# Matrix Multiplication: Techniques and Applications in Linear Algebra

1. WEEK TO : 3
2. SOFTWARE : Maple 13
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4. OBJECTIVES :

After following this module, students are expected to be able to perform operations on matrices such as addition, subtraction, multiplication and can determine other operations that can only be performed on matrices such as determinant, transpose, and matrix inverse.

5. **INTRODUCTORY** THEORYDefinition of **Matrix**

A matrix is a set of numbers arranged in a rectangular shape based on the order of rows and columns. The numbers that make up the matrix are called elements. Matrices are expressed in the form mxn where m denotes rows and n denotes columns.Example of a 3x3 ordered matrix A

$$A = \begin{matrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{matrix}$$

$a\, a_{1112} \ldots a_{33}$ are referred to as elements of matrix A while the indices 11 ... 33 indicate the row and column of element a, for example $a_{11}$ means element a is in row 1 and column 2.

**Types of Matrices**

1. **Row matrix**

   P =( 2 -1 3)

2. **Column matrix**

$$\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

3. **Zero matrix**

000
( 000)
000

4. **Square matrix**

-21
( )
23

5. **Upper triangular matrix**

**1-12**
**(0 3 5)**
**0 0 4**

6. **Lower triangular matrix**

**1 0 0 (-2**
**5 0 )**
**3 4-2**

7. **Diagonal matrix**

**1 0 0**
**(0 3 0)**
**0 0 5**

8. **Identity matrix**

**1 0 0**
**I = (0 1 0)**
**0 0 1**

$I = ($ 10
)
01

9. **Singular matrix**

Matrix with zero determinant

$P = ($ 24
)
48

**Operations on Matrices**

1. **Matrix Summation**

For example, if there are two matrices, namely matrix A and matrix B. if matrix C is the sum matrix of A and B, then matrix C can be obtained by summing up each element in matrix A that is aligned with each element in Matrix B. Therefore, the requirement for two or more matrices to be summed is that they must have the same order.

Matrix addition has the following properties:

- Commutati
ve
A+B=B+A
- Associative

(A+B)+C=A+(b+C)

- Zero matrix

  A+0=A

**2. Matrix Reduction**

Suppose there are two matrices, namely matrix A and matrix B. if matrix C is the subtraction matrix of A with B then matrix C can be obtained by subtracting each element in matrix A that is aligned with each element in Matrix B. therefore the requirement that two or more matrices can be subtracted is that they must have the same order.

Matrix subtraction has the same properties as matrix addition.

**3. Matrix Multiplication**

**a. Matrix multiplication with scalars**

Suppose there is a matrix A of order m X n and a real number (scalar) that is k. the multiplication between matrix A and scalar k is denoted kA which is obtained by multiplying each element of matrix A by scalar k. all matrices of arbitrary order can be multiplied by a real number (scalar).

$$kA = k\begin{bmatrix} a11 & \cdots & a1n \\ \vdots & \ddots & \vdots \\ am1 & \cdots & amn \end{bmatrix} = \begin{bmatrix} ka_{11} & \cdots & ka_{1n} \\ \vdots & \ddots & \vdots \\ ka_{m1} & \cdots & ka_{mn} \end{bmatrix}$$

**b. Multiplication between matrices**

Suppose there are two matrices, namely matrix A with order m x p and matrix B with order p x n. The multiplication of matrices A and B is denoted A x B which is obtained from the sum of the product of the corresponding elements in the i-th row of matrix A with the j-th column in matrix B with i = 1,2,3,...,m and j = 1,2,3,...,n.

The requirement for two matrices to be multiplied is that the first matrix must have the same number of columns as the number of rows in the second matrix. The order of the matrix resulting from the multiplication of two matrices is the number of first rows times the number of second columns, for example the multiplication of a 2x3 matrix with a 3x2 matrix.

Multiplication between two matrices has the following properties:

- Associative

  (AxB)xC = Ax(BxC)

- Distributive

  Ax(B+C) = (AxB)+(AxC)

- Multiplication with a zero matrix will result in a zero matrix

- Ax0 = 0

## 4. Matrix Transpose

The transpose of a matrix is a new matrix generated by moving a row element to a column element, denoted $A^T$ e.g.

$$A = \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} \rightarrow A^T = \begin{matrix} a & d & g \\ b & e & h \\ c & f & i \end{matrix}$$

## 5. Matrix Determinant

Determinant is the value that can be calculated from the elements of a square matrix. The determinant of matrix A is denoted **det(A), det A, or $|A|$.**

Determinant of 2x2 order matrix

Suppose the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

## 6. Matrix Inverse

Inverse of 2x2 order matrix

Suppose $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $ad - bc \neq 0$, the inverse of matrix A can be found by :

$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

6. STEP WORK

Matrix writing can be written as follows

```
> c := <<1, 2, 3>|<4, 5, 6>>;
```

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

```
A := Matrix([[1, 3, 5], [2, 4, 6], [3, 7, 9]])
```

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 7 & 9 \end{bmatrix}$$

```
B := Matrix([[1, 2], [2, 4]])
```

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$E := array([[1, 0, 0], [0, 1, 0], [0, 0, 1]])$

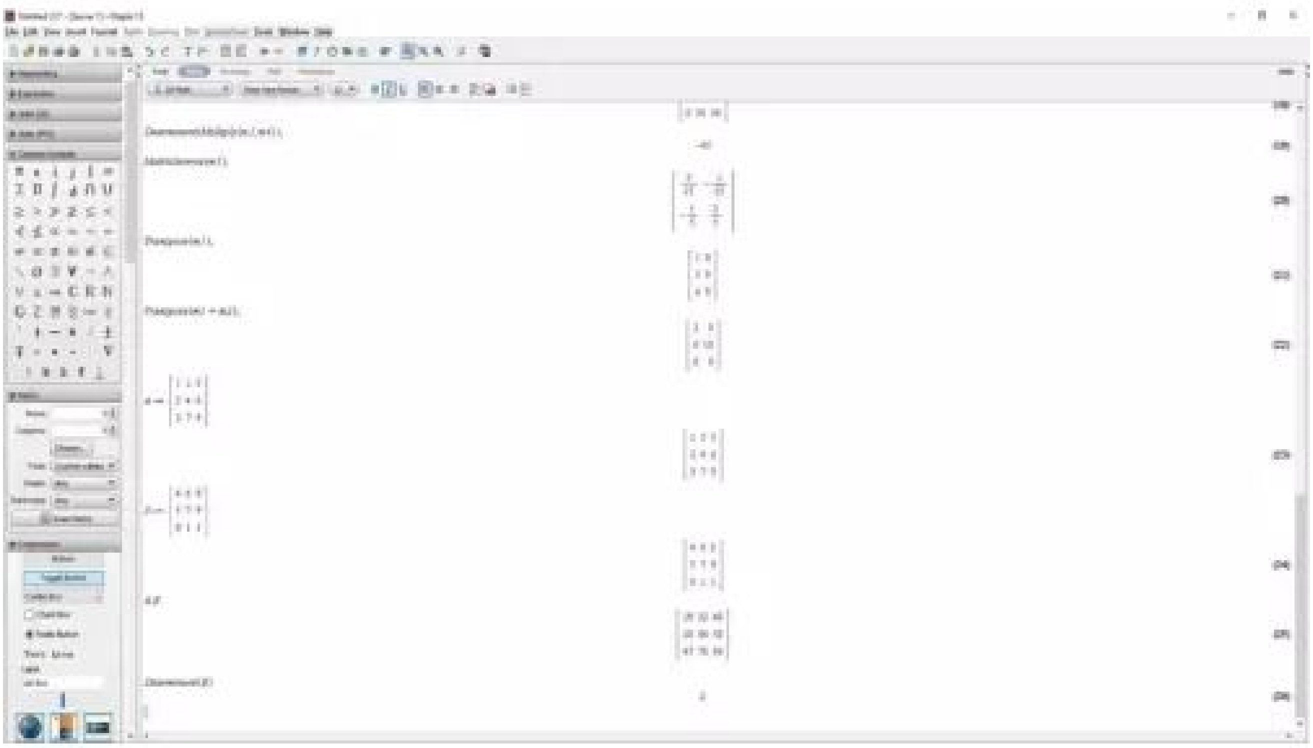
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$F := Matrix([[2, 2, 2], [0, 3, 0], [1, 0, 1]])$

$$\begin{bmatrix} 2 & 2 & 2 \\ 0 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Or you can also input the matrix in the maple menu on the left under "common symbols".

*Note that some operations on matrices larger than 2x2 in maple 13 can only be applied when creating the matrix through this menu such as the determinant multiplication operation etc.



How to write matrix operations commands in Maple 13

> $m1 := Matrix([[1, 5, 6], [0, 9, 9]])$

$$\begin{bmatrix} 152 \\ 0 & 9 & 9 \end{bmatrix}$$

$m2 := Matrix([[1, 1, 0], [0, 1, 0]])$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$m3 := Matrix([[9, 1], [3, 2]])$

$$\begin{bmatrix} 9 & 1 \\ 3 & 2 \end{bmatrix}$$

$m4 := Matrix([[1, 4], [1, 1]]);$

$$\begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$$

$m1 - m2$

$$\begin{bmatrix} 2 & 6 & 6 \\ 0 & 10 & 9 \end{bmatrix}$$

$m1 - m2$

$$\begin{bmatrix} 0 & 4 & 6 \\ 0 & 8 & 9 \end{bmatrix}$$

$6 \cdot m1$

$$\begin{bmatrix} 6 & 30 & 36 \\ 0 & 54 & 54 \end{bmatrix}$$

$Determinant(Multiply(m3, m4));$

$$-45$$

$MatrixInverse(m3);$

$$\begin{bmatrix} \frac{2}{15} & -\frac{1}{15} \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix}$$

$Transpose(m1);$

$$\begin{bmatrix} 1 & 0 \\ 5 & 9 \\ 6 & 9 \end{bmatrix}$$

$Transpose(m1 + m2);$

$$\begin{bmatrix} 2 & 0 \\ 6 & 10 \\ 6 & 9 \end{bmatrix}$$

## 7. TASK

$$m_1 := \begin{bmatrix} 57 & -76 & -32 \\ 27 & -72 & -74 \\ -93 & -2 & -4 \end{bmatrix} \quad m_2 := \begin{bmatrix} 10 & -9 \\ -16 & -50 \end{bmatrix} \quad m3 := \begin{bmatrix} 43 & 12 \\ 25 & -2 \\ 94 & 50 \end{bmatrix} \quad m_4 := \begin{bmatrix} -48 & 9 & -50 \\ 77 & 31 & -80 \end{bmatrix}$$

Specify

(1) Inverse of matrix m1 and then its determinant

(2) Multiplication of m2 matrix with itself

(3) Transpose m2

(4) The result of number 3 is then summed with m4

(5) Determinant of no.(2)