Trigonometric Function Conversion to Identity: Indeterminate Constraint Handling

1. MODULE : 1
2. ORGANIZER: Pasti Menang
3. SOFTWARE : Maple 13
4. OBJECTIVES:

   Students can understand the concept of limit function in problem solving and students are expected to be able to calculate the limit value of a trigonometric function both manually and computationally using Maple 13 *software*.
5. INTRODUCTORY THEORY

a. Concept of Limit of Trigonometric Functions

A trigonometric limit is the closest value to an angle of a trigonometric function. Calculation

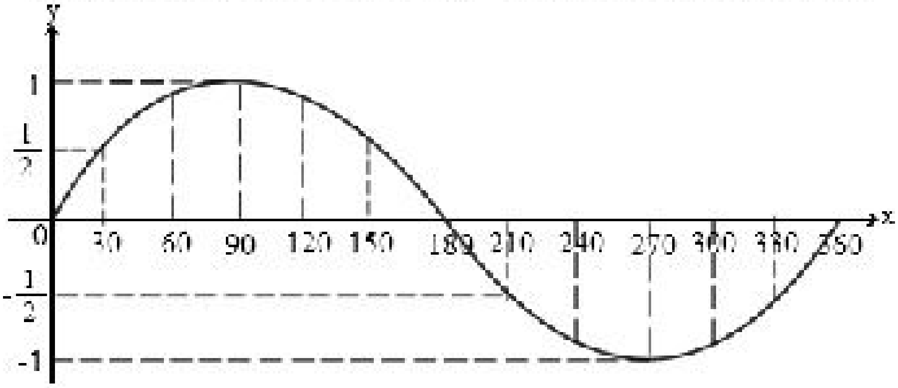
The limit of this function can be directly substituted as for example the limit of an algebraic function but there is a trigonometric function that must first be converted to a trigonometric identity for an indeterminate limit, namely a limit that if directly substituted the value is 0, it can also be for an indeterminate limit does not have to use identity but uses the trigonometric limit theorem or some use identity and theorem. So if a trigonometric limit function is substituted for a value that approaches it produces and then it must be solved in another way.

To get started with the concept of trigonometric function limits, consider the table of values and graph of the function $f(x) = sin\ x$ below.

Table of function values $f(x) = sin\ xx$

x	29,6°	29,7°	29,8°	29,9°	30°	30,1°	30,2°	30,3°	30,4°
$f(x) = sin\ x$	0,494	0,495	0,497	0,498	0,5	0,502	0,503	0,505	0,507

The graph of the function y = $f(x) = sin\, x$ is presented as follows.



Based on the observation of the values of the table and graph above, it can be seen as follows.

a. For values of x approaching $30°$ from the left (written: $x \to 30°$), the function value $f(x) = \sin$ x is close to the value of 0.5 so that it can be written $\lim_{x \to 30^-} \sin x = 0.5$. Form

This kind of **limit** is called the **left limit.**

b. For values of x approaching $30°$ from the right (written: $x \to 30°$), the function value $f(x) =$ $\sin x$ is close to the value of 0.5 so that it can be written $\lim_{x \to 30^+} \sin x = 0.5$. Form

This kind of **limit is** usually called the **right limit.**

Because the value of $\lim_{x \to 30^-} \sin x = \lim_{x \to 30^+} \sin x = 0.5$ then $\lim_{x \to 30} \sin x = 0.5$.

So the limit can be defined as follows: a function $f(x)$ is defined

for x around a, then $\lim_{x \to a} f(x) = L$ if and only if

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$$

b. Properties of Trigonometric Function Limits

The properties of limits of trigonometric functions are the same as those of limits of algebraic functions. Suppose f and

g is a function whose limit value x approaches a (written: x->a), where k and a are real

numbers and n is a positive integer, as follows:

1. $\lim_{x \to a} k = k$

2. $\lim_{x \to a} f(x) = f(a)$
3. $\lim_{x \to a} k.f(x) = k.\lim_{x \to a} f(x)$
4. $\lim_{x \to a}(f(x) + g(x)) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
5. $\lim_{x \to a}(f(x) - g(x)) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
6. $\lim_{x \to a}(f(x) . g(x)) = \lim_{x \to a} f(x) . \lim_{x \to a} g(x)$
7. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$
8. $\lim_{x \to a}(f(x))^n = (\lim_{x \to a} f(x))^n$

c. How to Solve Limit Trigonometric Functions

First, the limit problem of trigonometric functions can be solved by **substituting Direct**. If the result obtained is not an indeterminate form 0/0, then the result is the limit value sought. If the result is an indeterminate form 0/0, then you can use **substitution, the basic formula for the limit of trigonometric functions, factoring, simplifying it, and multiplying by a pair**.

1) **Substitution**

If there is a form $\lim_{x \to c} f(x) = f(c)$, then the limit of an itrigonometric function $f(x)$ is the result of substituting the value of $c \to x$ from trigonometry. Therefore, the following formula can be used:

a. $\lim_{x \to c} \sin x = \sin c$
b. $\lim_{x \to c} \cos x = \cos c$
c. $\lim_{x \to c} \tan x = \tan c$
d. $\lim_{x \to c} cosec\ x = cosec\ c$
e. $\lim_{x \to c} \sec x = \sec c$
f. $\lim_{x \to c} cotan\ x = cotan\ c$

Example 1:

a. $\lim_{x \to \pi} \sin x = \sin(\pi) = \sin(180°) = 0$
b. $\lim_{x \to \frac{\pi}{2}} \cos \frac{1}{2} x = \cos\left(\frac{1}{2} \cdot \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{4}\right) = \cos 45° = \frac{1}{2}\sqrt{2}$ –

2) **Basic Trigonometric Limit Formula**

a. $\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{x}{\sin x} = 1$

b. $\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{x}{\tan x} = 1$

c. $\lim_{x \to 0} \frac{\sin ax}{bx} = \lim_{x \to 0} \frac{ax}{\sin bx} = \frac{a}{b}$

d. $\lim_{x \to 0} \frac{\tan ax}{bx} = \lim_{x \to 0} \frac{ax}{\tan bx} = \frac{a}{b}$

e. $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \lim_{x \to 0} \frac{\tan ax}{\tan bx} = \frac{a}{b}$

f. $\lim_{x \to 0} \frac{\sin ax}{\tan bx} = \lim_{x \to 0} \frac{\tan ax}{\sin bx} = \frac{a}{b}$

g. $\lim_{x \to 0} \frac{\sin ax + \tan bx}{cx - \sin dx} = \frac{a + b}{c - d}$

h. $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

Example 2:

a. $\lim_{x \to 0} \frac{\sin 5x}{x} = \lim_{x \to 0} \frac{\sin 5x}{x} \cdot \frac{5}{5}$

$= 5 . \lim_{x \to 0} \frac{\sin 5x}{5x}$

$= 5 . \lim_{x \to 0} \frac{\sin 5x}{5x}$

$= 5 . (1)$

$= 5$

3) **Factoring**

Example 3:

a. $\lim_{x \to 1} \frac{(x^2 - 1) . \tan(2x - 2)}{\sin^2 (x - 1)} = \lim_{x \to 1} \frac{(x - 1)(x + 1) \tan 2(x - 1)}{\sin^2 (x - 1)}$

$= \lim_{x \to 1} \left(\frac{(x - 1)}{\sin(x - 1)} (x + 1) . \frac{\tan 2(x - 1)}{\sin(x - 1)} \right)$

$= \lim_{x \to 1} \frac{(x - 1)}{\sin(x - 1)} . \lim_{x \to 1} (x + 1) . \lim_{x \to 1} \frac{\tan 2(x - 1)}{\sin(x - 1)}$

$= 1 . \lim_{x \to 1} (x + 1) . 2$

$= 1 . (1 + 1) . 2$

$= 4$

4) **Simplify it**

In simplifying the limit problem of trigonometric functions, you must remember and use the trigonometric identity formulas.

Example 5:

$$\lim_{x\to\frac{\pi}{2}} \frac{3(\sin x - \cos x)}{1 - \sin 2x}$$

To work on the problem above, we must use trigonometric identities, so that it can make it easier to work on the trigonometric limit problem. We recall that

$$1 - \sin 2x = (\sin x - \cos x)^2$$

Then,

$$\lim_{x\to\frac{\pi}{2}} \frac{3(\sin x - \cos x)}{1 - \sin 2x} = \lim_{x\to\frac{\pi}{2}} \frac{3(\sin x - \cos x)}{(\sin x - \cos x)^2}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{3}{(\sin x - \cos x)}$$

$$= \frac{3}{\left(\sin\frac{\pi}{2} - \cos\frac{\pi}{2}\right)} = \infty$$

5) **Multiply by a Thousand**

Example 5:

a. $$\lim_{x\to 2} \frac{\sin(2x-4)}{2-\sqrt{6-x}} = \lim_{x\to 2} \frac{\sin(2x-4)}{2-\sqrt{6-x}} \cdot \frac{2+\sqrt{6-x}}{2+\sqrt{6-x}}$$

$$= \lim_{x\to 2} \frac{\sin(2x-4)}{(x-2)} \cdot 2 + \sqrt{6-x}$$

$$= \lim_{x\to 2} \frac{\sin(2x-4)}{(x-2)} \cdot \lim_{x\to 2} 2 + \sqrt{6-x}$$

$$= 2.\left(2 + \sqrt{6-2}\right)$$

$$= 8$$

6. STEP WORK

Writing commands with Maple 13 to solve the limit of trigonometric functions with the following steps:

1. We can use the formula where >limit $(f, x = a, dir)$

   f - algebraic expression

xx - variable

a - limit point

dir - (optional) the selected limit direction, left limit or right limit.

2. Then to enter the function, replace f with a trigonometric function for which the limit will be sought.
3. Click enter.

For Example 1(a), if it is done using Maple 13, the working steps are as above and then replace f with $\sin x$ and a with π, then the result from Maple 13 computation is 0.

For Example 1(b), if done using Maple 13, the step work as above then replace f with $\cos \frac{1}{2}x$ and a with $\frac{\pi}{2}$, then obtained from Maple 13 computation, namely $\frac{1}{2}\sqrt{2}$.

For Example 2, if worked out using Maple 13 then the step work as above then replace f with $\frac{\sin 5x}{x}$ and a with 0, then the result from Maple 13 computation is 5.

For Example 3, if done using Maple 13, the step work as above then replace f with $\frac{(x2-1)\tan(2x-2)}{\sin^2(x-1)}$ and a with -1, then the result of Maple 13 computation is 4.

For Example 4, if done using Maple 13, the step work as above and then replace f with the result $\frac{3(\sin xx - \cos xx)}{1-\sin 2x}$ and a with $\frac{\pi}{2}$, then obtained from Maple 13 computation which is ∞.

For Example 5, if done using Maple 13, the step work as above then replace f with $\frac{\sin(2x-4)}{2-\sqrt{6-x}}$ and a with 2, then The result obtained from Maple 13 computation is 8.