°10
Then, $e^{\pi x} = e^{-\frac{1}{2}x} e^{i\frac{\pi}{2}x} = e^{-\frac{\pi}{2}x} (\cos \frac{\pi}{2}x + i\sin \frac{\pi}{2}x)$ $\Rightarrow Re(e^{\pi x}) = e^{-\frac{1}{2}x} \cos \frac{\pi}{2}x$ $Im(e^{\pi x}) = e^{-\frac{1}{2}x} \sin \frac{\pi}{2}x$ General soln: $y(x) = e^{-\frac{\pi}{2}} \left(a \cos \frac{\pi}{2}x + \beta \sin \frac{\pi}{2}x \right)$ Since $y(0) = 0$, $0 = \alpha$ $\Rightarrow y(x) = \beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x$ $y(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x + \frac{13}{2}\beta e^{-\frac{\pi}{2}} \cos \frac{\pi}{2}\pi$ Since $y(0) = 1$, $\beta = \frac{2\pi}{3}$ $\therefore \Rightarrow PS. y(x) = \frac{2\pi}{3} e^{-\frac{\pi}{2}} \sin(\frac{\pi}{2}x)$ $f^{-2} 2Inhomogeneous Equations$ $y'' + ay' + by = f(x)$ $y'' + ay'_{+} + by_{+} = f(x)$ $y'' + ay'_{+} + by_{+} = f(x)$ Take the difference: Note: $(y_{+} \pm y_{-})^{*} = y_{-}^{*} \pm y_{-}^{*}$			
$e^{\lambda x} = e^{-\frac{1}{2}x} e^{\frac{1}{2}x} = e^{-\frac{1}{2}x} (\cos \frac{\pi}{2}x + i\sin \frac{\pi}{2}x)$ $\Rightarrow Re (e^{\lambda x}) = e^{-\frac{1}{2}x} \cos \frac{\pi}{2}x$ $Im (e^{\lambda x}) = e^{-\frac{1}{2}x} \sin \frac{\pi}{2}x$ General soln: $y(x) = e^{-\frac{\pi}{2}} \left(\alpha \cos \frac{\pi}{2}x + \beta \sin \frac{\pi}{2}x \right)$ Since $y(0) = 0$, $0 = \alpha$ $\Rightarrow y(x) = \beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x$ $y'(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x + \frac{\pi}{2}\beta e^{-\frac{\pi}{2}} \cos \frac{\pi}{2}x$ $y'(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} \sin \frac{\pi}{2}x + \frac{\pi}{2}\beta e^{-\frac{\pi}{2}} \cos \frac{\pi}{2}x$ Since $y'(0) = 1$, $\beta = \frac{2\pi}{2}$ $\therefore \Rightarrow PS. y(x) = \frac{2\pi}{3} e^{-\frac{\pi}{2}} \sin(\frac{\pi}{2}x)$ $e^{-\frac{1}{2}x} = \frac{1}{3} e^{-\frac{\pi}{2}} \sin(\frac{\pi}{2}x)$ $y'' + ay' + by = f(x)$ $y'' + ay' + by = f(x)$ $y'' + ay' + by_{1} = f(x)$ $y'' + ay'_{2} + by_{2} = f(x)$		$\lambda = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$	an a
$\begin{array}{c} \overrightarrow{P} \ Re \ (e^{\Delta x}) = e^{-\frac{1}{2}x} \cos \frac{15}{2}x \\ Im \ (e^{\Delta x}) = e^{-\frac{1}{2}x} \sin \frac{15}{2}x \\ General \ soln: \\ y(x) = e^{-\frac{x}{2}} \left(\alpha \cos \frac{15}{2}x + \beta \sin \frac{15}{2}x \right) \\ Since \ y(0) = 0 \ , \\ 0 = \alpha \\ \Rightarrow y(x) = \beta e^{-\frac{x}{2}} \sin \frac{15}{2}x \\ y'(x) = -\frac{1}{2}\beta e^{-\frac{x}{2}} \sin \frac{15}{2}x + \frac{15}{2}\beta e^{-\frac{x}{2}} \cos \frac{15}{2}x \\ Since \ y'(0) = 1 \ , \\ \beta = \frac{215}{3} \\ e^{-\frac{x}{3}} \sin \frac{15}{2}x + \frac{15}{2}\beta e^{-\frac{x}{2}} \cos \frac{15}{2}x \\ General \ y'(x) = -\frac{1}{2}\beta e^{-\frac{x}{2}} \sin \frac{15}{2}x + \frac{15}{2}\beta e^{-\frac{x}{2}} \cos \frac{15}{2}x \\ Since \ y'(0) = 1 \ , \\ \beta = \frac{215}{3} \\ e^{-\frac{x}{3}} \sin \frac{15}{2}x + \frac{15}{2}\beta e^{-\frac{x}{2}} \sin \frac{15}{2}x \\ e^{-\frac{x}{3}} \cos \frac{15}{2}x \\ General \ y'(x) = \frac{2.6}{3} e^{-\frac{x}{2}} \sin \frac{15}{2}x \\ e^{-\frac{x}{3}} \cos \frac{15}{2}x \\ General \ y'(x) = \frac{2.6}{3} e^{-\frac{x}{2}} \sin \frac{15}{2}x \\ e^{-\frac{x}{3}} \cos \frac{15}{2}x \\ General \ y'(x) = \frac{2.6}{3} e^{-\frac{x}{3}} \sin \frac{15}{2}x \\ e^{-\frac{x}{3}} \cos \frac{15}{2}x \\ general \ y'(x) = \frac{1}{3} e^{-\frac{x}{3}} \sin \frac{15}{2}x \\ e^{-\frac{x}{3}} \cos \frac{15}{2}x \\ f^{-\frac{x}{3}} e^{-\frac{x}{3}} \sin \frac{15}{2}x \\ general \ y'' + by = f(x) \\ g'' + ay' + by = f(x) \\ g'' + ay'_{1} + by_{1} = f(x) \\ g''' + ay'_{2} + by_{3} = f(x) \\ Take \ the \ difference : \\ Note. \ (y_{1} \pm y_{2})'' = y_{1}^{+} \pm y_{2}^{+} \end{array}$		Then,	(1999)(1997)(1997)(1997)(1997)(1997)(1997)
$I_{m} (e^{xx}) = e^{-\frac{1}{2}x} sin \frac{13}{2}x$ $General soln:$ $y(x) = e^{-\frac{\pi}{2}} \left(a \cos \frac{13}{2}x + \beta sin \frac{13}{2}x \right)$ $Since y(0) = 0,$ $0 = \alpha$ $\Rightarrow y(x) = \beta e^{-\frac{\pi}{2}} sin \frac{13}{2}x$ $y'(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{\pi}{2}} cos \frac{13}{2}x$ $y'(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{\pi}{2}} cos \frac{13}{2}x$ $Since y(0) = 1,$ $\beta = \frac{213}{3}$ $\Rightarrow PS. y(x) = \frac{2\sqrt{3}}{3} e^{-\frac{\pi}{2}} sin (\frac{13}{2}x)$ $6 \cdot 2 \cdot 2 Inhomogeneous Equations$ $y'' + ay' + by = f(x) (*)$ $\int Suppose that y_1 \& y_2 are solns.$ $y'' + ay'_1 + by_1 = f(x) (1)$ $y''_2 + ay'_2 + by_3 = f(x)$ $Take the \ difference : \qquad Note. \ (y_1 \pm y_2)'' = y_1'' \pm y_2''$			
$\begin{array}{c} \text{General soln:} \\ y(x) = e^{-\frac{x}{2}} \left(a \cos \frac{\pi}{2} x + \beta \sin \frac{\pi}{2} x \right) \\ \text{Since } y(0) = 0 , \\ 0 = \alpha \\ \Rightarrow y(x) = \beta e^{-\frac{x}{2}} \sin \frac{\pi}{2} x \\ y'(x) = -\frac{1}{2} \beta e^{-\frac{x}{2}} \sin \frac{\pi}{2} x + \frac{\pi}{2} \beta e^{-\frac{x}{2}} \cos \frac{\pi}{2} x \\ y'(x) = -\frac{1}{2} \beta e^{-\frac{x}{2}} \sin \frac{\pi}{2} x + \frac{\pi}{2} \beta e^{-\frac{x}{2}} \cos \frac{\pi}{2} x \\ \text{Since } y'(0) = 1 \\ \beta = \frac{2\pi}{3} \\ \Rightarrow \rho \text{S. } y(x) = \frac{2\pi}{3} e^{-\frac{x}{2}} \sin (\frac{\pi}{2} x) \\ 6 \cdot 2 \cdot 2 \\ \text{Inhomogeneous Equations} \\ y'' + ay' + by = f(x) \\ \text{V Suppose that } y_1 \& y_2 \text{ are solns.} \\ y'' + ay'_1 + by_1 = f(x) \\ y'' = y''_1 + y'_2 + by_2 = f(x) \\ \text{Take the difference : Note, } (y_1 \pm y_2)'' = y''_1 \pm y'_2 \\ \end{array}$		$\Rightarrow \operatorname{Re}\left(e^{\lambda x}\right) = e^{-\frac{1}{2}x} \cos \frac{15}{2}x$	
$y(x) = e^{-\frac{x}{2}} \left(\alpha \cos \frac{13}{2} x + \beta \sin \frac{15}{2} x \right)$ Since $y(0) = 0$, $0 = \alpha$ $\Rightarrow y(x) = \beta e^{-\frac{x}{2}} \sin \frac{13}{2} x$ $y'(x) = -\frac{1}{2}\beta e^{-\frac{x}{2}} \sin \frac{13}{2} x + \frac{13}{2}\beta e^{-\frac{x}{2}} \cos \frac{13}{2} x$ Since $y'(0) = 1$, $\beta = \frac{213}{3}$ $\Rightarrow PS$. $y(x) = \frac{249}{5} e^{-\frac{x}{2}} \sin(\frac{15}{2}x)$ $6 \cdot 2 \cdot 2$ Inhomogeneous Equations y'' + ay' + by = f(x) (*) \checkmark Suppose that $y_1 \ \& \ y_2$ are solns. $y''_1 + ay'_1 + by_1 = f(x)$ (1) $y''_2 + ay'_2 + by_3 = f(x)$ Take the difference : Note. $(y_1 \pm y_2)'' = y''_1 \pm y''_2$		$Im(e^{\lambda x}) = e^{-\frac{1}{2}\lambda} \sin \frac{\pi}{2} x$	
Since $y(0) = 0$, $0 = \alpha$ $\Rightarrow y(x) = \beta e^{-\frac{\pi}{2}} \sin \frac{15}{2} x$ $y'(x) = -\frac{1}{2}\beta e^{-\frac{\pi}{2}} \sin \frac{15}{2}x + \frac{15}{2}\beta e^{-\frac{\pi}{2}} \cos \frac{15}{2}x$ Since $y'(0) = 1$, $\beta = \frac{215}{3}$ $\Rightarrow PS.$ $y(x) = \frac{2\sqrt{5}}{5}e^{-\frac{\pi}{2}} \sin(\frac{15}{2}x)$ $6\cdot 2\cdot 2$ Inhomogeneous Equations y'' + ay' + by = f(x) (*) \checkmark Suppose that $y_1 \& y_2$ are solns. $y'' + ay'_1 + by_1 = f(x)$ (1) $y''_2 + ay'_2 + by_3 = f(x)$ Take the difference : Note. $(y_1 \pm y_2)'' = y_1'' \pm y_2''$] m.j.e.] ann an ¹ an an t-an a a a a a a a a a a
$0 = \alpha$ $\Rightarrow y(x) = \beta e^{-\frac{x}{2}} \sin \frac{15}{2} x$ $y'(x) = -\frac{1}{2}\beta e^{-\frac{x}{2}} \sin \frac{13}{2} x + \frac{13}{2}\beta e^{-\frac{x}{2}} \cos \frac{13}{2} x$ Since $y'(0) = 1$, $\beta = \frac{215}{3}$ $\cdot \Rightarrow PS. y(x) = \frac{2.5}{3} e^{-\frac{x}{2}} \sin(\frac{15}{2}x)$ 6.22 <u>Inhomogeneous Equations</u> $y'' + ay' + by = f(x) \qquad (*)$ $\int \text{Suppose that } y_1 \& y_2 \text{ are solns.}$ $y''_{1} + ay'_{1} + by_{1} = f(x) \qquad (1)$ $y_{2}'' + ay'_{2} + by_{2} = f(x)$ Take the difference : Note. $(y_{1} \pm y_{2})'' = y_{1}'' \pm y_{2}''$	11	$y(x) = e^{-\frac{\pi}{2}} \left(\alpha \cos \frac{13}{2} x + \beta \sin \frac{13}{2} x \right)$	2007 d
$\Rightarrow y(x) = \beta e^{-\frac{x}{2}} \sin \frac{15}{2} x$ $y'(x) = -\frac{1}{2}\beta e^{-\frac{x}{2}} \sin \frac{15}{2}x + \frac{13}{2}\beta e^{-\frac{x}{2}} \cos \frac{15}{2}x$ Since $y'(0) = 1$, $\beta = \frac{215}{3}$ $\Rightarrow P.S. y(x) = \frac{2.5}{3} e^{-\frac{x}{2}} \sin(\frac{15}{2}x)$ 6.22 Inhomogeneous Equations $y'' + ay' + by = f(x) \qquad (*)$ $\checkmark Suppose that y_1 \& y_2 are solns.$ $y''_1 + ay'_1 + by_1 = f(x) \qquad (1)$ $y''_2 + ay'_2 + by_3 = f(x)$ Take the difference : Note. $(y_1 \pm y_2)'' = y_1'' \pm y_2''$		Since $y(0) = 0$,	ananan Saada II mada In Inna Sad
$y'(x) = -\frac{1}{2}\beta e^{-\frac{x}{2}} \sin \frac{13}{2}x + \frac{13}{2}\beta e^{-\frac{x}{2}} \cos \frac{13}{2}x$ Since $y'(0) = 1$ $\beta = \frac{213}{3}$ $\Rightarrow PS. y(x) = \frac{2.13}{3} e^{-\frac{x}{2}} \sin(\frac{13}{2}x)$ 6.2.2 Inhomogeneous Equations $y'' + ay' + by = f(x) \qquad (*)$ $\checkmark \text{ Suppose that } y_1 \& y_2 \text{ are solns.}$ $y''_1 + ay'_1 + by_1 = f(x) \qquad (1)$ $y''_2 + ay'_2 + by_3 = f(x)$ Take the difference : Note: $(y_1 \pm y_2)'' = y_1'' \pm y_2''$			
Since $y'(0) = 1$ $\beta = \frac{2I_3}{3}$ $\Rightarrow P.S. y(x) = \frac{2J_3}{3}e^{-\frac{x}{2}}\sin(\frac{J_3}{2}x)$ 6.2.2 Inhomogeneous Equations y'' + ay' + by = f(x) (*) $\checkmark Suppose that y_1 \& y_2 are solns.$ $y_1'' + ay_1' + by_1 = f(x)$ (1) $y_2'' + ay_2' + by_3 = f(x)$ Take the difference : Note: $(y_1 \pm y_2)'' = y_1'' \pm y_2''$		$\Rightarrow y(x) = \beta e^{-2} \sin \frac{14}{2} x$	ana dana dan karangan ang sana sana sana sana sana sana
$\beta = \frac{2I_3}{3}$ $\Rightarrow P.S. y(x) = \frac{2.43}{3} e^{-\frac{x}{2}} \sin(\frac{43}{2}x)$ $6.2.2 Inhomogeneous Equations$ $y'' + ay' + by = f(x) \qquad (*)$ $\checkmark Suppose that y_1 \& y_2 are solns.$ $y_1'' + ay_1' + by_1 = f(x) \qquad (1)$ $y_2'' + ay_2' + by_2 = f(x)$ $Take the difference : \qquad Note: (y_1 \pm y_2)'' = y_1'' \pm y_2''$	n tree want tan dan had ta fa anna e ta ta ta fa	$y'(x) = -\frac{1}{2}\beta e^{-\frac{1}{2}}\sin \frac{1}{2}x + \frac{1}{2}\beta e^{-\frac{1}{2}}\cos \frac{1}{2}x$	
$ \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \rightarrow P.S. y(x) = \frac{2 \sqrt{3}}{3} e^{-\frac{X}{2}} \sin(\frac{\sqrt{3}}{2}x) \end{array} \end{array} \end{array} \\ \hline 6\cdot2\cdot2 & \underline{Inhomogeneous} Equations \\ \hline y'' + ay' + by = f(x) & (*) \end{array} \\ \hline \\ \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} y'' + ay' + by = f(x) \\ \hline \\ y_1'' + ay_1' + by_1 = f(x) \\ \hline \\ y_2'' + ay_2' + by_2 = f(x) \end{array} \end{array} \end{array} \end{array} $			Selaman en
6.2.2 Inhomogeneous Equations y'' + ay' + by = f(x) (*) \checkmark Suppose that $y_1 \& y_2$ are solns. $y_1'' + ay_1' + by_1 = f(x)$ (1) $y_2'' + ay_2' + by_2 = f(x)$ Take the difference : Note: $(y_1 \pm y_2)'' = y_1'' \pm y_2''$	(Hanson an ann an ann an an an an an an an an		~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
$y'' + ay' + by = f(x) \qquad (*)$ $\int \text{Suppose that } y_1 \& y_2 \text{ are solns.}$ $y_1'' + ay_1' + by_1 = f(x) \qquad (1)$ $y_2'' + ay_2' + by_2 = f(x)$ Take the difference: Note: $(y_1 \pm y_2)'' = y_1'' \pm y_2''$		$\cdot \Rightarrow P.S. y(x) = \frac{2A^3}{3} e^{-2} \sin(\frac{A^3}{2}x)$	
$\int \text{Suppose that } y_1 \& y_2 \text{ are solns.}$ $y_1'' + ay_1' + by_1 = f(x) \qquad (1)$ $y_2'' + ay_2' + by_2 = f(x)$ Take the difference: Note: $(y_1 \pm y_2)'' = y_1'' \pm y_2''$	6.2.2	Inhomogeneous Equations	
$\begin{array}{ll} y_{1}'' + ay_{1}' + by_{1} = f(x) & (1) \\ y_{2}'' + ay_{2}' + by_{2} = f(x) \\ \end{array}$ Take the difference: Note: $(y_{1} \pm y_{2})'' = y_{1}'' \pm y_{2}''$		y'' + ay' + by = f(x) (*)	an frantska annanska angela
$\frac{y_2'' + ay_2' + by_2}{Take the difference} = f(x)$ Note: $(y_1 \pm y_2)'' = y_1'' \pm y_2''$	19 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	✓ Suppose that y₁ & y₂ are solns.	
Take the difference: Note: $(y_1 \pm y_2)'' = y_1'' \pm y_2''$		$y_{i}'' + ay_{i}' + by_{i} = f(x)$ (1)	
	111 11 12 12 12 12 12 12 12 12 12 12 12		a sana a anna an falan dan falan
$(y_1'' - y_2'') + a(x_1)(y_1' - y_2') + b(x_1)(y_1 - y_2) = 0$	n, comme fra 11154519 (februar e e e e e e e e e e e e e e e e e e e	Take the difference: Note: $(y_1 \pm y_2)'' = y_1'' \pm y_2''$	
	n an		
$(y_1 - y_2)'' + \alpha(x)(y_1 - y_2)' + b(x)(y_1 - y_2) = 0$	ու ուսուս ու դերերնութուր ավելու ու դեսուսություն	$(y_1 - y_2)'' + \alpha(x)(y_1 - y_2)' + b(x)(y_1 - y_2) = 0$	nam an ar an an an an Eirin Lannana

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•••	\checkmark Any soln. of (*) is the sum of any particular soln. of (*) plus a soln. of the homogeneous eqn.
	Mon. 28/11/16 MATH1401: Mathematical Methods 1
	Prof. Halburd
Wijiquu is isaa aa	\sqrt{Any} soln. of (*) is of the form
eet 200 eestermaan da aan da armada amamada amamada ahaa ah	Yer + Yhom
eenstelististatististististististaan johan maanaa maanaa aa	particular integral complementary function (CF)
	(PI) soln of the homogeneous egn'
	'a particular soln. of (1)'
haladala kalendara ka	• EXAMPLE.
Margan and a state of the	Find the general soln of $y''-3y'+2y=x+\sin x$
enernandari for a for a formal and the art of the second of	Soln: <u>Step 1</u> : Solve the homogeneous egn.
198919 Mart 4 and 5 a	$y_{h}'' - 3y_{h}' + 2y_{h} = 0$
adalladigu, adaabadi aha mada jajamijan, yaya ijajana mama ya	characteristic egn:
	$\pi^2 - 3\pi + 2 = 0$
	$(\lambda - 1)(\lambda - 2) = 0$
1999 1999 1999 1999 1999 1999 1999 199	λ= 1, 2
NAJOS (MARINA AND AND AND AND AND AND AND AND AND A	$\therefore CF. y(h) = \alpha e^{x} + \beta e^{2x}$
Ø	Step 2: Find PI.
68/1886.616(186111)(12)1111(16)/02/0222222,00000111222)221111	$y(x) = G + Gx + C_2 \sin x + C_3 \cos x$, then
······	$y'(x) = C_1 + C_2 \cos \chi - C_3 \sin \chi$
1,011,0111,111111111111111111111111111	$y''(x) = -C_2 \sin x - C_3 \cos x$
annan an a	substitute :
	$(-C_{2}Sin\chi - C_{3}COS\chi) - 3(C_{1} + C_{2}COS\chi - C_{3}Sin\chi) + 2(C_{0} + C_{1}\chi + C_{3}Sin\chi + C_{3}COS\chi) = \chi + Sin\chi$
	equate coefficients.
	x° : $-3C_{1}+2C_{0}=0$ $C_{1}=\frac{1}{2}$
	$\chi': \qquad 2C_{r}=1 \qquad \Rightarrow C_{s}=\frac{3}{2}$
	$\sin \chi$: $-C_2 + 3C_3 + 2C_2 = 1$ $C_8 = \frac{3}{4}$
	$cos \chi_{1} = -C_{3} - 3C_{4} + 2C_{3} = 0 \qquad C_{3} = \frac{3}{10}$
n na	

	So the general soln is	
	$\frac{y(x) = y_p(x) + y_h(x)}{3 + 1 + x + 1 + (r; r; r + 2cr; r) + (r; r)^2 + 2cr; r)}$	
	$=\frac{3}{4}+\frac{1}{2}x+\frac{1}{10}(\sin x+3\cos x)+\alpha e^{x}+\beta e^{2x}$	
• Trial Eunction (
f(x)	Trial Function	
	ciebx if b is not a soln. of AE	
a e ^{bx}	cixebx if b is a non-repeated root,	
	i.e. e ^{bx} solves homogeneous egn	
	c.x2ebx if b is a repeated root	
polynomial in ∝ of degree n	general polynomial of degree n	
a cos (bx)	acos(bx) + Bsin(bx) provided this does not solve the	Ŵ
and/or asin(bx)	homogeneous eqn.	
	$x[acos(bx)+\beta sin(bx)]$ if $acos(bx)+\beta sin(bx)$ solves	
	the homogeneous egn	
e ^{ax} .sin(bx)	Write a Re/Im part of e ^{(a+ib)x} .	
COS(b×)	Soln solve with this & take Re or Im part	
C08 ² X _	» Rewrite as sums of trig fns	
· · · · · · · · · · · · · · · · · · ·	Rewrite in terms of exponentials	
	•	
		·····
Fri. 02/12/16		~
M	ATH1401 : Mathematical Methods 1	
	Prof. Halburd	
6.2.3 Euler's Equation		
2	$x^2y'' + Qxy' + by = f(x)$ (#)	
√ For homogeneou	s case, i.e. $f(x) = 0$	
U	oins of the form $y(x) = x^n$.	
Then,		
y'= א:	χ^{n-1} , $\chi'' = \lambda(\lambda-1) \chi^{\lambda-2}$	
Then,		
	$y_h'' + \alpha x y_h' + b y_h = 0$	Ű

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Then it is transformed into the form $\frac{dy}{dx} = \varphi(\frac{y}{x})$.

	l,	
•		Wed. 30/11/16
0		MATHI401 Help Class
		Prof. Wilson
*******		linear 1 st order 2 nd order
		$g(x) = f_{0}(x)y + f_{1}(x)\frac{dy}{dx} + f_{2}(x)\frac{d^{2}y}{dx^{2}}$
an a sa s	1,1,1,2,000 And \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$	homogeneous: g(x) = 0
	an a	for a homogenous eqn, if $y=h(x)$ is a soln, then y=Ah(x) is also a soln
	1.	1 st order linear integrating factor
		FXAMPLE (D).
		$\frac{dy}{dx} + \frac{3}{x}y = \sin x \qquad \text{when} x = \pi, y = 1$
		Integrating factor
		$I(x) = e^{\int p(x) dx}$
		[Here $P(x) = \frac{3}{2}$]
	*******************************	$J(x) = \exp\left(\int \frac{3}{2\pi} dx\right) = \exp\left(3\ln x\right) = x^3$
		Then
	1919 - 1979 - 1979 - 1979 - 1970 - 1970 - 1970 - 1970 - 1970 - 1970 - 1970 - 1970 - 1970 - 1970 - 1970 - 1970 -	$x^{3}\frac{dy}{dx} + 3x^{2}y = x^{3}\sin x$
		$\frac{d}{dx}(x^3y) = x^3 \sin x$
		$\Rightarrow x^3y = \int x^3 \sin x dx$
	u = x3	
	n.= 3x;	$u = 3x^{1} - x^{3}\cos x + 3x^{3}\sin x - \int 6x\sin x dx \qquad u' = 6x v = \sin x$
	U= 6X	BICOBAGA
	u'=6	V = -cos x = -x ³ cos x + 3x ³ sin x + 6x cos x - 6 sin x + C
		Therefore,
-()A-1-($\chi^{3}Y = -\chi^{3}COS\chi + 3\chi^{2}Sin\chi + 6\chiCOS\chi - 6Sin\chi + C$
		$y = -\cos x + \frac{3}{x}\sin x + \frac{6}{x^2}\cos x - \frac{6}{x^3}\sin x + \frac{c}{\pi^3}$
****		<u> </u>
	n d 10 million de la desta	유 +
	·····	⇒ C = 61ī
		$\therefore y = -\cos x + \frac{3}{2}\sin x + \frac{6}{2}\cos x - \frac{6}{2}\sin x + \frac{6\pi}{3}\sin x + $
		The very last thing we do is use the initial condition
	2.	2 nd order (constant coefficients)

EXAMPLE.			
	$\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = xe$	× y(0)=2	
		y'(0)=1	
~CF&PI" me	thod .		
0 ÇF	is the soln of the ho	mogeneous egn.	
compiemer	stary function	-	
If y	$=e^{\lambda x}$, then $\frac{dy}{dx} = \lambda e$	እኣ	
	$\frac{d^2 y}{dx^2} = \lambda^2 \epsilon$	2yx	
sub :	$\lambda^2 e^{\lambda x} - 3 \lambda e^{\lambda x} + 2e^{\lambda}$	* = 0	
	$e^{\lambda x} (\lambda^{2} - 3\lambda + 2)$	= 0	
	e ^{nx} (n-2)(n-1)	=0	
	<u>×</u>	=1 or 2	
So 2	solns, $e^{x} \& e^{2x}$, t	o the hom egn.	
⇒ gener	ral soln of the homoge	neous egn:	
	$y(x) = A e^{x} + B e^{2x}$		
÷	nd any one soln to t		
particular Tr integral	rial & error : try a y	that "looks like" the RHS	
Our RHS	is xe*. So try y	= axe ^x	,,,
	$\Rightarrow \frac{dy}{dx}$	$= \alpha e^{x} + \alpha x e^{\infty}$	
	$\Rightarrow \frac{d^2 Y}{d\chi^2}$	$= \alpha e^{x} + \alpha e^{x} + \alpha x e^{x} = 2\alpha e^{x} + \alpha x e^{x}$	
substitut	te:		
2ae'	*+αxe [*] -3(αe [*] +αxe [*])	$+ 2\alpha x e^{x} = x e^{x}$	
	20Ce*-	$3ae^{x} = xe^{x}$	
This DOE	S NOT work !!!		
 Systematic 	: method:		
CF: C*,	e ^{2x} pick <u>one</u> and for	actor it out, i.e.	
	$y = e^{x}f(x)$, st	art again.	
	$\frac{dy}{dx} = e^{x}f(x) + e^{x}\frac{dy}{dx}$	du	
-	$\frac{d^{2}y}{dx^{2}} = e^{x}f(x) + 2e^{x}\frac{dy}{dx} + e^{x}$	$\left[e^{x}f(x) + e^{x}\frac{dy}{dx}\right] + 2e^{x}f(x) = xe^{x}$	Ő
	t(x) +2E* += + e* + + 1 - 37	$1e^{T(x)} + e^{X} + 2e^{T(x)} = Xe^{T(x)}$	3

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$f''-f'=x$ Now set $g=\frac{dx}{dx}$ $\frac{dy}{dx}-g=x$ This is a t^{tt} order. $I(x)=e^{-x} is ext{ the integrating factor.}$ Then, $e^{-x}\frac{dx}{dx}-e^{-x}g=xe^{-x}$ $\frac{dx}{dx}(e^{-x}g)=xe^{-x}$ $e^{-x}g=\int xe^{-x}dx u=x y'=e^{-x}$ $e^{-x}g=\int xe^{-x}dx u'=t v=-e^{-x}$ $e^{-x}g=-xe^{-x}+c$ $g=-xe^{-x}-e^{-x}+c$ $g=-x-t+ce^{x}$ $\Rightarrow \frac{dt}{dx}=-x-t+ce^{x}$ $\Rightarrow \frac{dt}{dx}=-x-t+ce^{x}$ $\Rightarrow f(x)=-\frac{1}{2}x^{2}-x+ce^{x}+De^{x}$ $f(x)=-\frac{1}{2}x^{2}-x+ce^{x}+De^{x}$ $f(x)=-\frac{1}{2}x^{2}-x+ce^{x}+De^{x}$ $f(x)=-\frac{1}{2}x^{2}-xe^{-x}+ce^{2x}+De^{x}$ $f(x)=-\frac{1}{2}x^{2}-xe^{-x}+ce^{2x}+De^{x}$ $f(x)=-\frac{1}{2}x^{2}-xe^{-x}+ce^{2x}+De^{x}$ $f(x)=-\frac{1}{2}x^{2}-xe^{-x}+ce^{2x}+De^{x}$ $f(x)=-\frac{1}{2}x^{2}-xe^{-x}+ce^{2x}+De^{x}$ $f(x)=-\frac{1}{2}x^{2}-xe^{-x}+ce^{2x}+De^{x}$ $f(x)=-\frac{1}{2}x^{2}-xe^{-x}+ce^{-x}+de^{-x}$	
Now set $g = \frac{df}{dx}$ $\frac{dg}{dx} - g = x$ This is a 4^{at} order. $I(x) = e^{-x}$ is the integrating factor. Then, $e^{-x} \frac{dg}{dx} - e^{-x}g = xe^{-x}$ $\frac{dg}{dx} (e^{-x}g) = xe^{-x}$ $e^{-x}g = \int xe^{-x} dx$ $u = x$ $v' = e^{-x}$ $e^{-x}g = -xe^{-x} + \int e^{-x} dx$ $u' = 1$ $v' = -e^{-x}$ $e^{-x}g = -xe^{-x} + \int e^{-x} dx$ $u' = 1$ $v' = -e^{-x}$ $e^{-x}g = -xe^{-x} + e^{-x} + c$ $g = -x - 1 + ce^{x}$ $\Rightarrow \frac{df}{dx} = -x - 1 + ce^{x}$ $\Rightarrow f(x) = -\frac{1}{2}x^{2}e^{x} - xe^{-x} + ce^{-2x} + De^{x}$ fT $CF\bigcirc Initial conditions (LAST step!)g(0) = 2$, $g'(0) = 1\cdots \Rightarrow c = 0, D = 2Trial & error axe^{-x} failedNext try ax^{i}e^{-x} + \beta xe^{-x}if \lambda is repeated in CF, then useg_{ce} = Ae^{-hx} + Bxe^{-hx} = (A + Bx)e^{-hx}If \lambda is a pair of complex roots, i.e. \lambda = a \pm tb, then$	
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$I(x) = e^{-x} is ext{ the integrating factor.}$ $I(x) = e^{-x} is the integrating factor.$ $Then, e^{-x} dg = -e^{-x} g = xe^{-x}$ $\frac{d}{dx} (e^{-x}g) = xe^{-x}$ $e^{-x}g = \int xe^{-x} dx u = x v' = e^{-x}$ $e^{-x}g = -xe^{-x} + \int e^{-x} dx u' = i v = -e^{-x}$ $e^{-x}g = -xe^{-x} - e^{-x} + c$ $g = -x - i + ce^{x}$ $\Rightarrow df = -x - i + ce^{x}$ $\Rightarrow f(x) = -\frac{1}{2}x^{2} - xe^{-x} + ce^{-x} + D$ $\Rightarrow f(x) = -\frac{1}{2}x^{2} - xe^{-x} + ce^{-x} + D$ $\Rightarrow y = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{-x} + D$ $\Rightarrow y = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{2x} + De^{-x}$ $f(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{2x} + De^{-x}$ $f(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{2x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{-x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{-x} + De^{-x}$ $f(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{-x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{-x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{-x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{-x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{-x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{-x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{-x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{-x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{-x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + ce^{-x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + e^{-x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} - xe^{-x} + e^{-x} + De^{-x}$ $g(x) = -\frac{1}{2}x^{2}e^{-x} + e^{-x} $	Ţ#₩₩\$ ₩ \$29\$.###################################
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$e^{-x} \frac{dq}{dx} - e^{-x}g = xe^{-x}$ $\frac{d}{dx} (e^{-x}g) = xe^{-x}$ $e^{-x}g = \int xe^{-x} dx$ $u = x v' = e^{-x}$ $e^{-x}g = -xe^{-x} + \int e^{-x} dx$ $u' = 1 v' = -e^{-x}$ $e^{-x}g = -xe^{-x} - e^{-x} + c$ $g = -x - 1 + ce^{x}$ $\Rightarrow \frac{df}{dx} = -x - 1$	an a
$\frac{d}{dx} (e^{-x}g) = xe^{-x}$ $e^{-x}g = \int xe^{-x} dx$ $u=x$ $v'=e^{-x}$ $e^{-x}g = -xe^{-x} + \int e^{-x} dx$ $u'=1$ $v'=-e^{-x}$ $e^{-x}g = -xe^{-x} + e^{-x} + e^{-x}$ $g = -x - 1 + e^{x}$ $\Rightarrow \frac{df}{dx} = -x - 1 + e^{x}$ $\Rightarrow f(x) = -\frac{1}{2}x^{2} - x + e^{x} + D$ $\Rightarrow y = -\frac{1}{2}x^{2}e^{x} - xe^{x} + e^{2x} + De^{x}$ $f(x) = -\frac{1}{2}x^{2}e^{x} - xe^{x} + e^{2x} + De^{x}$ $g(x) = -\frac{1}{2}x^{2}e^{x} - xe^{x} + e^{2x} + De^{x}$ $g(x) = -\frac{1}{2}x^{2}e^{x} - xe^{x} + e^{2x} + De^{x}$ $g(x) = -\frac{1}{2}x^{2}e^{x} + e^{2x} + De^{x}$ $g(x) = 2, y'(x) = 1$ $-x \Rightarrow C = 0, D = 2$ $Trial \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	
$e^{-x}g = -xe^{-x} + \int e^{-x}dx \qquad u'=i \qquad v=-e^{-x}$ $e^{-x}g = -xe^{-x} - e^{-x} + c$ $g = -x - i + ce^{x}$ $\Rightarrow \frac{df}{dx} = -x - i + ce^{x}$ $\Rightarrow f(x) = -\frac{1}{2}x^{2} - x + ce^{x} + D$ $\Rightarrow y = -\frac{1}{2}x^{2}e^{x} - xe^{x} + ce^{2x} + De^{x}$ $\exists initial \ conditions \ (LAST \ step !)$ $y(0) = 2, y'(0) = i$ $\Rightarrow c = 0, D = 2$ $Trial \ & error axe^{x} failed$ $Next \ try \qquad ax^{2}e^{x} + \beta xe^{x}$ $If \ & \lambda \ is \ repeated \ in \ CF, \ then \ ase$ $y_{cF} = Ae^{hx} + Bxe^{hx} = (A + Bx)e^{hx}$ $If \ & \lambda \ is \ a \ pair \ of \ complex \ roots \ , \ i.e. \ h = a \pm ib \ , \ then$	And Manufold exemple for disordine environmentations of a second re-
$e^{-x}g = -xe^{-x} + \int e^{-x}dx \qquad u'=1 \qquad v=-e^{-x}$ $e^{-x}g = -xe^{-x} - e^{-x} + c$ $g = -x-1 + ce^{x}$ $\Rightarrow \frac{df}{dx} = -x-1 + ce^{x}$ $\Rightarrow f(x) = -\frac{1}{2}x^{2} - x + ce^{x} + D$ $\Rightarrow y = -\frac{1}{2}x^{2}e^{x} - xe^{x} + ce^{2x} + De^{x}$ $\exists initial \ conditions \ (LAST \ step !)$ $y(0) = 2, y'(0) = 1$ $= \Rightarrow C = 0, D = 2$ $Trial \ & error \alpha xe^{-x} failed$ $Next \ try \qquad \alpha x^{2}e^{-x} + \beta xe^{-x}$ $If \ \ \ \ is \ \ repeated \ in \ CF, \ \ then \ use$ $y_{CF} = Ae^{-h^{x}} + Bxe^{-h^{x}} = (A + Bx)e^{-h^{x}}$ $If \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	namen of summarian data and a summarian data data and a gla ana ang da da data gang da data data gang da data d
$g = -x - 1 + ce^{x}$ $\Rightarrow \frac{df}{dx} = -x - 1 + ce^{x}$ $\Rightarrow f(x) = -\frac{1}{2}x^{2} - x + ce^{x} + D$ $\Rightarrow y = -\frac{1}{2}x^{2}e^{x} - xe^{x} + ce^{2x} + De^{x}$ $PT \qquad CF$ $(a) initial conditions (LAST step!)$ $y(0) = 2, y'(0) = 1$ $\dots \Rightarrow C = 0, D = 2$ $Trial \& error : \alpha xe^{x} failed$ $Next try \qquad \alpha x^{2}e^{x} + \beta xe^{x}$ $If \lambda is \ repeated \ in \ CF, \ then \ use$ $y_{cF} = Ae^{\Lambda x} + Bxe^{\Lambda x} = (A + Bx)e^{2\pi}$ $If \Lambda is \ \alpha \ pair \ of \ complex \ roots, \ i.e. \Lambda = a \pm ib, \ then$	Nelekteennen een anter muster muster en een een en en een een een een een
$ \Rightarrow \frac{df}{dx} = -x - 1 + ce^{x} $ $ \Rightarrow f(x) = -\frac{1}{2}x^{2} - x + ce^{x} + D $ $ \Rightarrow y = -\frac{1}{2}x^{2}e^{x} - xe^{x} + ce^{2x} + De^{x} $ $ \Rightarrow y = -\frac{1}{2}x^{2}e^{x} - xe^{x} + ce^{2x} + De^{x} $ $ \Rightarrow y = -\frac{1}{2}x^{2}e^{x} - xe^{x} + ce^{2x} + De^{x} $ $ \Rightarrow y = -\frac{1}{2}x^{2}e^{x} - xe^{x} + ce^{2x} + De^{x} $ $ \Rightarrow y = -\frac{1}{2}x^{2}e^{x} + ce^{2x} + De^{x} $ $ \Rightarrow y = -\frac{1}{2}x^{2}e^{x} + \beta xe^{x} $ $ \Rightarrow z = 0, D = 2 $ $ \Rightarrow z = -2 $ $ \Rightarrow z =$	
$\Rightarrow f(x) = -\frac{1}{2}x^{2} - x + ce^{x} + D$ $\Rightarrow y = -\frac{1}{2}x^{2}e^{x} - xe^{x} + ce^{2x} + De^{x}$ PI CF (a) initial conditions (LAST step!) y(0) = 2, y'(0) = 1 $\dots \Rightarrow C = 0, D = 2$ Trial & error αxe^{x} failed Next try $\alpha x^{2}e^{x} + \beta xe^{x}$ If γ is repeated in CF, then use $y_{CF} = Ae^{\gamma x} + Bxe^{\gamma x} = (A + Bx)e^{\gamma x}$ If γ is a pair of complex roots, i.e. $\gamma = a \pm ib$, then	
$\Rightarrow y = -\frac{1}{2}x^{2}e^{x} - xe^{x} + Ce^{2x} + De^{x}}{cF}$ (a) initial conditions (LAST step!) $y(0) = 2, y'(0) = 1$ $\dots \Rightarrow C = 0, D = 2$ Trial & error $\therefore \alpha xe^{x}$ failed Next try $\alpha x^{2}e^{x} + \beta xe^{x}$ If \land is repeated in CF, then use $y_{CF} = Ae^{\Lambda x} + Bxe^{\Lambda x} = (A + Bx)e^{\Lambda x}$ If \land is a pair of complex roots, i.e. $\Lambda = a \pm ib$, then	1944 Million Milanian Jaman
Initial conditions (LAST step!) $ \begin{array}{c} y(0) = 2, y'(0) = 1 \\ \dots \Rightarrow C = 0, D = 2 \end{array} $ Trial & error: $\alpha x e^{x}$ failed Next try: $\alpha x^{2}e^{x} + \beta x e^{x}$ If γ is repeated in CF, then use $ \begin{array}{c} y_{CF} = Ae^{\Lambda x} + Bxe^{\Lambda x} = (A + Bx)e^{\Lambda x} \end{array} $ If γ is a pair of complex roots, i.e. $\gamma = a \pm ib$, then	an a
Initial conditions (LAST step!) $ \begin{array}{c} y(0) = 2, y'(0) = 1 \\ \dots \Rightarrow C = 0, D = 2 \end{array} $ Trial & error: $\alpha x e^{\chi}$ failed Next try: $\alpha x^{2}e^{\chi} + \beta x e^{\chi}$ If Λ is repeated in CF, then use $ \begin{array}{c} y_{CF} = Ae^{\Lambda \chi} + Bxe^{\Lambda \chi} = (A + B\chi)e^{\Lambda \chi} $ If Λ is a pair of complex roots, i.e. $\Lambda = a \pm ib$, then	art and a manufacture of ferming a 1 a march 1 a grant 2 a g
$\begin{array}{c} y(0) = 2 , y'(0) = 1 \\ & \cdots \Rightarrow C = 0 , D = 2 \\ \hline & & \\ \hline \hline \hline & & \\ \hline \hline & & \\ \hline \hline \hline & & \\ \hline \hline \hline & & \\ \hline \hline \hline \hline$	
$ \begin{array}{c} \cdots \Rightarrow c = 0 \ , \ D = 2 \\ \hline \end{array}$ $ \begin{array}{c} \text{Trial & error} & axe^{\times} & \text{failed} \\ \text{Next try} & ax^2e^{\times} + \beta xe^{\times} \\ \hline \end{array}$ $ \begin{array}{c} \text{Next try} & ax^2e^{\times} + \beta xe^{\times} \\ \hline \end{array}$ $ \begin{array}{c} \text{If } \lambda \text{ is repeated in } CF, \text{ then use} \\ & y_{cF} = Ae^{\Lambda^{\times}} + Bxe^{\Lambda^{\times}} = (A + Bx)e^{\Lambda^{\times}} \\ \hline \end{array}$ $ \begin{array}{c} \text{If } \lambda \text{ is a pair of complex roots}, \text{ i.e. } \lambda = a \pm ib, \text{ then} \\ \end{array}$	unted de d'antes (n de al d
• Trial & error axe^{*} failed Next try $ax^{2}e^{*} + \beta xe^{*}$ • If γ is repeated in CF, then use $y_{CF} = Ae^{n^{*}} + Bxe^{n^{*}} = (A + Bx)e^{n^{*}}$ If γ is a pair of complex roots, i.e. $\gamma = a \pm ib$, then	
Next try $\alpha x^2 e^{x} + \beta x e^{x}$ • If γ is repeated in CF, then use $y_{CF} = Ae^{\gamma x} + Bxe^{\gamma x} = (A + Bx)e^{\gamma x}$ If γ is a pair of complex roots, i.e. $\gamma = a \pm ib$, then	
• If γ is repeated in CF, then use $y_{cF} = Ae^{\gamma x} + Bxe^{\gamma x} = (A + Bx)e^{\gamma x}$ If γ is a pair of complex roots, i.e. $\gamma = a \pm ib$, then	1
$y_{cF} = Ae^{\Lambda x} + Bxe^{\Lambda x} = (A + Bx)e^{\Lambda x}$ If Λ is a pair of complex roots, i.e. $\Lambda = a \pm ib$, then	tendeparteriten of the physical data department of the second second second second second second second second
If γ is a pair of complex roots, i.e. $\gamma = a \pm ib$, then	1444/14411151115111511151115111511151115
	gennestroomtaktistatumanan om unassetserennnakasta
$y_{cF} = e^{\alpha x} (A \cos b x + B \sin b x)$	10000100000000000000000000000000000000
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Wed. 30/11/16 Applied Tutacal	hm () mm og før frankle (mm en en mm i ser er samst mandelske før de
Applied Tutoria) 1. Find the soln of $\frac{dy}{dx} = \frac{4x+y}{x+y}$ that passes through (1,1).	nakonorona na sana ana ang pang pang pang pang pang pan
General Method: $\frac{dy}{dx} = \Psi(\frac{y}{x})$ in that only depends on Let $z = \frac{y}{x}$, then	<u>y</u> Z

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.gg.eeeeegeeaaanaareeeeesterii Cüüra (willinkaand Cülliil)	y = zx	
	$\Rightarrow \frac{dy}{dx} = z + x \frac{dz}{dx} = \psi(\frac{y}{x})$	الاستراسين و در المراسين معروف المراسين المراسين و در المراسين المراسين المراسين و در المراسين المراسين و معروف
094441111111111111111111111111111111111	$\frac{d\Psi}{dx} = \varphi(xy)$	
2.		11949-4711535417778791153541727 244
and a second	Let $Z = \chi y$, then $\frac{dz}{dx} = y + \chi \frac{dy}{dx}$	effekfelen fan et ffelfan far ferminek anna er er samekk
30042-27-27-29-20-20-27-2-20-20-20-20-20-20-20-20-20-20-20-20-2	$\Rightarrow \frac{dx}{dx} = \frac{1}{x} \left(\frac{dz}{dx} - y \right)$	aan maada ah waxaa dagaa ah da ah
an(n).anm,b).ang);223/246(2)/2/2222/24444444444444444444444444444	$= \frac{1}{x} \frac{dz}{dx} - \frac{z}{x^2}$	III maarin aa maa maa maa maa maa maa maa maa maa
		n;mmqn;mm2221;m3/224/nq/mq/qd/202pmmn/202m
	$\frac{dy}{dx} = \frac{x+y+3}{x-y+5} = \frac{(x+4)+(y-1)}{(x+4)-(y-1)}$	oraneological deepoint
,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	Let $u = x + a$, $v = y + b$.	11129-04-04-05-0111-14-02-0-14-13-0-0-04-04-06-05-04-011-13-0-04-04-06-05-04-011-13-0-04-04-06-05-04-011-13-0-0
	So $\frac{dv}{dx} = \frac{dy}{dx}$	
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gaa ay ay ay ahaa ahaa ahaa ahaa ahaa ah		nammi Mandol Art Schenika dinmini (dal Sak Dar) ang ini
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Defense fan en en en ferte de la coma an	MATH1401: Mathematical Methods 1
	Prof. Halburd
amputation operated [totals/accounted=20]=355	Chapter 7 § Probability §
7.1	Sample Space & Set Operations
<u>Fili</u>	Introduction
felefeletus III muunumpudes egimus ejim seeda saadagaa gaa	• Def
ethaladdd carbard babaan b Lann Saebherean a baan by Argen	A sample space is a set representing the possible outcomes of an
Andready (1999 and 1	experimental / trial etc.
	V EXAMPLE.
and the second	Two coins are tossed. The possible sample space is
	(H&H, H&T, T&T)
nijenskooraan on musickeelajojouraalajooraan ooraannaa	Another possibility is
a ta an	(HH, HT, TH, TT)
	This means the 1 st coin = H & the 2 nd coin = T
. Construction for the programming of the product o	Def.
n matala Hanna (a Indonesia a Annonesia da Interneti y mana s	Subsets of a sample space are called events. They represent collection of outcome
	V EXAMPLE:
	Consider the set $S = \{0, 1, 2\}$
aanaa ahaa ahaa ahaa ahaa ahaa ahaa aha	The elements of S are 0,1 and 2.
17mm \$ 10mm (\$	Subsets of s are
<u> </u>	$\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}$
######################################	empty set is a subset of every set every set is a subset of itself
estandaren ezartatura gar provinsi estaturatur estaturatur estaturatur estaturatur estaturatur estatur estatur	Note: Ø, 0 and {0} are completely different objects
1111111/101111111111111111111111111111	empty set a number a set consisting of one element (0)
7.1.2	Set Operations
4000/0000 112/00,011 120000/001/0000000/ALp/14/24/24/24/24/24/24/24	S

	A TESTB
······································	is an element of '
n samta ina	O Intersection: $A \cap B = \{x : x \in A \ \& x \in B\}$
V	<pre>② Union: AUB = {x : x ∈ A or x ∈ B}</pre>

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* \$*`

		/S
	(a) Complement of A: $A^{c} = A' = \overline{A} = \{x : x \in S, x \notin A\}$	A B
ann an		uthing that is not in A
	A and B are disjoint (mutually exclusive)	o •
<u>A\B'</u>	S The relative complement	
-A-8'	$A \cdot B = \{x \in A, x \notin B\}$	A (B)
´A without	- 8	
11111111111111111111111111111111111111		
umenen andere	Given a sample space S.	**************************************
- mand we like an employment of the like sector of the like	A probability on S is a fn that assigns a	
	subset $A \subseteq S$. is a subset of '	
	✓ Note: Don't distinguish between ⊂ and	<u><u> </u></u>
	✓ <u>Properties</u> :	n an de se la constant (nom a se la constant de la Nom de se la constant de la constant
	1) P(A)≥0 ¥A⊂S	<mark>n a na mana la bana a manana</mark> na ang ang ang ang ang ang ang ang ang
	2) P(S) = 1	50
saak sakaanke ee soo maanka soo ka soo a aa	3) If $A \cap B = \emptyset$, then	
	$P(A \cup B) = P(A) + P(B)$ A or B'	
- managar ang sama sa sa sang sang sang sang sang sang sa	Let A and B be events in S (not nec	essarilu disioint), then
1999 - 1999 - Concentration (Concentration of Concentration of Concentrati	(1) $P(A') = 1 - P(A)$ (not A'	
an a	(2) $P(\emptyset) = 0$	
	(3) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ 'A and	
	Proof of (1):	
	$A \cap A' = \emptyset$	₩₽ ₽₩₽₽₽₽₩₽₽₩₽₽₩₩₩₽₩₽₽₩₽₽₩₽₽₩₽₽₩₽₩₩₽₩₽₽₩₽
	$By 2^{2}, 1 = P(S)$	um blevilleren en openen blevel mune ein min die die beerbeerbeerbeerbeerbeerbeerbeerbeerbe
	$= P(A \cup A')$	
	$= P(A) + P(A') \qquad by$.
1999 - 1999 () 1999 () 1999 (1999 - 1999 (1999 (1999 (1999 (1999 (1999 (1999 (1999 (1999 (1999 (1999 (1999 (199	$\Rightarrow P(A') = I - P(A)$	
	$\frac{P \operatorname{coof} of (\mathcal{Q})_{:}}{P(\mathcal{Q}) = 1 - P(\mathcal{Q}^{c})}$	
.	$\frac{P(\emptyset) = 1 - P(\emptyset)}{= 1 - P(S)}$	
. <u>19</u> 10-1917 - 1910-1910 - 1910 - 1910 - 1910 1910 - 1910 - 1910 - 1910 - 1910 - 1910 1910 - 1910 - 1910 - 1910 - 1910 - 1910 - 1910 - 1910		

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$$= 1-1$$

$$= 0$$

$$= 0$$

$$A^{3}B \text{ and } B \text{ are disjoint.}$$

$$A^{3}B \text{ and } B \text{ are disjoint.}$$

$$A^{3}B \text{ and } B \text{ are disjoint.}$$

$$P(A \cup B) = P(A \setminus B) \cup B)$$

$$= P(A \setminus B) + P(B) \quad by = 3)$$

$$A^{3}B \text{ and } A^{3}B \text{ are disjoint.}$$

$$P(A) = P((A \cup B) \cup (A \cap B))$$

$$= P(A \setminus B) + P(A \cap B)$$

$$= P(A \cap B) + P(A \cap B) + P(A \cap B)$$

$$= P(A \cap B) + P(A \cap B) + P(A \cap B)$$

$$= P(A \cap B) + P(A \cap B) + P(A \cap B)$$

$$= P(A \cap B) + P(A \cap B) + P(A \cap B) + P(A \cap B) + P(A \cap B)$$

$$= P(A \cap B) + P(A \cap B$$

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A fair coin is tossed twice. We use the sample space S= {HH, HT, TH, TT} (equally - likely) 1st: use words to describe events; 2nd: use sets Soln: $P(Two heads appear) = P([HH]) = \frac{1[HH]}{|S|} = \frac{1}{4}$ P(One head and one tail appear) = P({HT,TH}) = $\frac{|{HT,TH}|}{|S|} = \frac{2}{4} = \frac{1}{2}$ P(At least one tail) = P({HT, TH, TT}) = <u>| {HT, TH, TT}|</u> ISI Or = I - P(no tail) $= 1 - P({HH})$ = 1-4 $=\frac{3}{4}$ V EXAMPLE @: A fair die is rolled twice and the numbers are recorded Let A =first roll is a 5. B = the largest number shown is 4. C = the sum of numbers is prime. Calculate P(A), P(B) and P(C). Soln: Sample space $(1,1), (1,2), \dots, (1,6)$ $S=\left\{\begin{array}{c} (2,1),(2,2),\dots,(2,6)\\ \vdots\end{array}\right\}$ |S| = 36 $(6,1), (6,2), \dots, (6,6)$ Then, $P(A) = P(\{(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}) = \frac{|A|}{|S|} = \frac{6}{36} = \frac{1}{6}$ $P(B) = P\left(\left\{(1, 4), (2, 4), (3, 4), (4, 4), (4, 3), (4, 2), (4, 1)\right\}\right) = \frac{|B|}{|S|} = \frac{7}{36}$ $P(C) = P(\{(1,1), (1,2), (1,4), (1,6), (2,1), (2,3), (2,5$ (3,2), (3,4), (4,3), (4,1), (5,2), (5,6), (6,1), (6,5)ISI $=\frac{5}{12}$

7.1.4	Discrete Sample Space
	Def O:
	A sample space is discrete if it is finite or it has a countable infinite
New Sector Concerned and Sector Se	number of elements.
•	Def 0:
Among a subject and subject to a	✓ A set is countably infinite if you can list the elements
	1. X1
	$2. \chi_2 \longrightarrow \sum_{j=1}^{\infty} P(\chi_j) = 1$
Amattel: ;	i.e. there is a one-to-one correspondence between the elements of S and N
unterten ¹ ennegisetetetetetetetetetetetetetetetetetetet	✓ If S is countable, we can write sum over everything in set s $\sum_{x \in S} P(x) = 1$
	in set s $(x,y) = 1$

	Mon. 05/12/16
Andre Sector and the	MATH1401: Mathematical Methods 1
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142/00/01/1750-01/01/01/01/01/01/01/01/01/01/01/01/01/0	V EXAMPLE .
	- Consider a game in which a fair coin is tossed until the first head
■2000000000000000000000000000000000000	appears when the game ends.
below water and the second	- S= {H, TH, TTH,}
Annual physical and a second	This is countably infinite because we can list the elements.
	- Let $P_n = probability$ that the game ends on the nth row, then
#1464484414400000000000000000000000000000	$P_{n} = (\pm, \pm, \pm, \pm, \pm) \times \pm = (\pm)^{n}$
	$P_n = (\underbrace{\pm}, \underbrace{\pm}, \pm$
The sum of	nrabahilities the first (0-1) rolls
of all event	s (subsets) Check: A geometric series with first term $\frac{1}{2}$
adds up to	$\sum_{n=1}^{\infty} p_n = \sum_{n=1}^{\infty} (\frac{1}{2})^n \text{ and ratio } r = \frac{1}{2}$
·····	$=\frac{1}{1-1/2}=1$
199-189-19-19-19-19-19-19-19-19-19-19-19-19-19	For geometric series,
	$ r < l \Rightarrow \lim \rightarrow \frac{a}{1-r}$
	— Aside, a coin flipped infinitely many times.
W	S= {HTTHHTHT }

* *

9/94/001114/00119799797974/8/99910/00149	This is uncountable / not discrete.	
	Conditional Probability	200000000000000000000000000000000000000
	V EXAMPLE:	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
	An urn contains 3 black balls and 2 white balls. Two balls	surfere survey and and st
	are removed in order (without being put back). Find the	****
aaaaaaaa dhada da baaliyaa qorqoo aadaa da da da	probability that	quue a constant and a constant a c
nan se te sen a se	1.) The first ball is black.	gy particular propriate data
sentenhiltitningsgentmetermeterhet	2) The second ball is black	
	3.) The two balls have the same colour.	
ann a ann an bhliachta ann ann an bhliacht	Soln: \bullet \bullet \bullet \bullet \bullet	
,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	$\rho(1^{st} black) = \frac{3}{5}$ $1^{st} black, 2^{nd}$ white	
		,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
isterari etiti (Carta anti a Carta)	$\frac{1/2}{3/5} \rightarrow B \xrightarrow{1/2} BW \qquad \bullet \not $	room Carlonne Creent of Mad
and the second statement of the second statement of the second statement of the second statement of the second	$2/5 \rightarrow W \xrightarrow{3/4} WR$	
eggennennissenset van de staar va	1/4 = WW	

agaa jaanaa ah a	E elemente allt of 4 elements	
ooreeeonnaarinatatematorrameda	Therefore, $P(BB) = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$	2em=(-(-(-(-2e)/2e)
n politika na katala katal	$P(BW) = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$	****
***************************************	$P(W\theta) = \frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$	tenenetten er
ana ang ang ang ang ang ang ang ang ang	$P(WW) = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$	
	Then clearly,	
una (a dado o Parigra a materio (alla da	$P(2^{nd} \text{ ball is black}) = P(\{8B, WB\})$	
	= $P(\{BB\}) + P(\{WB\})$ since they're disjoint	1922-2022-2022-2022-2022-2022-2022-2022-
mean (1420/04) Manual An Aslanda (An Analaina		wayaanadad maddattatta
		nenterinen teknologischer
una a comunit d'al 1922 i no comunit a manifest d	P(same colour) = P([BB, WW])	244.00022224.444444
	$= P(\{BB\}) + P(\{WW\}) $ since disjoint	*****
	$=\frac{3}{10}+\frac{1}{10}$,
oonaan amaan daddaa daalaa daa daalaa		
annan an tao amin' a	✓ General Case :	

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P(A) P(B A* AnB* P(A) P(B A* A^* ∩ B complement of A * $P(B A*] A^* ∩ B^*$ Def Conditional Probabilities P(B A) is the conditional probability that 6 accurs, given that A has occurs P(B A) is the conditional probability that 6 accurs, given that A has occurs P(B A) P(B A) Since (BnA) P(B) P(B A) P(B A) P(B A) P(B A) P(B A) P(B A) P(B A) P(B A) P(B A) <th>٠</th> <th>P(BA) AOB</th>	٠	P(BA) AOB			
$P(A^{S}) = \frac{P(B A^{S})}{P(B A^{S})} A^{S} \cap B$ complement of $A = \frac{P(B A^{S})}{P(B A^{S})} A^{S} \cap B^{S}$ Def Conditional Probabilities $P(B A) = \frac{P(B A)}{P(A)} = \frac{P(B A)}{P(A)}$ $P(B A) = \frac{P(B A)}{P(A)} = \frac{P(B A)}{P(A)}$ $P(B A) = \frac{P(B A)}{P(A)} = \frac{P(B A)}{P(A)}$ $P(B A) = \frac{P(B A)}{P(A)} = \frac{P(B A)}{P(A)} = \frac{P(B A)}{P(A)}$ $P(B A) = \frac{P(B A)}{P(A)} = \frac{P(B A)}{P(B A)} = \frac{P(B A)}{P(A)} = \frac{P(B A)}{P(B A)} = \frac{P(B A)}{P(B A)} = \frac{P(B A)}{P(A)} = $	0				
<pre>complement of A P(B' A' ∩ B' Conditional_Probability_that_B occurs, given that A has occurs P(B A) is the conditional probability_that_B occurs, given that A has occurs P(B A) = P(B ∩ A) P(A) / Clearly, we can write B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A) ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A') B ∩ B = (B ∩ A') ∪ (B ∩ A'</pre>					
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P(B A) is the conditional probability that B accurs, given that A has occurs P(B A) $\frac{P(B\cap A)}{P(A)}$ B = (BnA) u (BnAs) A Since (BnA)(BnAs) B = (BnA) u (BnAs) BonA B = (BnA) u (BnAs) BonA P(B) = P(BnA) + p(BnAs) [P(B) = P(B A) P(A) + p(B As)P(A^{5})] Important ! Important ! 7:3 Counting Let S be a set with n elements. • (Drdered samples, repetition allowed If the number of samples is r , then there are n' different orderings of length r from n objects. V EXAMPLE. Haw many 4-digit numbers can be constructed using the digits 1.2 & 3 ? (Repeats allowed) 'Soln: $3 \times 3 \times 3 \times 3 = 3^4$ • (Ordered samples, with no repeats Choose any of the n objects 1 st Choose any of the n -1 remaining objects 2 rd i i Therefore, there are $[n(n-U(n-2) - (n-r+U)]$ called number (#) of permutations of length r		complement of A * P(BC AC) : ACOBC			
$\begin{array}{c c} P(B A) = \frac{P(B A)}{P(A)} & B \cap A^{c} & B \cap A^{c} \\ \hline P(B A) = \frac{P(B A)}{P(A)} & A & B \cap A^{c} \\ \hline B \cap A^{c} & B = (B \cap A) \cup (B \cap A^{c}) \\ \hline B = (B \cap A) \cup (B \cap A^{c}) \\ \hline B = (B \cap A) \cap (B \cap A^{c}) = \emptyset & we have \\ \hline B \cap A^{c} & B & B \\ \hline P(B) = P(B \cap A) + P(B \cap A^{c}) \\ \hline P(B) = P(B \cap A) + P(B \cap A) \\ \hline P(B) = P(B \cap A) + P(B \cap A) \\ \hline P(B) = P(B \cap A) + P(B \cap A) \\ \hline P(B \cap A) + P(B \cap A) \\ \hline P(B) = P(B \cap A) + P(B \cap A) \\ \hline P(B) = P(B \cap A) + P(B \cap A) \\ \hline P(B) = P(B \cap A) + P(B \cap A) \\ \hline P(B \cap A) + P(B \cap A) \\ \hline P(B \cap A) + P(B \cap A) \\ \hline P(B \cap A) + P(B \cap A) \\ \hline P(B \cap A) + P(B \cap A) \\ \hline P(B \cap A) + P(B \cap A) \\ \hline P(B \cap A) \\ \hline P(B \cap A) + P(B \cap A) \\ \hline P(B \cap $	11,000 11,000 1000 000 000 000 000 000 0	. Def. Conditional Probabilities			
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How many 4-digit numbers can be constructed using the digits 1, 2 & 3 ? (Repeats allowed) Soln: 3×3×3×3 = 3 ⁴ ? Ordered samples, with no repeats Choose any of the n objects 1 st Choose any of the n-1 remaining objects 2 nd : Therefore, there are [n(n-1)(n-2)(n-r+1)] called number (#) of permutations of length r	****	of length r from n objects.			
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Soln: $3 \times 3 \times 3 \times 3 = 3^{4}$ (2) <u>Ordered samples, with no repeats</u> Choose any of the n objects 1 st Choose any of the n-1 remaining objects 2 nd : Therefore, there are [n(n-1)(n-2)(n-r+1)] called number (#) of permutations of length r	andra Antonio antonio antonio antonio antonio				
 Ordered samples, with no repeats Choose any of the n objects 1st Choose any of the n-1 remaining objects 2nd I I Therefore, there are [n(n-1)(n-2) (n-r+1)] called number (#) of permutations of length r 	edentanakija (an o emotekako (keder) o (
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$\frac{n(n-1)(n-2)(n-r+1)}{r}$ called number (#) of permutations of length r	1	i			
$\frac{n(n-1)(n-2)(n-r+1)}{r}$ called number (#) of permutations of length r	1999,999,979,979,979,979,979,979,979,979	Therefore, there are			
	ng anna e a a anna a suas na anna an anna an				
Notation: " $P_r = n(n-1)(n-2)(n-r+1) = \frac{n!}{(n-r)!}$					
		Notation: " $P_r = n(n-1)(n-2)(n-r+1) = \frac{n!}{(n-r)!}$			

③ Unordered samples, with no repeats Consider a set S of n elements. let "Cr be the number of subsets of r elements. Note: sets & subsets do not have orders Each such subset could be written in Pr = r! different ordered ways. e.g. {1,2,3}, {1,3,2}, {2,3,1}, ... 3! ways So $(r!)^{n}C_{r} =$ number of permutations of length r, chosen from n objects. $\Rightarrow (r!)^{n}C_{r} = {}^{n}P_{r} = \frac{n!}{(n-r)!}$ $\Rightarrow \boxed{n - r} = \frac{n!}{(n - r)! r!} = \binom{n}{r}$ 'n choose r': the order does not matter Fri. 09/12/16 MATH1401: Mathematical Methods 1 Prof. Halburd Recap: $S = \{1, 2, 3, 4\}$ o find the number of all 3-digit numbers, using S where we can repeat. 4 * 4 * 4 = 64 @ find the number of all 3-digit numbers, using S where we cannot repeat. $4 \times 3 \times 2 = \frac{4!}{1!} = {}^{4}P_{3}$ order matters \Rightarrow "Pc = $\frac{n!}{(n-r)!}$ ③ order does not matter (combination) - Pascal's Triangle ${}^{n}C_{r} = \frac{n!}{(n-r)!(r)!} = \binom{n}{r}$ 121 1331 Note: $\binom{n}{r}$ = binomial coefficients * 641 $(1+\chi)^n = \sum_{r=0}^n {n \choose r} \chi^r$ \checkmark EXAMPLE \bigcirc : Find the total number of subsets of a set of size n. $S = \{1, 2, 3, ..., n\}$

¹	For each element in S, for in given subset A of S. I put a tick
	or a cross depending on whether the element is in A.
\$\$\$\$\$\$\$	1^{st} approach : A = (1, 2, 4)
a ta sa	$S = \{1, 2, 3, 4, 5, 6,, n\}$ list all the elements
·······	√√× √ × × … ×
nangagi unu unu dai dajadajat dada manjaginang pamang punang su unu su pu	counting subsets = counting ticks & crosses
۵۰۰۰ ماریخ و در در ماریخه و مانور میشود میشود میشود از مانو میشود و مانو میشود. ماریخ و این می و در ماریخه و م	$2^{\circ} = # \text{ of subsets}$
stattist H H H H H H H H H H H H H H H H H H H	2^{nd} approach: subsets with 0 element : $\binom{n}{0} = 1$
11,0111 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	l element: $\binom{n}{l} = n$
	2 elements: $\binom{n}{2} = \frac{n(n-1)}{2}$
	3 elements: $\binom{n}{3} = \frac{n(n-1)(n-2)}{3}$
	total # of subsets = $\sum_{r=0}^{n} {\binom{n}{r}} r^{r}$ binomial expansion
	= (1+1) ⁿ
. para para ana ang ang ang ang ang ang ang ang an	
142 open 100 open after den der de Spektige konstanten er eine open	VEXAMPLE Q:
19999900000000000000000000000000000000	In a group of r people, find the probability that at least 2 share
i Parani III. I IIII wana waka waka waka ya 2000 kwana waka waka	the same birthday (ignore 29th Feb.).
and a second	Soln: prob = 1-prob (all b'days are different)
Manangan ang ang ang ang ang ang ang ang	Since $P(all b'days are different) = \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{365}{365} \cdot \frac{365}$
Antonia antona antona antona antona	₹ <u>1</u> <u>365!</u> (<u>365)</u> . <u>(365-r)</u> .
1,0100,01,01,01,01,000 (0,01,01,01,000)	we have P(r) [at least 2 share b'days]
	$= 1 - \frac{365!}{(365)!(365-r)!}$
Nederstammanska i samet forfertel forsæl formanskammerska	Then.
	P(2) = 0.003, $P(3) = 0.005$, $P(22) = 0.476$, $P(23) = 0.507$.
7.4	Independence l Bayes' Formula
	Independence
	Def.

naðhaða Veðana annað þan einnar í eine eftir dannar deman	2 events are independent if and only if
*******	$P(A \cap B) = P(A)P(B)$ Recall: $P(B A) = \frac{P(A \cap B)}{P(A)}$
in a transformed of the second	
	So independent $\Rightarrow P(B A) = P(B)$

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	A fair coin is tossed 3 times.	×	
	Consider the 2 events		
	a throwing at least one head & one tail		
	(b) throwing at most one head		
	Q. (1) Are these events independent?		
	(2) Are they still independent if the coin is tossed	4 times?	
	Soln: (1) $S = \{HHH, HHT, HTH,\}$ $ S = 2^3 = 8$		
	$A = \{ at least IH \& IT \}$		
	= {HHT, HTH, THH, TTH, THT, HTT} [A]=	6	
	= S. {HHH, TTT}		
	$B = \{at most H\}$		
	= {TTT, TTH, THT, HTT} [8]=	:4	
	$\Rightarrow P(A \cap B) = \frac{ A \cap B }{ S } = \frac{3}{8}$		
	$P(A)P(B) = \frac{ A }{ S } \frac{ B }{ S } = \frac{6}{8} \frac{4}{8} = \frac{3}{8}$		
4 	· · · · · · · · · · · · · · · · · · ·		
	(2) $S = \{HHHH,\}$ $ S = 2^4 = 16$		
	$A = S \{ HHHH, TTTT \} A = 16 - 2 = 14$		
	$B = \{TTTT, HTTT, THTT, TTHT, TTTH\} B =5$		
100	$A \cap B = \{HTTT, THTT, TTHT, TTTH\} $ $ A \cap B = 4$		
	$\Rightarrow P(A \cap B) = \frac{1}{4}$		Ø
	$P(A) P(B) = \frac{14}{16} \times \frac{5}{16} \neq \frac{1}{4}$		
	So A & B are dependent.		
	<u>Bayes' Formula</u>		
	Recall: Conditional Probability $P(B[A) = \frac{P(A \cap B)}{P(A)}$ (*)		
	$P(B) = P(B A)P(A) + P(B A^{c})P(A^{c}) (\#)$		
	Note: $P(A B) = \frac{P(A \cap B)}{P(B)}$		
	$\frac{\text{Def.}}{P(A B)} = \frac{P(B A)P(A)}{P(B A^c)P(A^c)} = \frac{from (*)}{from (#)}$		
	$P(A O) = P(B A) P(A) + P(B A^{c}) P(A^{c}) \leftarrow \text{from (#)}$		
	This is called Bayes' Formula		

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	This problem involves 2 coins: one is fair & the other has two heads.
	A coin is selected at random and tossed, and the result is a head. Find the probability that the coin was a few acie
- Хрануда, 2017 (Майайндонун са 1917) султабуласт стутуулын араа	the probability that the coin was a fair coin. Soln: Let F be the event that the fair coin was selected.
maantaa aa ahaa ahaa ahaa ahaa ahaa ahaa	Let H, be the event that a head is chosen.
	$P(F H_i) = \frac{P(H_i F)P(F)}{P(H_i F)P(F) + P(H_i F^c)P(F^c)}$
1 a 2 a 11 a 1 a 2 a 2 a 2 a 2 a 2 a 2 a	$= \frac{\frac{1}{2}, \frac{1}{2}}{\frac{1}{2}, \frac{1}{2} + 1, \frac{1}{2}} = \frac{1}{3}$
	If the coin is tossed a 2nd time & again reveals a H, what is the
Angeleg (announce) in a subjectively, side and announce) as a ge	probability that it is fair ?
	Soln: Let $H_2 = event$ that 2H's are shown. $P(F H_2) = \frac{P(H_2 F)P(F)}{P(H_2 F)P(F^2)}$
10000000000000000000000000000000000000	$= \frac{\frac{1}{4 \cdot 2}}{\frac{1}{4 \cdot 2} + \frac{1}{5}} = \frac{1}{5}$
e form you wan e is a star of the manufacture of the form of the form of the star of the star of the star of th	<u><u><u></u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u></u>
10000000000000000000000000000000000000	Binomial Distribution X~B(n,p) [n × Bernoulli Trials]
mijija dummu po pri statamu je jezu jaum e stratam	√ a fixed number of independent trials.
al fan Tennen oan de fan de	✓ on each trial, there are two outcomes. A-level
	$\sqrt{\text{the probability of success remains constant.}}$ prob = $^{n}C_{x} p^{x}(1-p)^{n-x}$
utudadada	✓ EXAMPLE:
	A multiple-choice exam consists of 25 questions, each with 4 possible
Methoda and a second and a second	answers. A student guesses at random. Find the probability that
	(1) every answer is correct
mmen i i i udenteta provi tar secondareljan / major y opos	(2) exactly one answer is correct
····	(3) at least two answers are correct
	(4) exactly 7 answers are correct
·	(5) the student passes, given the pass mark is 40%.
יינע איז	Soln: (1) $(\frac{1}{4})^{25} \approx 8.88 \times 10^{-16}$
	(2) $P(exactly \mid correct) = 25. (\frac{1}{4}). (\frac{3}{4})^{24}$
	(25) the correct the remaining 24 questions are wrong answer

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(3) P (at least 2 correct) = 1- P(0 correct) - P(exactly 1 correct) $= \left| - \left(\frac{3}{4}\right)^{25} - 25\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^{24} \right|$ = 0.993 (4) P (exactly 7 correct) = $\binom{25}{4} (\frac{1}{4})^{3} (\frac{3}{4})^{18} = 0.165$ # of ways of selecting which 7 questions are correct (5) 40% of 25 (questions) is 10. P(pass) = prob(10 or more correct) = 1- prob (at most 9 correct) = 1 - [P(0 correct) + P(1 correct) + ... + P(9 correct)] $= 1 - \binom{25}{9} (\frac{3}{4})^{25} - \binom{25}{1} (\frac{1}{4}) (\frac{3}{4})^{24} - \binom{25}{2} (\frac{1}{4})^2 (\frac{3}{4})^{23} - \dots - \binom{25}{9} (\frac{1}{4})^9 (\frac{3}{4})^{16}$ = 0.0713 7.6 Bernoulli Trials . Def. A repeated event/ experiment with 2 possible outcomes (success/ failure) is called a **Bernoulli** Trial If p is the probability of success and g=1-p is the probability of failure, then r successes in n trials is $h(\mathbf{r}) = \begin{pmatrix} \mathbf{n} \\ \mathbf{r} \end{pmatrix} \mathbf{p}^{\mathbf{r}} \mathbf{g}^{\mathbf{n} - \mathbf{r}}$ · Mean Value √ Def. Suppose that the outcomes of a sequence of experiments or events are numbers (e.g. rolling a die) If each outcome is x_i , and occurs with probability $P(x_i)$, then the mean is $\overline{\chi} = \sum \chi_i P(\chi_i)$ V EXAMPLE. Fair die. $x_1 = 1$, $x_2 = 2$, ..., $x_6 = 6$ Soln: $P(x_j) = \frac{1}{6}$ $\sum_{i=1}^{6} \chi_i P(\chi_i) = (1+2+3+4+5+6), \frac{1}{6} = \frac{7}{2}$ Then the mean is

EXAMPLE: - To any sequence of n. Bernoulli trials, we associate the probability of r successes $b(r) = \binom{n}{r} p^{r} q^{n-r}$ - The mean (or average) of successes is <u><u>pr.b(r)</u> probability of r successes</u> # of successes $\hat{\Sigma}_{\Gamma,b(\Gamma)} = \hat{\Sigma}_{\sigma} \Gamma \begin{pmatrix} n \\ \Gamma \end{pmatrix} \rho^{r} \hat{g}_{\sigma}^{r-r}$ take out $g^{n} :: it does not contain r$ $= \left[\underline{g}^{n}\right] \sum_{r=1}^{n} r\binom{n}{r} \left(\frac{P}{g}\right)^{r} \qquad (sum on r)$ $= \underbrace{\operatorname{Period}_{r=0}^{n} (r)(\underline{q})}{\operatorname{period}_{r=0}^{n} (1+x)^{n}} = \underbrace{\operatorname{Period}_{r=0}^{n} \binom{n}{r} x^{r}}{\operatorname{period}_{r=0}^{n} (\frac{n}{r}) x^{r-1}} \quad \text{differentiate wrt } x$ $\Rightarrow \operatorname{n}(1+x)^{n-1} = \underbrace{\operatorname{Period}_{r=0}^{n} \binom{n}{r} x^{r-1}}{\operatorname{period}_{r=0}^{n-1} (1+x)^{n-1}}$ $= \underbrace{\operatorname{Period}_{r=0}^{n} \binom{p}{q} (1+\frac{p}{q})^{n-1}}{\operatorname{period}_{r=0}^{n-1} (x=\frac{p}{q})}$ $= \underbrace{\operatorname{Period}_{r=0}^{n} \binom{p}{q} \cdot \frac{1}{q^{n-r}} (q+p)^{n-1}}{\operatorname{period}_{r=0}^{n-1} (q+p)^{n-1}}$) looks similar $= np(q+p)^{n-1}$ = np77 Paisson Distribution We want to approximate the binomial distribution for large n, where the mean $\lambda = np$ is held fixed (so p is small). 12/12/16 Mon. MATHI401: Mathematical Methods 1 Prof. Halburd Recall: · Bernoulli Trials: ✓ probability of success: P ✓ probability of failure : g=rp \checkmark probability of r successes from n trials: $b(r) = \binom{n}{r} p^r g^{n-r}$ • mean or average # of successes for binomial distribution:

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 $n = \sum_{n=1}^{n} r \cdot b(r) = np \quad (*)$ 7.7 Poisson Distribution n fixed, consider the limit as $n \rightarrow +\infty$. $(x): P = \frac{\lambda}{n} \rightarrow 0$ Then, we have $b(r) = \binom{n}{r} p^{r} g^{n-r}$ $= \binom{n}{r} \left(\frac{\lambda}{n}\right)^{r} \left(1 - \frac{\lambda}{n}\right)^{n-r}$ $= \frac{n!}{r!(n-r)!} \left(\frac{\lambda}{n}\right)^{r} \left(1 - \frac{\lambda}{n}\right)^{n-r}$ $b(\mathbf{r}) = \frac{\lambda^{\mathbf{r}}}{\mathbf{r}!} \cdot \frac{\mathbf{n}!}{(\mathbf{n}-\mathbf{r})!} \cdot \frac{1}{\mathbf{n}^{\mathbf{r}}} \left(1 - \frac{\lambda}{\mathbf{n}}\right)^{\mathbf{n}-\mathbf{r}'}$ So, $\frac{\lambda^{r}}{r!} \cdot \frac{n(n-1)(n-2)(n-3)}{n(1-n)} \cdot \frac{(1-\frac{\lambda}{n})^{n}}{(1-\frac{\lambda}{n})^{-r}}$ $= \frac{\lambda^{r}}{\Gamma!} \cdot \left(\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)\left(1 - \frac{3}{n}\right) \dots \left(1 - \frac{p+1}{n}\right) \cdot \left(1 - \frac{\lambda}{n}\right)^{n} \left(1 - \frac{\lambda}{n}\right)^{-r}\right)$ $\frac{n!}{(n-r)!} \xrightarrow{n \to \infty} 1$ Also, similarly, $\lim_{n \to \infty} (1 - \frac{\lambda}{n})^r = 1$ Let $a_n = (1 - \frac{\lambda}{n})^n$, then $\log a_n = \ln \log \left(\left(-\frac{\lambda}{n} \right) \right)$ $\log(1+\chi) = \chi + ?\chi^2 + \omega$ $= \Pi \left(-\frac{\lambda}{n} + \frac{2}{n} (\frac{1}{n})^2 \right) \xrightarrow{n \to +\infty} -\lambda$ \Rightarrow time = e^{-\lambda} So, $\lim_{r \to r} b(r) = \frac{\gamma r}{r!} = \rho(r)$ Poisson Distribution ✓ EXAMPLE: An insurance company pays £500,000 to each client who experiences a fire. The company has 5,000 clients. The probability of a client having a fire in one year is 10⁻⁴. Find the probability that the company pays at least £2000.000 in a year. Soln: Assume no client has more than 1 fire. n = 5000 this is large $p = 10^{-4}$ So the mean number of fires is <u>∧=np = 0:5</u>

 $\pm 2,000,000 = 4$ fires $prob (\ge f_{2,000,000}) = prob (at least 4 fires)$ = 1 - P(0) - P(1) - P(2) - P(3)t prob (2 fires) Since $P(r) = \frac{e^{-n}n^{r}}{r!} = e^{-\frac{1}{2}} \cdot \frac{(\frac{1}{2})^{r}}{r!}$ prob $(\geq \pm 2,000,000) = 1 - e^{-\frac{1}{2}} \left[1 + \frac{(\frac{1}{2})^2}{1!} + \frac{(\frac{1}{2})^2}{2!} + \frac{(\frac{1}{2})^3}{3!} \right]$ = $1 - \frac{79}{48} e^{-\frac{1}{2}}$ 0.00175 = 0.2% = • Check $\sum_{r=0}^{\infty} P(r) = \sum_{r=0}^{\infty} \frac{e^{-\lambda} \lambda^r}{r!}$ = e⁻, e = 1 Events occurring over intervals of time or space √ EXAMPLE. cars passing at a pt X over an hour. (Let n = average # passing in an hour) Assume O no 2 cars pass X at the same time @ the time that a car passes is independent of the other cars. Omin 1 hour - Place a mark on the line for each time a car passes. For sufficiently large (small enough) (s.t.) n, split the interval into n sub-intervals r containing 0 or 1 mark. - View that as Bernoulli trials with probability $P = \frac{\pi}{n}$ that a given sub-interva has a mark, then prob (r cars pass in the hour) = prob (r successes) $-b(r) = {\binom{n}{r}}\rho^{r}g^{n-r} = {\binom{n}{r}}(\frac{\lambda}{n})^{r}(1-\frac{\lambda}{n})^{n-r}$ - To make the subintervals arbitrarily small (i.e. to allow the time to be arbitrarily close), take lim. This is the calculation we just did. So we have the Poisson distribution $P(r) = e^{-n} \cdot \frac{n^r}{r!}$

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Fri. 16/12/16 Mathematical Mathematical Mathematical	America de la facta de la compara que
MATH1401: Mathematical Methods I Prof. Halburd	
Def. Poisson Distribution	
The probability of r events occurring in some interval (of sp	ace or
time) with an average of A events in the interval is	eenen an
$P(r) = e^{-\gamma} \frac{\Delta r}{P}$	elander benefense ander freigt freider i man frei einen eine eine freider in der eine eine eine eine freider ei
(independent events)	
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An office receives on average 3 calls per hour. Find the	probability
that in a particular hour	inaliterature production of the second s
(a) no calls are received and	aren en antieren antieren 1112 mehr beset beseten die seinen die seinen die seinen die seinen die seinen die s Seine seine sein
(b) exactly 3 calls are received.	1999-11999-1999-1997-1997-1997-1997-199
Soln: average calls per hour = $7 = 3$	an a
(a) prob (no calls) = $P(0)$	r
$=e^{-3}\cdot\frac{3^{\circ}}{0!}$	eneranı energinda dağında ber ana dağında aşlamın dağında göre
$=e^{-3}=0.0498$ (5%)	
(b) prob (3 calls) = P(3) = $e^{-3} \cdot \frac{3^3}{31}$	anahata balan kuna munananan papapana bahanan karana
$=\frac{q}{2}e^{-3}$ (22%)	sama-toottotesteennn teatatotetotetotet
VEXAMPLE Q:	*****
A roll of ribbon contains one detect per metre on av	-
A 50 cm piece is cut. What is the probability that it contr	ains
at least one defect ?	£4
Soln: $n = \frac{1}{2}$ defect per metre	ana, and an anna belancifa [52] fa tanàna ao kaona dan dara fa
prob (at least one defect) = 1 - prob (no defect)	angan mandalah kanadan Singgan Singga ng mananan sanan si sana
= 1 - P(0) = $1 - e^{-\frac{1}{2}} \cdot \frac{(\frac{1}{2})^{\circ}}{0!}$	neventum (1944) nämn for finsk et memmin funska († 1945)
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• Def Suppose that a particle can be anywhere on the real line IR, we represent the probability that the particle is between x=a and x=b as $P(a < x < b) = \int_{a}^{b} f(x) dx$, where f(x) is called a probability density • Def. f(x) is a piecewise continuous function. i.e. $\chi_1 < \chi_2 < \chi_3 < \dots$ s.t. $f(\chi)$ is continuous on (χ_n, χ_{n+1}) - ← fi≈i can have several jumps compared to f(x) = { ' x irrational is not piecewise continuous × rational \checkmark Properties: $(f_{-\infty}^{\infty} f(x) dx = 1)$ (3) The mean of probability distribution is $\mu = \int_{-\infty}^{\infty} x f(x) dx$ (like the discrete case $\mu = \sum_{i=1}^{\infty} P(x_{i})$) ✓ EXAMPLE : The probability density describing the location of a particle is $f(x) = \begin{cases} C(x-x^3) \\ c \end{cases}$ 0 < x < otherwise Find (i) the normalisation constant c. (ii) the mean (iii) the probability that the particle is between x=0 and $x=\frac{1}{2}$ Soln: (i) $\int_{-\infty}^{\infty} f(x) dx = 1$ $c \int_{0}^{1} (x - x^{3}) dx = 1$ since f(x) = 0otherwise c (三十) = | C=4 Therefore, when 0 < x < 1, $f(x) = 4(x - x^3)$ $= 4 \chi (1 - \chi^2)$ $= 4 \times (1 + x)(1 - x) > 0$

(i)
$$\mu = \int_{-\infty}^{\infty} x freedx$$

 $= 4\int_{0}^{\pi} (x^{-}x^{+}) dx$
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 $g(f-x)=-g(g)$$

Med. 07/12/16 MATH1401 Help Class Prof. Wilson § Probability § Choosing r -from n (1) Replacement allowed, order matters. (2) No replacement order matters Number = $n \times (n-1) \times (n-2) \times \dots \times (n+1-p) = \frac{p_1}{(n-p_2)} = \frac{p_p}{p_p}$ $y = \frac{1}{1} \times \frac{1}{2^{nd}} \times \frac{1}{3^{nd}} + \frac{1}{p^{th}} = \frac{p_p}{(n-p_2)}$ BNe replacement order does not matter If there are M ways to do it, each gives r! ordered sets. So Mxr! = "P, = n-13! Therefore, $M = \frac{n!}{(n-1)!r!} = {}^{n}C_{r} = \binom{n}{r}$ $M = \frac{n!}{(n-1)!r!} = {}^{n}C_{r} = \binom{n}{r}$ HW8-9 Q10. · Each possible ordering is equally-likely There are n! ordering. So the probability of each occurring is $\frac{1}{n!}$ Therefore. P(Exactly r are in the right place) Number of orderings in which exactly r are in the right place n! \bigcirc <u>r=n</u>. no. of orderings = 1 $\Rightarrow P(n \text{ correct}) = \overline{n!}$ 2 r=n-1 It is impossible to get just one book wrong (where would it go?) $\Rightarrow P((n-1) \text{ connect}) = 0$

3 r=n-2 Number of ways to get exactly 2 wrong = number of ways to choose 2 books (which we swap) = ⁿC₂ <u>n!</u> 21(n-2)) Therefore $P(2 \text{ wrong}) = \frac{nC_2}{n!} = \frac{1}{2!(n-2)!} = \frac{1}{(n-2)!2!}$ (A) r=n-3 Number of ways to get exactly 3 wrong number of ways to X get these 3 wrong -A-B-C-X all in right places $\binom{n}{3}$ $\frac{A \ C \ B \ X \ A \ right}{B \ A \ \subseteq \ X \ C \ right}$ $\frac{A \ C \ B \ A \ \subseteq \ X \ C \ right}{C \ A \ B \ V \ works}$ C B A X B regist Thurefore, $P(3 \text{ wrong}) = \binom{n}{3} \times \frac{1}{2} \times \frac{n!}{n!} \times \frac{2}{3!(n-3)!} \frac{1}{n!} (n-3)!3$ $\int \frac{\Gamma = n - 4}{2},$ Number of ways to get exactly 4 wrong number of ways to number of ways to choose 4 books get these 4 wrong 0 wrong 9=41-1 1 2 wr $\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right)^{-1} \left(\frac{1$ h 2 wrong 4! permutations 3 wrong O Wrong * ABCD DABC ADBC ABDC 2 WRANG ACBD DACB ADCB ACDB DBAC BDAC BADC ℲℲ℮ -DBCA WICH -BEAD BCDA -BDCA-DCAB J CABD CADB CDAB DCBA (BDA-CDBA -CBAD' Therefore, $p(4 \text{ wrong}) = {\binom{n}{4}} \times 9 \times \frac{1}{n!} = \frac{n!}{4!(n-4)!} = \frac{3}{(n-4)!8}$ - QUESTION 1. 50 red balls of 50. black balls V ß

Want to maximise chance of getting a red Choose un -> choose ball -> maximise P(red) 25 R 2SR 25B 25B R α $\frac{1}{2} \qquad R \qquad B \qquad H \qquad B$ $P(R) = P(\alpha \cap R) + P(\beta \cap R)$ $= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$ = 1/2 2 50R SOB ß $\frac{1}{2} \alpha - \frac{\beta}{1} R$ $P(R) = P(Q(R)) = \frac{1}{2} \times 1 = \frac{1}{2}$ 49R 3 IR 50 B α ρ 12 α B B F R R R $P(R) = P(\alpha \cap R) + P(P(R))$ $= \frac{1}{2} \times 1 + \frac{1}{2} \times \frac{49}{99}$ $= \frac{74}{99}$ - QUESTION 2 Given that I have a boy, what's the probability that the other child is a girl? $\frac{1}{2}$ B = G {(B,B), (G,B), (G,B), (G,G)} $\frac{1}{2}$ G = B $P(\text{the other child is a girl } | \text{ I have a boy}) = \frac{2}{3}$

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