Finding the Global Maximum in a Function with Constraints on Intervals

Module to: 1

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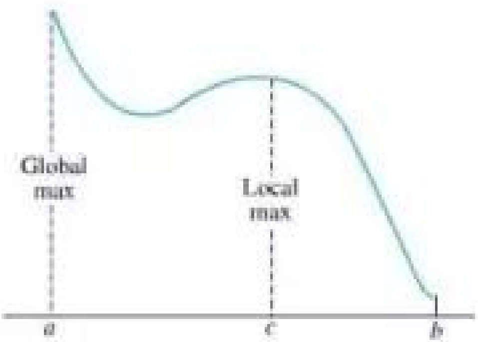
Software : Maple 13

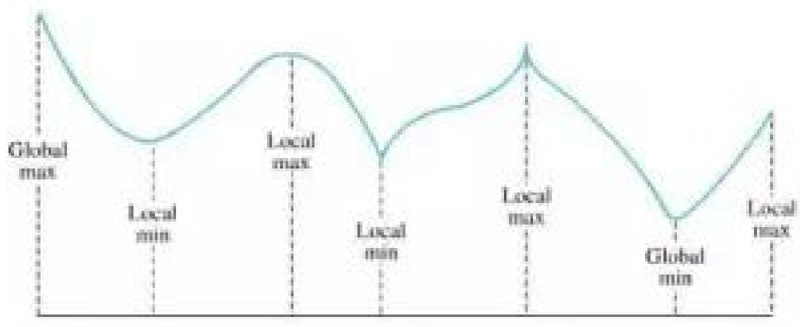
Objective: After this practicum students are expected to.

1. Determine local maximum and minimum values using the First Derivative Test.
2. Determine local maximum and minimum values using the Second Derivative Test.
3. Determining extreme values on open intervals.

Theory Introduction

The maximum value of a function f on a set S is the largest value of f over the entire set S. It is sometimes seen as the **global maximum** or *absolute maximum value* of f. So for a function f with origin $S = [a, b]$ is the maximum value of
but what about $f(c)$? It may not be a global maximum but a maximum in its own range not a king but a chief in his own area so we call $f(c)$ a **local maximum** or *relative maximum*. Similarly, the **global minimum value** is the smallest value among the local minimum values.





Definition

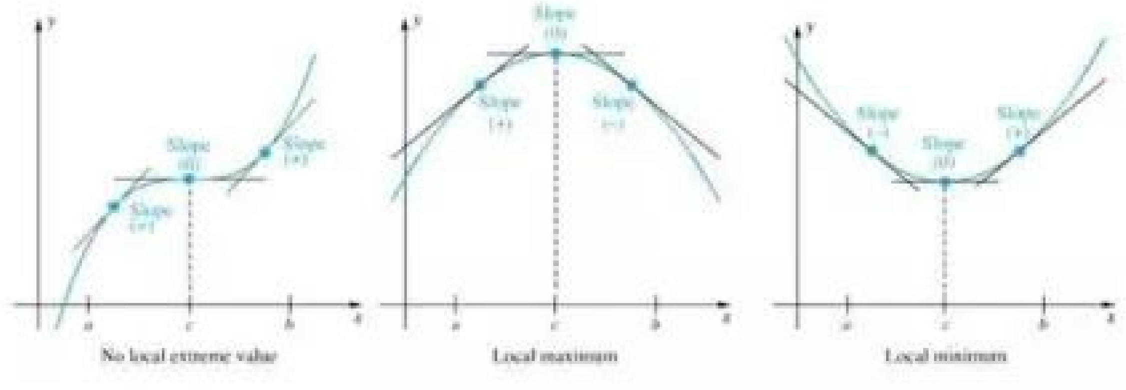
Let S be the origin of f. It contains the point c. We say that:

i. $f(c)$ the **local maximum value of** f if there exists an interval (a, b) containing c such that $f(c)$ is the maximum value of f on $(a, b) \cap S$

ii. $f(c)$ the **local minimum value of** f if there exists an interval (a, b) containing c such that such that $f(c)$ is the minimum value of f on $(a, b) \cap S$

iii. $f(c)$ the **local extreme** value **of** f if it is either a local maximum or a local minimum.

Where is the Local Extreme Value? The Critical Point Theorem applies except that in this case it will be replaced by *local extreme values*. So critical points (endpoints, stationary points, and singular points) are candidates for points where local extremes are likely to occur. We say candidate because we are not demanding that every point must be a local extreme as in the left part of the graph in Figure below. However, if the derivative is



positive on one side of the critical point and negative on the other, then we have local extremes as shown in the center and right graphs of Fig. above...

Theorem A
First Derivative Test

Suppose f is continuous on an open interval (a, b) containing a critical point c

i. If $f'(x) > 0$ for all x in (a, c) and $f'(x) < 0$ for all x in (c, b), then $f(c)$ is the local maximum value of f.
ii. If $f'(x) < 0$ for all x in (a, c) and $f'(x) > 0$ for all x in (c, b), then $f(c)$ is the local minimum value of f.
iii. If $f(x)$ is equal on both sides of c, then $f(c)$ is not a local extreme value of $f(x)$.

Example 1 and testing with **Maple 13** ***software***

Find the local extreme value of the function $f(x) = x^2 - 6x + 5$ at $(-\infty, \infty)$

Answer
Since f is continuous everywhere, f has a derivative $f'(x) = 2x - 6$ so the critical point is a single solution, $x = 3$.

With Maple 13 Define the function first

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f:=(x)->x^2 - 6x + 5
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$$x \to x^2 - 6x + 5$$

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plot(f(x))   #f(x) is a pre-defined function
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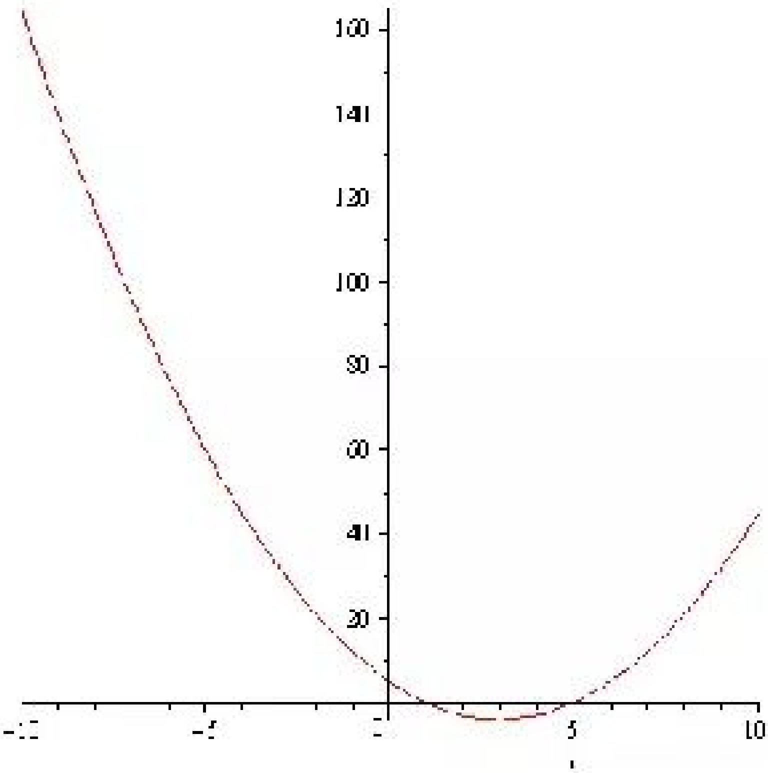


$D(f)(x)$;#commands for deriving functions

$$2x - 6$$

$solve(D(f)(x) > 0, x)$; #commands to find monotonous rise

$$RealRange(Open(3), \infty)$$

On this interval the function is monotonically increasing

$solve(D(f)(x) < 0, x)$;#commands for finding monotone descent

$$RealRange(-\infty, Open(3))$$

On this interval the function is monotone descent

> Since there is only a minimum value, x=3 is an extreme value.

Theorem B
Second Derivative Test

Suppose that f' and f'' exist at any point of the open interval (a, b) containing c and suppose that $f'(c) = 0$

i. If $f''(c) < 0$ then $f(c)$ is the local maximum value of f.
ii. If $f''(c) > 0$ then $f(c)$ is the local minimum value of f.

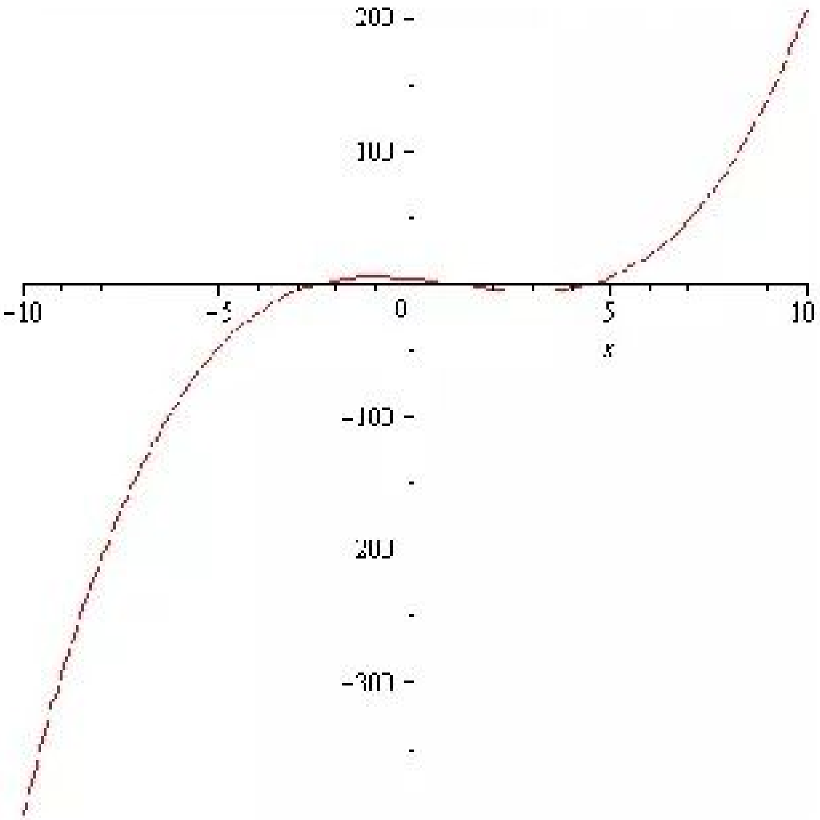
Example 2 and testing with **Maple 13 *software***

Find the local extreme by testing the second derivative of the function $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 4$

> $f := (x) \rightarrow \frac{1}{3}x^3 - x^2 - 3x + 4$

$$x \to \frac{1}{3}x^3 - x^2 - 3x + 4$$

plot(f(x))



#First derivative looking for critical points
D(f)(x)

$$x^2 - 2x - 3$$

solve(D(f)(x));

$$3, -1$$

#Second Derivative Test looking for relative maximum and minimum values
(D@@2)(f)(x);

$$2x - 2$$

(D@@2)(f)(3);

$$4$$

(D@@2)(f)(-1);

$$-4$$

f(3)

$$-5$$

f(-1)

$$\frac{17}{3}$$

From the second derivative test, it can be concluded that f(3) is the maximum value and f(-1) is the minimum value in the interval [-5,5].

Extremes on Open Intervals When looking for a local maximum or minimum value, the given interval is usually a closed interval but it is possible that the given interval can also be open, so keep in mind that a maximum (minimum) without a certain description means a global maximum (minimum).

Tasks

Find every maximum and minimum value or extreme value in the given function and interval if no interval is given then find the global maximum and minimum values!

1. $f(x) = x^4 + 4$
2. $f(x) = f(x) = 6\sqrt{x} - 4x\ pada\ [0,4]$
3. $g(x) = x^4 + x^2 + 3$

Reference

Varberg, Purcell, Rigdon, 2010, *Calculus volume 1 Ninth Edition*, Erlangga