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Q 1(a) Express $x^2 - 8x + 11$ in the form $(x + p)^2 + q$

where p and q are constants

(b) Hence find the exact solutions of the equation

$x^2 - 8x + 11 = 1$

Solution

(a) $x^2 - 8x + 11 = x^2 - 8x + 16 - 16 + 11$

$x^2 - 8x + 11 = (x^2 - 8x + 16) - 16 + 11$

$x^2 - 8x + 11 = (x - 4)^2 - 5$

Comparing it with the equation $(x + p)^2 + q \implies$

$p = -4, q = -5$

(b) $x^2 - 8x + 11 = 1$

$x^2 - 8x + 11 = (x - 4)^2 - 5 = 1$

$(x - 4)^2 - 5 = 1$

$(x - 4)^2 - 5 + 5 = 1 + 5$

$(x - 4)^2 = 6$

$x - 4 = \pm\sqrt{6}$

$x = 4 \pm \sqrt{6}$

Q2 The thirteenth term of an arithmetic progression is 12 and the sum of the first 30 terms is -15. Find the sum of the first 50 terms of the progression.

Solution

$a_n = a + (n-1)d$

Put $a_n = 12, n = 13$

$12 = a + (13-1)d$

$12 = a + 12d$

$a + 12d = 12$ .... eq (1)

Formula to find the sum of an arithmetic progression

$s_n = \frac{n}{2}[2a + (n-1)d]$

Putting $S_n = -15, n = 30$

$-15 = \frac{30}{2}[2a + (30-1)d]$

$-15 = 15[2a + 29d]$

$-1 = [2a + 29d]$

$2a + 29d = -1 ....eq(2)$

Multiplying eq (1) by 2 and subtraction from equation (2)

$2a + 29d - 2a - 24d = -1 - 24$

$5d = -25$

$d = -5$

Putting in equation (1)

$a + 12(-5) = 12$

$a - 60 = 12$

$a = 72$

12

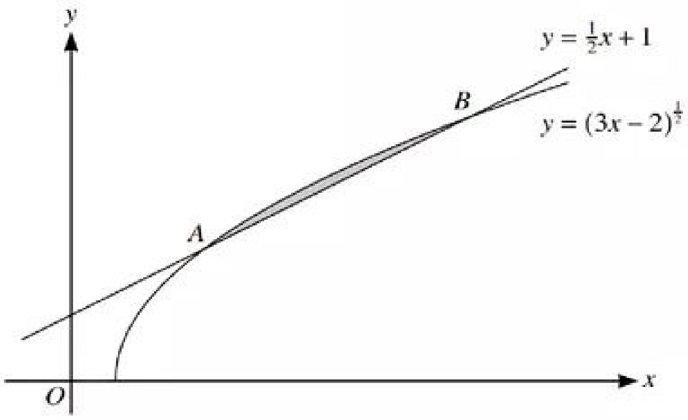


Figure 1:

Q7

The diagram shows the curve with equation $y = (3x-2)^{\frac{1}{2}}$ and the line $y = \frac{1}{2}x + 1$. The curve and the line intersect at points A and B

(a) Find the coordinates of A and B. (b) Hence find the area of the region enclosed between the curve and the line.

Solution

$y = (3x-2)^{\frac{1}{2}}$

$y = \frac{1}{2}x + 1$

$(3x-2)^{\frac{1}{2}} = \frac{1}{2}x + 1$

Squaring

$3x - 2 = \frac{1}{4}x^2 + x + 1$

$4(3x-2) = [\frac{1}{4}x^2 + x + 1] \times 4$

$12x - 8 = x^2 + 4x + 4$

$x^2 + 4x + 4 = 12x - 8$

$x^2 + 4x + 4 - 12x + 8 = 0$

$x^2 - 8x + 12 = 0$

$x^2 - 6x - 2x - 12 = 0$

$x(x-6) - 2(x-6) = 0$

$(x-2)(x-6) = 0$

$x - 2 = 0 \Longrightarrow x = 2$

$x - 6 = 0 \Longrightarrow x = 6$

Now

at $x = 6$

$y = \frac{1}{2}(6) + 1 = 4$

at $x = 2$

$y = \frac{1}{2}(2) + 1 = 2$

Hence points A and B are:

$A(2, 2), B(6, 4))$

(b) Area of the region enclosed between the curve and the line.

$= \int_2^6 [(3x-2)^{\frac{1}{2}} - (\frac{1}{2}x + 1)]dx$

$= \int_2^6 (3x-2)^{\frac{1}{2}} dx - \int_2^6 \frac{1}{2}x dx - \int_2^6 dx$

$= \frac{1}{3} \int_2^6 (3x-2)^{\frac{1}{2}}(3dx) - \frac{1}{2}\int_2^6 x dx - \int_2^6 dx$

$= (\frac{1}{3})(\frac{2}{3})|(3x-2)^{\frac{3}{2}}|_2^6 - (\frac{1}{2})(\frac{1}{2})|x^2|_2^6 - |x|_2^6$

$= (\frac{1}{3})(\frac{2}{3})|[(3 \times 6 - 2)^{\frac{3}{2}} - (3 \times 2 - 2)^{\frac{3}{2}}]| - (\frac{1}{2})(\frac{1}{2})|(6^2 - 2^2)| - |(6-2)|$

$= (\frac{1}{3})(\frac{2}{3})|[(16)^{\frac{3}{2}} - (4)^{\frac{3}{2}}]| - (\frac{1}{2})(\frac{1}{2})|(36-4)| - |4|$

$= (\frac{1}{3})(\frac{2}{3})|[(\sqrt{16})^3 - \sqrt{4}^3]| - (\frac{1}{2})(\frac{1}{2})|(36-4)| - |4|$

$= (\frac{1}{3})(\frac{2}{3})|[(\sqrt{16})^3 - \sqrt{4}^3]| - (\frac{1}{2})(\frac{1}{2})|(32)| - |4|$

$= \frac{2}{9}(4^3 - 2^3) - 8 - 4$

$= \frac{2}{9}(64 - 8) - 8 - 4$

$= \frac{2}{9}(56) - 8 - 4$

$= \frac{112}{9} - 12$

$= \frac{4}{9}$

Q8

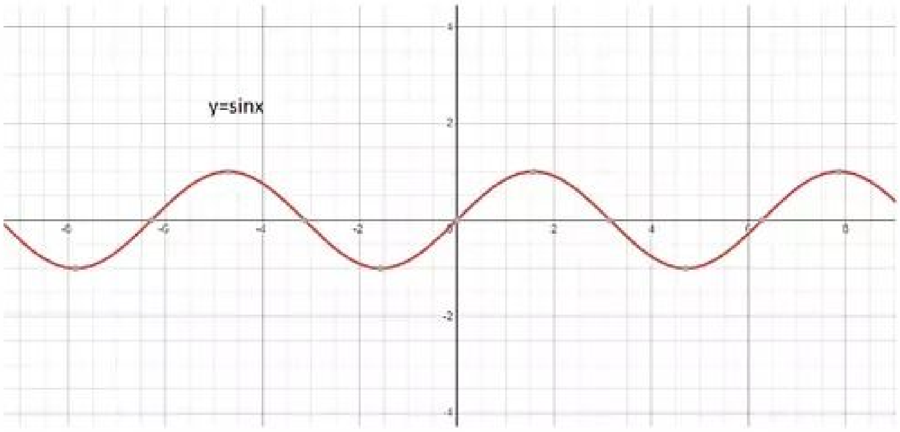


Figure 2:

(a) The curve $y = sinx$ is transformed to the curve

$y = 4\sin(\frac{x}{2} - 30°)$

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied.

(b) Find the exact solutions of the equation $y = 4\sin(\frac{x}{2} - 30°) = 2\sqrt{2}$ for $0° \leq x \leq 360°$

Solution

Note: Figure added in Q8 are neither the part of question nor the part of answer.These figure are drawn just to ensure visual learning.

Vertical stretches

$y = 4\sin x$

Horizontal Stretches

$y = 4\sin\frac{x}{2}$

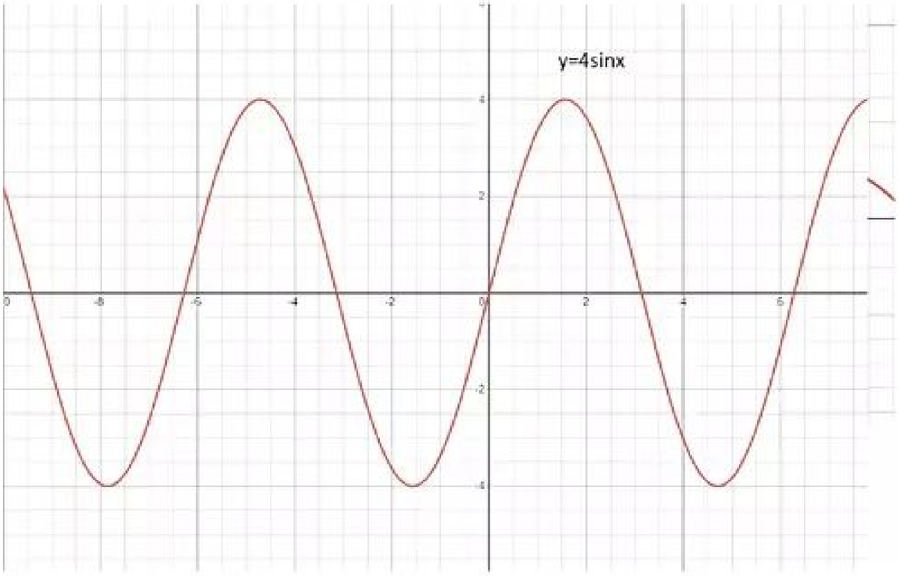


Figure 3:

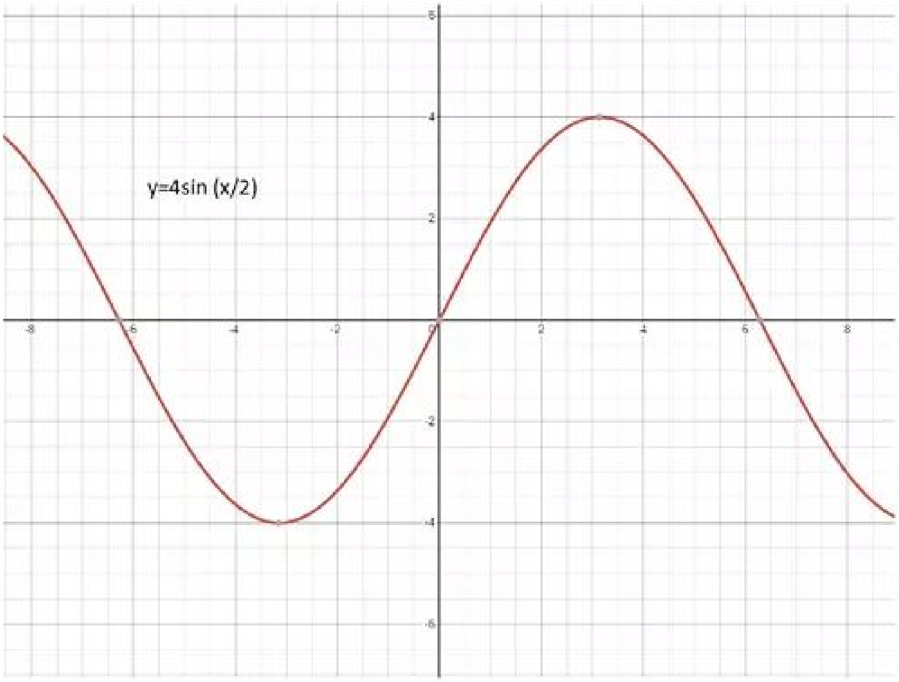


Figure 4:

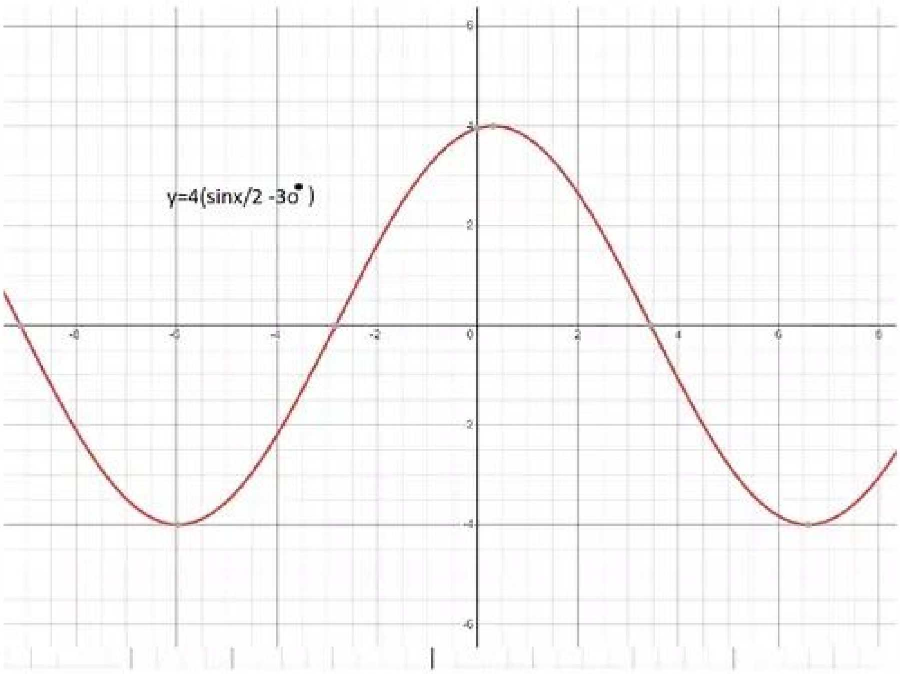


Figure 5:

Horizontal Translation

$y = 4\sin(\frac{x}{2} - 30°)$

(b)

$4\sin(\frac{x}{2} - 30°) = 2\sqrt{2}$

$\sin(\frac{x}{2} - 30°) = \frac{2\sqrt{2}}{4}$

$\sin(\frac{x}{2} - 30°) = \frac{\sqrt{2}}{2}$

$\sin(\frac{x}{2} - 30°) = \frac{\sqrt{2}}{2}$

$\frac{x}{2} - 30° = sin^{-1}(\frac{\sqrt{2}}{2})$

$\frac{x}{2} - 30° = 45°$

$\frac{x}{2} = 45° + 30°$

$\frac{x}{2} = 75°$

$x = 75° \times 2 = 150°$

Q9 The equation of a circle is $x^2+y^2+6x-2y-26 = 0$

(a) Find the coordinates of the centre of the circle and the radius. Hence find the coordinates of the lowest point on the circle.

(b) Find the set of values of the constant k for which the line with equation $y = kx - 5$ intersect the circle at two distinct points.

Solution

$X^2 + y^2 + 6x - 2y - 26 = 0$.....................$eq(1)$

$(x^2 + 6x) + (y^2 - 2y) = 2$

$(x^2 + 6x + 9) + (y^2 - 2y + 1) = 26 + 9 + 1$

$x-(-3)^2+(y-1)^2=36$

$\Longrightarrow$

$centre(-3,1), radius=6$

Taking derivative with respect to x of equation (1)

$\frac{d(x^2+y^2+6x-2y-26)}{dx}=\frac{d(0)}{dx}$

$2x+2y\frac{dy}{dx}+6-2\frac{dy}{dx}=0$

$2(y-1)\frac{dy}{dx}=-2x-6$

$\frac{dy}{dx}=\frac{-(x+3)}{y-1}$

Put

$\frac{dy}{dx}=\frac{-(x+3)}{y-1}=0$

$\frac{-(x+3)}{y-1}=0$

$-(x+3)=0$

$x+3=0$

$x=-3$

Putting in equation (1)

$(-3)^2+y^2+6(-3)-2y-26=0$

$9+y^2-18-2y-26=0$

$y^2-2y--35=0$

$y^2-7y+5y-35=0$

$y(y-7)+5(y-7)=0$

$(y+5)(y-7)=0$

$y+5=0\Longrightarrow y=-5$

$y-7=0\Longrightarrow y=7$

Hence

coordinates of the lowest point on the circle are: $(-3.-5)$

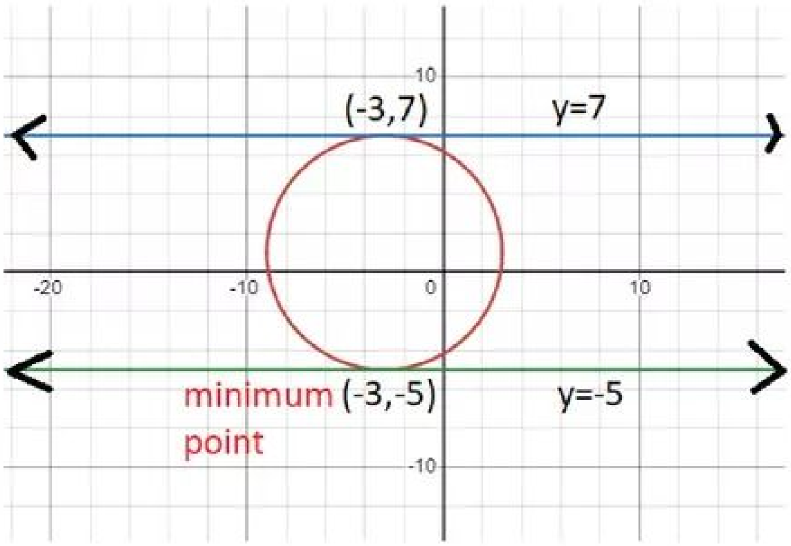


Figure 6:

(b) Putting $y = kx - 5$ in equation (1)

$x^2 + (kx-5)^2 + 6x - 2(kx-5) - 26 = 0$

$x^2 + k^2x^2 - 10kx + 25 + 6x - 2kx + 10 - 26 = 0$

$x^2 + k^2x^2 - 10kx + 25 + 6x - 2kx + 10 - 26 = 0$

$(1+k^2)x^2 - 12kx + 6x + 9 = 0$

$(1+k^2)x^2 + 6x(1-2k) + 9 = 0$

For real roots:

Discriminant :> 0

$6(1-2k)^2 - 36(1+k^2) > 0$

$36(1-4k+4k^2) - 36(1+k^2) > 0$

$36 - 144k + 144k^2 - 36 - 36k^2 > 0$

$108k^2 - 144k > 0$

$k(108K - 144) > 0$

$\Longrightarrow$ EITHER :

$k > 0 \qquad Or \qquad 108k - 144 > 0$

$k > 0 \qquad Or \qquad k > \frac{4}{3}$

OR

$k < 0 \qquad Or \qquad 108k - 144 < 0$

$k < 0 \qquad Or \qquad 108k < 144$

$k < 0 \qquad Or \qquad k < \frac{4}{3}$

Hence

$k < 0 \qquad Or \qquad k > \frac{4}{3}$

Putting $a = 72$, and $d = -5$, $n = 50$, in the equation $s_n = \frac{n}{2}[2a + (n-1)d]$

$s_{50} = \frac{50}{2}[2(72) + (50-1)(-5)]$

$s_{50} = 25[144 + 49(-5)]$

$s_{50} = -2525$

Q3 The coefficient of x^4 in the expansion of $(2x^2+\frac{k^2}{x})^5$ is a.The coefficient of x^2 in the expansion of $(2kx-1)^4$ is b

(a) Find a and b in terms of the constant k.

(b) Given that $a+b = 216$, find the possible values of k.

Solution

(r+1)th term in the expansion of $(2x^2 + \frac{k^2}{x})^5$

$T_{r+1} =^n C_r(2x^2)^{n-r}(\frac{k^2}{x})^r$

Put n=5

$T_{r+1} =^5 C_r(2x^2)^{5-r}(\frac{k^2}{x})^r =^5 C_r 2^{5-r}x^{10-2r}k^{2r}x^{-r}$

$T_{r+1} =^5 C_r 2^{5-r}x^{10-2r-r}k^{2r}$

$T_{r+1} =^5 C_r 2^{5-r}x^{10-3r}k^{2r}$

Now according to the given condition

$x^4 = x^{10-3r}$

$4 = 10 - 3r$

$3r = 10 - 4 = 6$

$r = 2$

Putting in the equation

Q10 The equation of a curve is such that $\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$. The curve has a stationary point at $(-1, \frac{9}{2})$.

(a) Determine the nature of the stationary point at $(-1, \frac{9}{2})$.

(b) Find the equation of the curve.

(c) Show that the curve has no other stationary points.

(d) A point A is moving along the curve and the y-coordinate of A is increasing at a rate of 5 units per second.

Find the rate of increase of the x-coordinate of A at the point where $x = 1$.

Solution

(a)

Putting $x = -1$ in the equation :

$\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$.

$\frac{d^2y}{dx^2} = 6(-1)^2 - \frac{4}{(-1)^3}$.

$\frac{d^2y}{dx^2} = 6 + 4 = 10 > 0$.

So there is minimum at $(-1, \frac{9}{2})$.

(b)

$\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$.

$d[\frac{dy}{dx}] = [6x^2 - \frac{4}{x^3}]dx$

Taking integral

$\int d[\frac{dy}{dx}] = \int [6x^2 - 4x^{-3}]dx$

$\frac{dy}{dx} = \int 6x^2dx - \int 4x^{-3}dx$

$\frac{dy}{dx} = 6\int x^2dx - 4\int x^{-3}dx$

$\frac{dy}{dx} = 6(\frac{x^3}{3}) - 4(\frac{x^{-2}}{-2}) + C$

$\frac{dy}{dx} = 2x^3 + 2x^{-2} + C$

$\frac{dy}{dx} = 2x^3 + \frac{2}{x^2} + C$

Since $(-1, \frac{9}{2})$ is a stationary point

$\Longrightarrow \frac{dy}{dx} = 0$ at $x = -1$

$0 = 2(-1)^3 + \frac{2}{(-1)^2} + C$

$0 = -2 + 2 + C$

$C = 0$

Now

$\frac{dy}{dx} = 2x^3 + \frac{2}{x^2}$

$dy = 2x^3dx + \frac{2}{x^2}dx$

$dy = 2x^3dx + 2x^{-2}dx$

Taking integral

$\int dy = \int 2x^3dx + \int 2x^{-2}dx$

$y = 2\int x^3dx + 2\int x^{-2}dx$

$y = \frac{2}{4}x^4 + \frac{2}{(-1)}x^{-1} + D$

$y = \frac{1}{2}x^4 - 2x^{-1} + D$

Putting $(-1, \frac{9}{2})$.

$\frac{9}{2} = \frac{1}{2}(-1)^4 - 2(-1)^{-1} + D$ $\frac{9}{2} = \frac{1}{2}(-1)^4 - 2(-1)^{-1} + D$

$\frac{9}{2} = \frac{1}{2} + 2 + D$

$\frac{9}{2} - \frac{1}{2} - 2 = D$

$D = 2$

Hence

$y = \frac{1}{2}x^4 - 2x^{-1} + 2$

(c)

Since

$y = \frac{1}{2}x^4 - 2x^{-1} + 2$

Taking derivative with respect to x

$\frac{dy}{dx} = 2x^3 + \frac{2}{x^2}$

At stationary points

$\frac{dy}{dx} = 2x^3 + \frac{2}{x^2} = 0$

$2x^3 + \frac{2}{x^2} = 0$

$2x^3 = -\frac{2}{x^2}$

$x^3 = -\frac{1}{x^2}$

$x^5 = -1$

$x^5 = e^{i(\pi+2k\pi)}$, k=0,1,2,3,4

$x = e^{i(\frac{\pi+2k\pi}{5})}$,k=0,1,2,3,4

Roots are:

$e^{i(\frac{\pi}{5})}$,$e^{i(\frac{3\pi}{5})}$,$e^{i(\frac{5\pi}{5})}$,$e^{i(\frac{7\pi}{5})}$,$e^{i(\frac{9\pi}{5})}$

Only real root:

$e^{i(\frac{5\pi}{5})} = e^{i\pi} = \cos\pi + i\sin\pi = -1$

Remaining are imaginary roots.So stationary point is only at $(-1, \frac{9}{2})$.

(d)

$y = \frac{1}{2}x^4 - 2x^{-1} + 2$

Taking derivative with respect to t $\frac{dy}{dt} = \frac{1}{2}\frac{d(x^4)}{dt} - 2\frac{d(x^{-1}}{dt} + \frac{d(2)}{dt}$

$\frac{dy}{dt} = \frac{1}{2}(4x^3)\frac{dx}{dt} + 2x^{-2}\frac{dx}{dt}$

$\frac{dy}{dt} = 2x^3\frac{dx}{dt} + 2x^{-2}\frac{dx}{dt}$

$\frac{dy}{dt} = [2x^3 + 2x^{-2}]\frac{dx}{dt}$

$\frac{dy}{dt} = [2x^3 + \frac{2}{x^2}]\frac{dx}{dt}$

Putting $x = 1$, $\frac{dy}{dt} = 5$

$5 = [2(1)^3 + \frac{2}{(1)^2}]\frac{dx}{dt}$

$5 = [2 + 2]\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{5}{4}$

$T_{r+1} = {}^5C_r 2^{5-r} x^{10-3r} k^{2r}$

$\implies$

$T_{2+1} = {}^5C_2 2^{5-2} x^{10-3(2)} k^{2(2)}$

$T_3 = 10(8)x^4k^4$

$T_3 = 80x^4k^4$

coefficient of $x^4 = 80k^4$

Hence

a=$80k^4$

(r+1)th term in the expansion of $(2kx-1)^4$

$T_{r+1} = {}^nC_r(2kx)^{4-r}(-1)^r$

$T_{r+1} = {}^nC_r 2^{4-r}k^{4-r}x^{4-r}(-1)^r$

According to the given condition

$x^{4-r} = x^2$

$\implies$

$4-r=2$

$r=2$

Putting $r=2$ in the equation

$T_{r+1} = {}^nC_r 2^{4-r}k^{4-r}x^{4-r}(-1)^r$

$T_{2+1} = {}^4C_2 2^{4-2}k^{4-2}x^{4-2}(-1)^2$

$T_3 = 6(4)k^2x^2 = 24k^2x^2$

Hence coefficient of $x^2 = 24k^2 = b$

(b)

$a+b=216$

Putting the values of a and b

$80k^4 + 24k^2 = 216$

$10k^4 + 3k^2 = 27$

$10k^4 + 3k^2 - 27 = 0$

$10k^4 + 18k^2 - 15k^2 - 27 = 0$

$2k^2(5k^2 + 9) - 3(5k^2 + 9)$

$(2k^2 - 3)(5k^2 + 9) = 0$

$2k^2 - 3 = 0$

$2k^2 = 3$

$k^2 = \frac{3}{2}$

$k = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{6}}{2}$

Similarly

$k^2 + 9 = 0$

$5k^2 = -9$

$k^2 = -\frac{9}{5}$

$k = \pm\frac{3\iota}{\sqrt{5}}$

Q4 (a) Prove the identity

$$\frac{\sin^3\theta}{\sin\theta - 1} - \frac{\sin^2\theta}{1+\sin\theta} \equiv -\tan^2\theta(1 + \sin\theta)$$

(b)Hence solve the equation

$$\frac{\sin^3\theta}{\sin\theta - 1} - \frac{\sin^2\theta}{1+\sin\theta} \equiv -\tan^2\theta(1 + \sin\theta) \text{ for } 0 < \theta < 2\pi$$

Solution

LHS= $\frac{\sin^3\theta}{\sin\theta-1} - \frac{\sin^2\theta}{1+\sin\theta}$

LHS= $\sin^2\theta[\frac{\sin\theta}{\sin\theta-1} - \frac{1}{1+\sin\theta}]$

LHS= $\sin^2\theta[\frac{\sin\theta}{\sin\theta-1} - \frac{1}{\sin\theta+1}]$

LHS= $\sin^2\theta[\frac{\sin\theta(\sin\theta+1)-(\sin\theta-1)}{(\sin\theta-1)(\sin\theta+1)}]$

LHS= $\sin^2\theta[\frac{(\sin^2\theta+\sin\theta)-\sin\theta+1}{(\sin^2\theta-1^2)}]$

LHS= $\sin^2\theta[\frac{(\sin^2\theta+1}{(\sin^2\theta-1^2)}]$

LHS= $-\sin^2\theta[\frac{(\sin^2\theta+1}{1-(\sin^2\theta}]$

LHS= $-\sin^2\theta[\frac{(\sin^2\theta+1}{\cos^2\theta}]$

LHS= $-\frac{sin^2\theta}{\cos^2\theta}(\sin^2\theta+1)$

LHS= $-\tan^2\theta(\sin^2\theta+1) = RHS$

LHS+RHS

(b)

Let

$-\tan^2\theta(1+\sin\theta) = 0$ for $0 < \theta < 2\pi$

$-\frac{\sin^2\theta}{\cos^2\theta}(1+\sin\theta) = 0$ for $0 < \theta < 2\pi$

$-\sin^2\theta(1+\sin\theta) = 0$ for $0 < \theta < 2\pi$

$\sin^2\theta(1+\sin\theta) = 0$ for $0 < \theta < 2\pi$

$\Longrightarrow$

$\sin^2\theta = 0$

$\sin\theta = 0$

$\theta = 0$

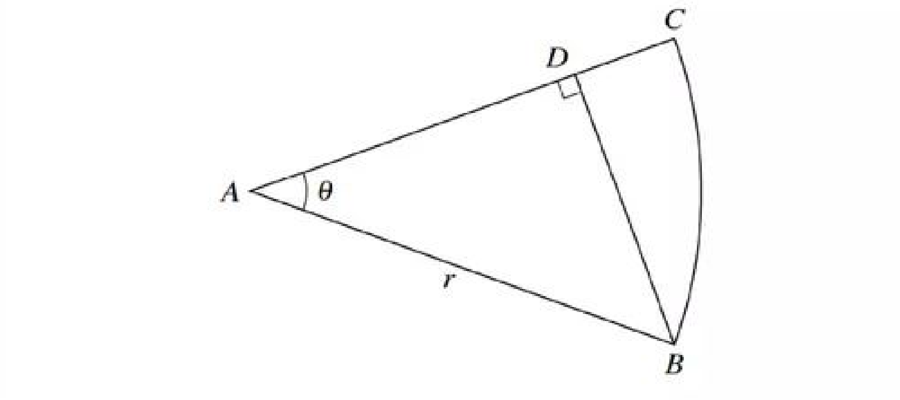
Since $0 < \theta < 2\pi$

$\sin\theta = 0$ has no solution in the given domain

$1 + \sin\theta = 0$

$\sin\theta = -1$

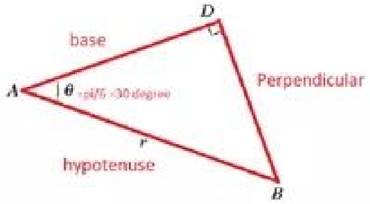
$\theta = \arcsin -1 = 3\frac{\pi}{2}$



Q5 The diagram shows a sector ABC of a circle with centre A and radius r. The line BD is perpendicular to AC. Angle CAB is θ radians.

(a) Given that $\theta = \frac{1}{6}\pi$, find the exact area of BCD in terms of r.

(b) Given instead that the length of BD is $\frac{\sqrt{3}}{2}r$, find the exact perimeter of BCD in terms of r



Reference to the right angled triangle ADB

$\theta = \frac{\pi}{6}$

$\sin\frac{\pi}{6} = \frac{BD}{r}$

$BD = r\sin\frac{\pi}{6} = \frac{r}{2}$

Similarly

$\cos\frac{\pi}{6} = \frac{AD}{r}$

$\frac{\sqrt{3}}{2} = \frac{AD}{r}$

$AD = \frac{\sqrt{3}}{2}r$

Now area of the right angled triangle ADB

$= \frac{1}{2}(AD)(BD) = \frac{1}{2}(\frac{\sqrt{3}}{2}r)(\frac{r}{2}) = \frac{\sqrt{3}r^2}{8}$

Area of the sector ACB$= \frac{1}{2}r^2\frac{\pi}{6} = \frac{\pi r^2}{12}$

Area of BCD=Area of sector ACB-Area of the right angled triangle ADB

Area of BCD=$\frac{\pi r^2}{12} - \frac{\sqrt{3}r^2}{8}$

Area of BCD=$(\frac{\pi}{12} - \frac{\sqrt{3}}{8})r^2$

(b) If

BD=$\frac{\sqrt{3}}{2}r$

$\implies$ AD=$\frac{r}{2}$

Now

DC=AC-AD=$r - \frac{r}{2} = \frac{r}{2}$

Arc BC=$\frac{1}{2}r\frac{\pi}{6} = \frac{\pi r}{12}$

Perimeter of BCD

=BD+DC+arc

=$\frac{\sqrt{3}}{2}r + \frac{r}{2} + \frac{\pi r}{12}$

Perimeter of BCD

=BD+DC+arc BC

=$(\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{\pi}{12})r$

Q6 The function f is defined as follows: $f(x) = \frac{x^2-4}{x^2+4}$ for $x > 2$.

(a) Find an expression for $f^{-1}(x)$

(b) Show that $1 - \frac{8}{x^2+4}$ can be expressed as $\frac{x^2-4}{x^2+4}$ and hence state the range of f

(c)Explain why the composite function ff cannot be formed.

Solution

$y = f(x) = \frac{x^2-4}{x^2+4}$

$y(x^2 + 4) = x^2 - 4$

$yx^2 - x^2 = -4y - 4$

$x^2(y-1) = -4(y+1)$

$x^2 = -4\frac{y+1}{y-1}$

$x^2 = 4\frac{1+y}{1-y}$

$x = \sqrt{4\frac{1+y}{1-y}}$ $x = 2\sqrt{\frac{1+y}{1-y}}$

$y = f(x) \Longrightarrow x = f^{-1}(y)$

$\Longrightarrow$

$f^{-1}(y) = 2\sqrt{\frac{1+y}{1-y}}$

Since y is a dummy variable .Replacing y by x

$f^{-1}(x) = 2\sqrt{\frac{1+x}{1-x}}$

(b)

$1 - \frac{8}{x^2+4}$

$\frac{x^2+4-8}{x^2+4}$

$\frac{x^2-4}{x^2+4}$

Since $f(x) = 1 - \frac{8}{x^2+4}$

As $x \longrightarrow \pm\infty$

$\frac{8}{x^2+4} \longrightarrow 0$

So $f(x) = 1 - 0 = 1$ as $x \longrightarrow \pm\infty$

Similarly

At $x = 0$, $f(0) = 1 - \frac{8}{0^2+4} = 1 - \frac{8}{0+4} = 1 - \frac{8}{4} = 1 - 2 = -1$

So range of f(x)= [-1,1]

(c)

Since

Range of $f = [-1, 1]$

Domain of $f = [2, \infty)$

Since

$f[-1, 1] \neq (2, \infty)$

ff cannot be formed.