Analyze Function Graphs: Testing Domain, Range, and Intersection Concepts with the x-axis and y-axisTheory Introduction

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Functions - one variable functions that have been studied in Differential Calculus and Integral Calculus courses such as algebraic functions, transcendent functions, trigonometric functions and other functions with advanced graph drawing concepts can be determined graphically. However, the results can be directly obtained with the help of Maple 13.

Example:

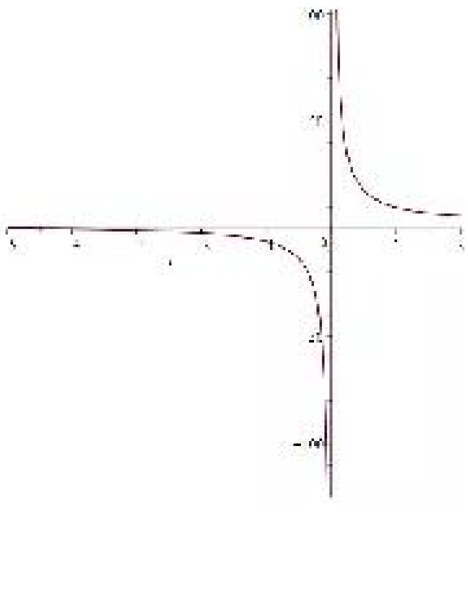
Given the function $f(x) = 2 + \frac{8}{x}$

Specify:

a. Draw a function graph
b. Domain and range of a function
c. Intersection point with x-axis and y-axis
d. First derivative of $f(x)$, monotonicity, stationary point, and singular point
e. Second derivative of the function $f(x)$, extreme point
f. Asimtot

Answer:

a. Draw a function graph in maple



b. Domain of $f(x)$: $_{Df} = (-\infty, 0) \cup (0, \infty)$

Range $f(x)$: $_{Rf} = (-\infty, 2) \cup (2, \infty)$

c. The coordinate of the intersection point with respect to the *x-axis* exists if $x = a$ is the zero value of the function *f(x)*, then

$(a, 0)$ is the coordinate of the intersection point with respect to the *x-axis.*

$y = 2 + \frac{8}{x} = 0$

$\frac{8}{x} = -2$

$x = -4$

then (-4,0) is the coordinate of the intersection point with respect to the *x-axis.*

Determining the intersection point on the x-axis with maple

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> with(student) :
> intercept(y = 2 + 8/x, y = 0)
                                        {x = -4, y = 0}                                        (1)
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The coordinate of the intersection with the *y-axis* exists if $x = 0$ and $f(x) = y$, then (0,*y*) is the coordinate of the intersection with the *y-axis.* While this function does not have a cut-off point with respect to the y-axis.

d. First derivative

$$f(x) = 2 + \frac{8}{x}$$

$$f'(x) = \frac{-8}{x2}$$

Determining the first derivative with maple

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> d/dx (2 - 8/x)
                                        -8/x^2                                        (2)
```

1) Monotony

Suppose f is continuous on the interval I and differentiable at any point in I.

- If $f'(x) > 0$ for all points in x from I, then f rises at I.
- If $f'(x) < 0$ for all points in x of I, then f is decreasing on I.

f(x) is monotone decreasing because $f'(x) < 0$ in the interval $(-\infty, 0) \cup (0, \infty)$. The graph also shows that $f(x)$ is monotonically decreasing.

2) Stationary point

A stationary point is a point on the graph where the first derivative of the curve equals zero or $f'(x) = 0$. There is no stationary point for this function.

3) Singular Points

A singular point is a point on the graph of f in a sharp angle, vertical tange nt, or jump. There are no singular points in this function because there are no sharp corners on the graph.

e. Second derivative

$$f(x) = 2 + \frac{8}{x}$$

$$f'(x) = -\frac{8}{x^2}$$

$$f''(x) = \frac{16}{x^3}$$

determine the second derivative on Maple

$$> \frac{d^2}{dx^2}\left(2 + \frac{8}{x}\right)$$

$$\frac{16}{x^3} \quad (3)$$

1) Extreme point

Suppose f is a continuous function in the interval (a, b). To find the extreme point, you can do the following steps.

a) Find the critical point, which is point c that satisfies $f'(c) = 0$

b) Calculate the value of $f''(c)$. There are three possibilities:

- If $f''(c) < 0$, then (c, $f(c)$) is a local maximum point
- If $f''(c) > 0$, then (c, $f(c)$) is a local minimum point.
- If $f''(c) = 0$, then (c, $f(c)$) then nothing can be concluded.

This function has no extreme points because there are no points that fulfill the critical point requirement.

f. Asimtot

Suppose a rational function is known:

$$y = \frac{(ax^n + bx^n - 1 + cx^n - 2 + \cdots + k)}{px^m + qx^m - 1 + rx^m - 2 + \cdots + z}$$

then:

- If $n < m$, then the flat asymptote is $y = 0$.

- If $n = m$, then the flat asymptote is $y = \frac{a}{p}$
- If $n > m$, then the asymptotes are either skewed asymptotes or curve asymptotes.

Determine the asymptotes of the function $y = 2 + \frac{8}{x}$

$$y = 2 + \frac{8}{x}$$

$$y = \frac{2x + 8}{x}$$

Upright asymptotes are obtained when the value of y approaches infinity and negative infinity for a given value of x. The value of y will be infinite if the denominator of the rational function approaches 0.

$y = 2 + \frac{8}{x}$ will be infinite when the value of x approaches 0. So, the vertical asymptote is $x = 0$

Because the numerator and denominator of the rational function are of the same degree, the asymptotes of

The flatness is $y = \frac{2}{1} = 2$.

Determining asymptotes in Maple

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> with(Student[Calculus1]) :
> Asymptotes(2 + 8/x)
                              [y = 2, x = 0]                              (5)
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